

Décembre 1973

*Spina
admission
(approved
for 8/4/76)*

ORGANISATION EUROPÉENNE D'ÉTUDES
PHOTOGRAMMÉTRIQUES EXPÉRIMENTALES

EUROPEAN ORGANIZATION FOR EXPERIMENTAL
PHOTOGRAMMETRIC RESEARCH



PUBLICATION OFFICIELLE

OFFICIAL PUBLICATION

N° 8

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DK 528.735-187(430-313.5)

Proceedings of the OEEPE Symposium on
Experimental Research on Accuracy of Aerial Triangulation
(Results of Oberschwaben Tests)

held at Brussels

from 12th to 14th June, 1973.

Après la page 350

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All discussions are edited by J. Kure, Enschede.

[altogether 119 figures, 68 tables and 1 map]

Préface

Cette publication réunit les travaux présentés au Symposium que l'Organisation d'Etudes Photogrammétriques Expérimentales (OEEPE) a tenu à Bruxelles les 12, 13 et 14 juin 1973 et les discussions qui ont résulté des exposés.

Le symposium était consacré à l'étude de la précision de la triangulation aérienne sur la base des résultats atteints, à la date de la réunion, dans les études expérimentales menées par l'OEEPE en utilisant les prises de vues réalisées sur le polygone d'essai d'Oberschwaben, en République Fédérale d'Allemagne.

Les domaines des recherches présentées au Symposium sont au nombre de trois. Le premier concerne l'étude de la précision des bandes et des blocs en fonction de la répartition des points d'appui. La précision atteinte expérimentalement est confrontée à celle théoriquement attendue. Il porte également sur la comparaison des différentes méthodes de compensation appliquées aux clichés grandangulaires et super-grandangulaires.

Le deuxième centre d'intérêt a trait à la recherche de méthodes de réduction des erreurs de toute nature au moyen de nouveaux procédés de prise de vues et d'exploitation des clichés.

Enfin, le troisième point traite de l'approche physique des processus générateurs d'erreurs.

La première étude, la plus avancée, a été exploitée par l'Institut de Photogrammétrie de Stuttgart sous la direction du professeur Ackermann. Elle a donné lieu à quatre communications. La deuxième a été développée aux Instituts de Photogrammétrie des Universités de Liège et de Mons, sous la direction du professeur Wiser (deux communications présentées). La troisième a été exécutée à l'Ecole Polytechnique de Milan, sous la direction du professeur Ciniatti. Ce dernier a exposé les résultats obtenus jusqu'à présent.

Les autres communications faites au Symposium se réfèrent à la préparation du champ d'essai d'Oberschwaben, à la possibilité d'élimination des erreurs systématiques dans des prises effectuées sur un champ d'essai (Rheidt), à la réduction des erreurs systématiques par compensation analytique d'un bloc et, finalement, aux résultats des expériences faites à l'ITC sur l'utilisation des réseaux.

Ces derniers travaux sont strictement liés aux recherches sur le champ d'Oberschwaben et aboutiront certainement à de nouvelles études très étendues.

La recherche la plus avancée, comme cela a été signalé ci-dessus, est celle du professeur Ackermann. Les résultats en sont d'un intérêt considérable. Certaines déductions théoriques ont été confirmées, mais, en même temps, de nouveaux problèmes ont surgi, par exemple celui de l'influence et du traitement des erreurs systématiques. Ils exigeront encore des études très approfondies et peut-être très longues.

Les discussions se sont déroulées d'une manière assez vive; leur niveau scientifique était élevé et elles ont mis en évidence les points les plus délicats de la triangulation aérienne. Cette dernière a d'ailleurs atteint une précision remarquable. Les discussions ont surtout permis aux spécialistes des différents pays de se livrer à un large échange d'idées à partir d'une quantité considérable de résultats concrets.

77 personnes ont participé au Symposium. Elles appartenaient à 12 nationalités différentes: 6 Pays non membres de l'OEEPE étaient présents. Leurs représentants ont pu se rendre compte tant de la méthode de travail appliquée au sein de l'OEEPE que de l'importance et des dimensions des questions qui sont abordées et, dans la limite du possible, résolues.

On peut finalement constater une fois de plus l'intérêt de ces réunions restreintes des spécialistes sur un thème bien défini, réunions qui ont un caractère complètement différent des grands Congrès internationaux.

Décembre 1973

L. Solaini
Président de l'OEEPE

Opening Addresses

Colonel Verberckt of the Institut Géographique Militaire welcomed the participants to the inaugural session of the Symposium and expressed his appreciation for the fact that so many diplomatic representatives of the OEEPE member countries were present, thus illustrating their interest in the work of the organisation.

The official opening was performed by *Général Dath*, Commandant of l'Ecole Royale Militaire and president of the Comité de Coordination des Activités Scientifiques du Ministère de la Défense Nationale, who welcomed the participants and expressed his best wishes for a successful symposium.

Prof. Solaini, President of the OEEPE, remarked that this symposium coincided with the 20th anniversary of the OEEPE and briefly outlined its inauguration, aims, structure and function. He proceeded to outline the tasks of the various scientific commissions and briefly summarised a number of the experiments executed by the OEEPE.

In conclusion, *Prof. Wiser*, president of the aerial triangulation commission, outlined the aims of the symposium. He summarised a number of problems occurring in the field of aerial triangulation and indicated the direction in which solutions would be sought for the elimination of triangulation errors. Finally, *Prof. Wiser* wished the participants a successful symposium and hoped that there would be a fruitful exchange of ideas.

DK 528.735.2-187(430-313.5):528.14

On Statistical Investigations into the Accuracy of Aerial Triangulation
The Test Project Oberschwaben

(with 2 figures)

By F. Ackermann, Stuttgart

(French version see p. 27)

1. Background and Scope of the Test

The topic of this symposium is the aerial triangulation test Oberschwaben. The test is concerned with the accuracy of aerial triangulation methods, and with its empirical evaluation in particular.

At the beginning of the technical part of the symposium it is expedient to sketch briefly the background, the organisation, and the scope of the Oberschwaben test.

1.1. Computational methods have achieved great progress in the field of aerial triangulation during the past decade. It is block-adjustment in particular which promoted the successful application of aerial triangulation very much.

We classify the methods for strip- and block-adjustment in 3 groups, according to the units of computation: the interpolation method with polynomials, the method of independent models, and the fully analytical or bundle method. The polynomial method, being the simplest, seems to be most widely used. The other two methods, being more rigorous and more demanding, are increasingly applied as computer programs have been developed in a number of places. The practical experience with computational methods for strip- and block-adjustment is very satisfactory, especially most recently. As a result, the practitioner has gained confidence in the accuracy capability and the performance of such methods.

Such confidence is supported by a number of theoretical studies into the accuracy of strips and blocks, which have been undertaken for a number of years and which have been completed, to some extent, rather recently. The theoretical studies have resulted in a number of rules concerning the dependence of accuracy on size and shape of blocks as well as on the distribution of control points. In practice such rules are applied, with seemingly good results.

1.2. If we look deeper into the matter of accuracy of aerial triangulation, however, the scientific state of knowledge is by far not as satisfactory as the wide practical application of aerial triangulation methods would suggest. In fact, very little is known about the comparison of the 3 groups of methods. The decision about which one to use is usually taken on very general grounds (apart from the availability of computer programs). There is considerable disagreement amongst experts about the relative evaluation of the various methods.

Although the polynomial methods are generally considered the least rigorous their users claim that the accuracy performance is far better than many experts prefer to believe. On the other hand everybody agrees that the bundle method gives the most rigorous and hence the most accurate approach to aerial triangulation. But it is still to a great extent a matter of opinion how the bundle method compares with the independent model method.

Our lack of knowledge concerning the comparison of methods is, of course, due to the fact, that until very recently nobody had computer programs available for more than one method.

1.3. In a different way, the situation of the theoretical accuracy studies concerning different control distributions is not too satisfactory either. It is true, that for both the bundle method and the independent model method quite a number of theoretical investigations have been undertaken which give a good insight into the effects of control on accuracy. It has been mentioned above, that in practice such theoretically derived rules are applied, seemingly with great success.

Nevertheless, it must be pointed out that all theoretical accuracy studies are based on simplified assumptions. In other words they refer, in principle, to simplified mathematical models rather than to reality-models. Usually plane coordinates or model coordinates are treated as observations to which are attributed independent random errors only, for lack of better knowledge and for reasons of simplicity. Such simplifying basic assumptions are by no means realistic. It is therefore, in principle, an open question, by how far the results of such theoretical accuracy studies can be considered sufficiently realistic. Do the errors of real photographs behave according to the theoretical assumptions? Does the mathematical model sufficiently predict reality? Such questions cannot be answered by theory alone.

The same gap between mathematical model and reality applies to recent theoretical investigations which consider systematic errors of image coordinates. It also holds for theoretical comparisons of methods.

The conclusion is, that the validity of theoretical models has to be evaluated and tested by experiment. Experimental testing of theoretical models is a fundamental necessity which photogrammetry shares with science and engineering in general.

1.4. There are quite a number of additional problems in aerial triangulation, related to accuracy, about which little is known or about which scientific confirmation is still lacking. Some of them are briefly listed:

- Comparison of wide-angle- and super-wide-angle photography in aerial triangulation,
- accuracy of blocks with 20 % versus 60 % side overlap,
- accuracy with signalized tie-points versus natural points and artificially marked points; the effect of the number of tie-points used,
- influence of laboratory camera calibration, test field calibration, and reseau-corrections on the accuracy of adjusted strips and blocks,
- analytical aerial triangulation versus semi-analytical procedures with model- and/or strip-formation by analogue instruments.

For a number of such questions some empirical or theoretical knowledge is available. But they cannot be considered solved.

1.5. When OEEPE considered the scientific situation, as sketched above, the need for extended empirical tests on aerial triangulation was evident. It was equally obvious, however, that great efforts would be required in order to get significant results, due to the size of projects aerial triangulation deals with.

After long discussions and preparations during 1967/68 the Comité Directeur of OEEPE decided in fall 1968 to take up the challenge and set up a new series of tests. Commission AB with its president, Prof. P. Wiser, was asked to investigate empirically, by controlled tests, the accuracy questions of strip- and block-triangulation.

The investigations were to be directed, in first instance, towards 3 groups of problems:

- to test by statistical methods the agreement or disagreement of empirical results with theoretical accuracy models (or accuracy hypotheses in statistical terminology), in view of 3 groups of parameters: overlap, block-size, and distribution of control points,
- to determine by experiment the accuracy relations of different methods of aerial triangulation and to test the results against theories, when available,
- to give empirical results for a number of accuracy problems which are not easily accessible to theoretical treatment, such as wide-angle versus super-wide-angle, signalized points versus natural or artificially marked points, planimetric versus spatial block-adjustment, different polynomial formulae, reseau photography, test field camera calibration etc.

The above list of problems relates in first instance to statistical investigations. It will be referred to as program I of the Oberschwaben test.

There is a second approach to study accuracy questions of aerial triangulation. It is the investigation of the complete error properties of the single photograph and of its measurements. If all error components of a photograph, including the correlation with the adjacent photographs, would be completely known the accuracy of any configuration of photographs (i. e. strips, blocks) could be derived by the theoretical method of propagation of errors and be relied upon. This kind of investigation will, perhaps, be the ultimate one. Whilst not pursuing the experimental investigation of the single photograph, nevertheless the Comité Directeur of OEEPE decided to make full use of the test material by initiating the Oberschwaben test program II. The intention was to study various error components of models and strips more thoroughly by using 90 % longitudinal overlap, repeated measurements etc., in order to obtain more detailed analysis of the effective errors and of their interrelation.

2. The Test Field Oberschwaben

2.1. Within Commission AB of OEEPE the specifications for a test field and for the aerial photography were worked out. A test area was selected which, of course, had to be a compromise of diverging conditions. According to its geographical location the test area was given the name "Oberschwaben" (Upper Suebia) which is the geographical and

historical name of the area between Danube and Lake of Constance in southern Germany, see fig. 1. The actual test area, delimited by various conditions, covers 2500 km². It is a rectangle, with a north-south extension of 62.5 km, and east-west extension of 40.0 km. The area is gently rolling or hilly, with heights above sea-level between about 400 m and 800 m. Due to the glacial and post-glacial history the surface features are moraines, moors, bands and patches of forests, and fertile agricultural areas, see aerial photograph on page 43.

2.2. From the national system of trigonometric points of first, second, and third order 548 points were selected and signalized; see fig. 2. They were to be used as control points and as check points in the tests. Of 484 of them also the heights are given.

The photo-scale was chosen to be 1 : 28 000, by compromising between contradictory specifications: The photo-scale had, on one hand, to be large enough to cover the area by several hundred photographs; on the other hand the photo-scale had to be small enough to avoid any noticeable influence of errors of the terrestrial coordinates of the trigonometric and height points.

In addition, all standard tie-points (6 per model) were targetted in the terrain, by double signals each. Thus the results of the triangulation tests would not be influenced by possibly poor point-transfer, the effects of which could be studied separately.

The Institut für Angewandte Geodäsie (IfAG), Frankfurt, agreed to signalize the test field and to do all preparatory work, in cooperation with the Landesvermessungsamt Baden-Württemberg. The paper of Mr. Belzner, page 35, will give more details about the test field and its preparation.

2.3. Aerial photographs were taken in spring 1969 by Firma Häussermann, Waiblingen, with the wide-angle camera Zeiss RMK A 15/23 and with the super-wide-angle camera Zeiss RMK A 8,5/23. As for both wide-angle and super-wide-angle photography the photo-scale 1 : 28 000 had to be obtained, the relative flying heights were 4284 m and 2380 m, respectively. With either camera a block of 15 strips with 60 % lateral overlap was flown. The strips run in north-south direction (or vice versa), each having 25 models. Such for each type of photography the test area was covered by 390 photographs.

Because of the targetted tie-points pin-point flying was asked, which caused some difficulties. During all photo-flights statoscope recordings were taken with the Zeiss Statoscope S 2.

The photography as described up to here refers to the test program I. Additional wide-angle photographs were taken from the eastern half of the test area for the test program II: strips with 90 % longitudinal overlap and with reseau photographs (Zeiss RMK AR 15/23).

Unfortunately there were bad weather conditions during spring 1969. Also because of the difficulties of the pin-point flying the total coverage was only reached by 4 days of flying during a period of 2 1/2 months. The long delay caused problems due to growing vegetation and destruction of signals.

In addition to the flying of the Oberschwaben test area a number of photographs were taken, with the same cameras, of the 2 km x 2 km camera calibration test field Rheidt, near Bonn. Unfortunately it was not possible to have the camera test field photographed immediately prior to, or after, each air survey flight of the Oberschwaben area. Thus the value of the Rheidt photographs for detecting and correcting systematic image errors is somewhat reduced.

3. Stereocomparator Measurements

3.1 After the processing of the films paper prints and glass-diatypes of the photographs were drawn.

The preparation of the measurements was done by the Institut für Angewandte Geodäsie (IfAG), Frankfurt. Each point was marked in the paper prints and given a number. Also lists of terrestrial coordinates were issued.

All the measurements for program I were done by stereocomparator. The principle was adopted to measure the photographs only once. The total measuring work was subdivided in 4 parts and given to 4 different photogrammetric centres of member countries of OEEPE. There were measured

- the 8 wide-angle strips no. 1, 3, 5, 7, 9, 11, 13, 15 by the Institut für Angewandte Geodäsie (IfAG), Frankfurt, with stereocomparator Zeiss PSK,
- the 7 wide-angle strips no. 2, 4, 6, 8, 10, 12, 14 by the Photogrammetric Institute of the Technical University of Vienna (in cooperation with the Bundesamt für Eich- und Vermessungswesen), with stereocomparator Wild StK 1,
- the 8 super-wide-angle strips no. 1, 3, 5, 7, 9, 11, 13, 15 by the Fotogrammetrische Dienst van het Cadaster, Den Haag, with stereocomparator Zeiss PSK,
- the 7 super-wide-angle strips no. 2, 4, 6, 8, 10, 12, 14 by the International Institute for Aerial Survey and Earth Sciences (ITC), Delft, with stereocomparator Wild StK 1.

Thus each of the 4 centres measured a complete (sub-)block of $8 \times 26 = 208$ or $7 \times 26 = 182$ photographs, respectively (equal to $8 \times 25 = 200$ or $7 \times 25 = 175$ models, respectively), each with 20 % lateral overlap. Those 4 blocks will be referred to as

- block Frankfurt,
- block Vienna,
- block The Hague,
- block Delft.

Of course, the blocks Frankfurt and Vienna combined form a wide-angle block with 60 % lateral overlap, of 390 photographs or 375 models. Accordingly the blocks The Hague and Delft combined form a super-wide-angle block with 60 % lateral overlap of 390 photographs or 375 models.

The preparation and the measurement of the photographs of program I lasted through 1969 and 1970.

3.2. The comparator-measurements, after reduction and correction, are directly used as observations for the bundle method of block triangulation.

For the method of independent models the stereocomparator measurements were processed by analytical relative orientation to models (= stereo-pairs). For the polynomial adjustments strips were formed, by computation, from the previously computed models. Thus, all later adjustments by different methods refer ultimately to the same measurements from stereocomparators. The IfAG, Frankfurt, provided the analytical model computations, except for the block Vienna, the models of which were computed by the Technical University of Vienna.

4. The Adjustments

4.1. The plate-coordinates and model-coordinates, obtained as described above, served as input-data for the series of strip- and block-adjustments of the test program I. For that program the Photogrammetric Institute of Stuttgart University acted as pilot centre. It did and still continues the computations of most of the strip- and block-adjustments of program I. They consisted first of a great number of strip adjustments, with the method of independent models, and with 8 versions of polynomial adjustments, lately supplemented by bundle adjustments. The total test program amounts to almost 1800 different strip adjustments. Parallel with the strip adjustments the block-adjustments commenced, concentrating at first on the method of independent models. More recently the block-adjustments extended to the bundle method, and during the last weeks to polynomials (by IfAG, Frankfurt).

4.2. The results of the strip- and block-adjustments of program I which are available by now will be presented during the first day and the morning of the second day of this symposium. The afternoon of the second day will be devoted to a description, and to the available results, of the test program II which is directed by Prof. Wiser and the University of Liège as pilot centre. On the third day reports will be presented on special investigations concerning reseau, test field camera calibration, and systematic image errors.

The test programs I and II have, as yet, not been fully completed. It will take a few more years to scientifically exploit the material of the test Oberschwaben to the full extent.

It is expected, however, that the results presented at this intermediate stage will be of sufficient interest to the aerial triangulation experts. Some of the results obtained are, indeed, somewhat surprising. They give sufficient occasion to reorient some views on aerial triangulation, and they will have influence on the future research in aerial triangulation.

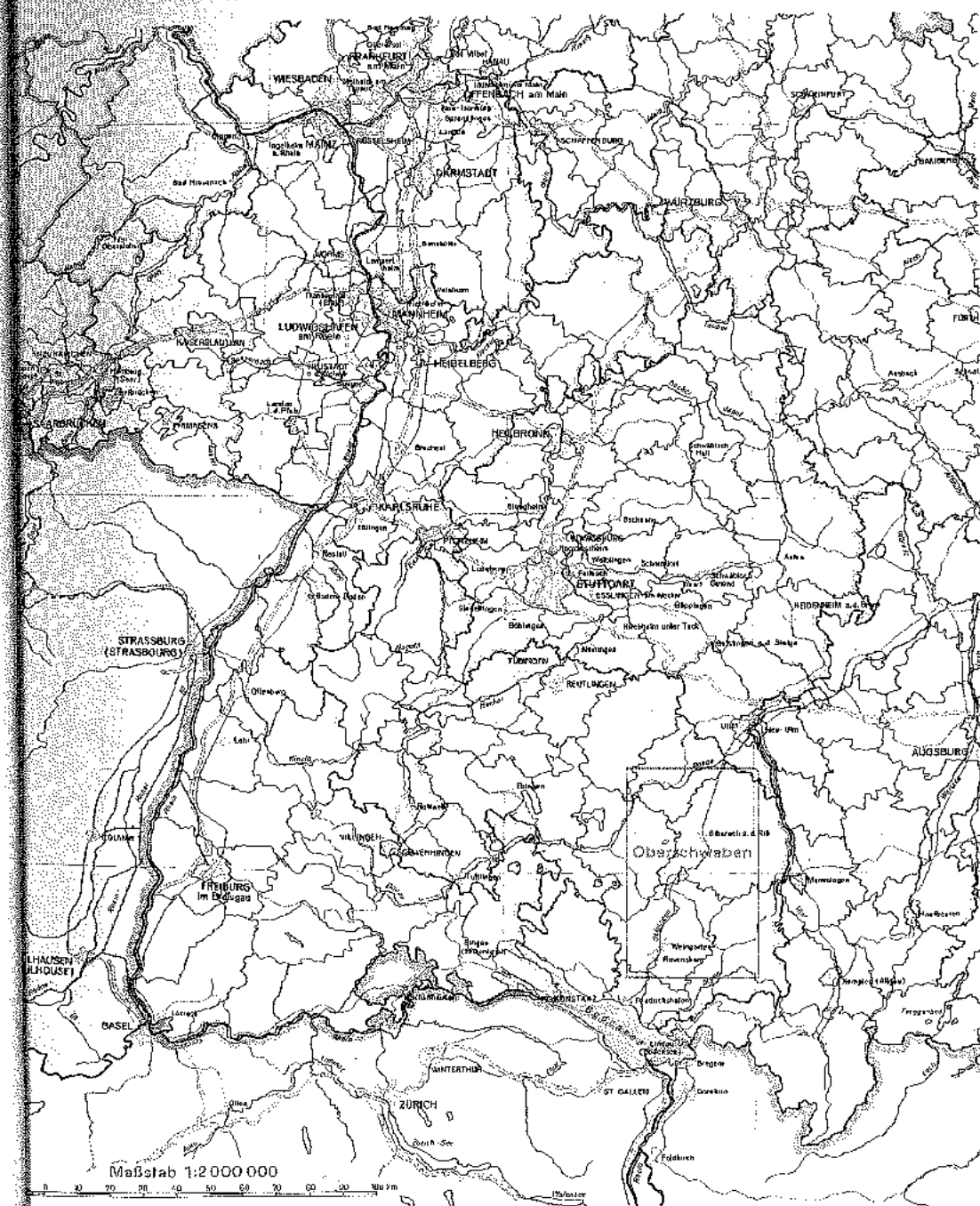
4.3. The presentations of this symposium are intended to give information and to be evaluated from 3 points of view:

- a demonstration of the high level of accuracy which aerial triangulation can obtain,
- an evaluation of the empirical results in view of current theories on the accuracy of aerial triangulation,
- conclusions about research, further development, and ultimate accuracy potential of aerial triangulation.

It is proposed that the panel discussion at the end of the symposium will consider and evaluate the presented results accordingly. It is hoped that there will be enough evidence to conclude on future research and progress in aerial triangulation.

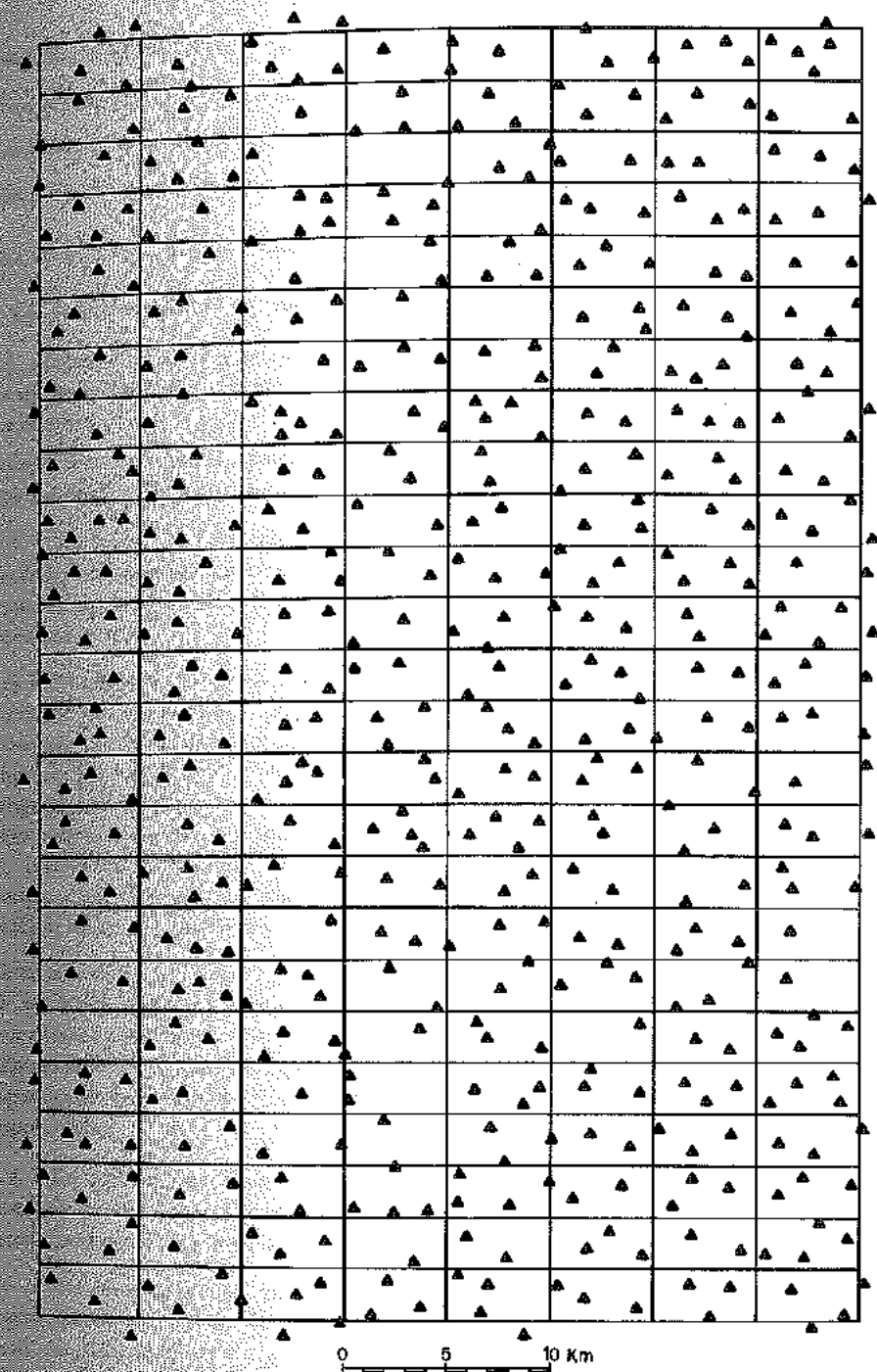
At the end of this introduction I may be permitted a personal note of thanks and of appreciation to all institutions and persons by whose efforts the remarkable results of the OEEPE test Oberschwaben have been obtained.

Discussion see p. 83.



Location of the OEEPE-Test-Field Oberschwaben in southern Germany
 Location géographique du champ d'essai Oberschwaben dans l'Allemagne du sud

Figure 1



OREPE-Test-Field Oberschwaben

Distribution of Control- and Check-Points

Distribution des points de contrôle et des points de comparaison

Figure 2

OR 528.738.2 : 971.430-313.5:528.14

Récherches statistiques sur la précision de l'aérottriangulation

Le champ d'essai Oberschwaben

Par L. Ackermann, Stuttgart

(La version anglaise, qui contient également les figures: p. 15)

1. Les fondements et l'ampleur de l'essai

Le projet d'aérottriangulation Oberschwaben est le sujet de ce symposium. Ce test examine la précision des méthodes de l'aérottriangulation et son évaluation empirique, en particulier.

Avant d'entrer dans la partie technique du symposium, il semble utile de donner un aperçu des fondements, de l'organisation et de l'ampleur de l'essai Oberschwaben.

1.1. Au cours de la dernière décennie un progrès remarquable a été constaté dans le sens de l'aérottriangulation en ce qui concerne les procédés numériques. C'est surtout la compensation d'un bloc de bandes qui a beaucoup avancé l'application de la triangulation aérienne.

Les méthodes de compensation sont classées en 3 groupes selon les unités numériques: la méthode d'interpolation polynômiale, la méthode des modèles indépendants et la méthode entièrement analytique ou méthode de gerbes. La méthode polynômiale étant la plus simple, elle semble être employée le plus souvent. Grâce au développement de programmes d'ordinateur cependant, on emploie à présent de plus en plus souvent les autres méthodes qui sont plus précises et plus exigeantes. L'expérience pratique avec les méthodes numériques de compensation est très satisfaisante, surtout ces derniers temps, d'où la grande confiance que l'on donne dans la capacité de précision et dans l'efficacité de ces méthodes.

Cette confiance se base sur plusieurs études théoriques concernant la précision des bandes et des blocs, expériences qui ont été entreprises il y a quelques années et qui ont été complétées récemment. On a déduit de ces études théoriques des règles concernant la dépendance de la précision que ce soit de la grandeur et de la forme des blocs ou de la répartition des points de contrôle. On applique ces règles en pratique et les résultats sont appréciables.

1.2. En examinant de plus près la question de la précision de l'aérottriangulation on constate toutefois que le niveau des connaissances scientifiques est loin d'être aussi satisfaisant que l'on pourrait croire.

En réalité nous ne savons que très peu de choses en ce qui concerne la comparaison des groupes de méthodes. Ce sont des raisons très générales qui déterminent le choix de l'une ou l'autre, d'autant plus que les différents programmes d'ordinateurs ne sont pas toujours disponibles. Aussi il n'y a toujours pas d'accord entre les spécialistes sur l'évaluation relative des différentes méthodes.

Bien que les méthodes polynômiales soient considérées en général comme les moins précises, ceux qui les emploient prétendent qu'elles atteignent un degré de précision beaucoup plus élevé que l'on pourrait croire. Par ailleurs, tout le monde affirme que la méthode des gerbes est la méthode la plus précise, donc la plus adéquate. Mais dans une certaine mesure, c'est toujours une question d'opinion personnelle pour comparer la méthode des gerbes à la méthode des modèles indépendants.

Notre manque de connaissances en ce qui concerne la comparaison des méthodes est naturellement dû au fait que jusqu'à présent personne ne disposait de programmes d'ordinateurs pour différentes méthodes.

1.3. Sur un autre plan, les études théoriques sur la précision concernant les différentes répartitions des contrôles ne sont pas très satisfaisantes non plus. Il est vrai que pour la méthode des gerbes aussi bien que pour la méthode des modèles indépendants on a entrepris de nombreuses recherches qui donnent une idée assez précise des effets des contrôles sur la précision. Comme il a été mentionné plus haut, ces règles théoriques sont appliquées en pratique avec un succès appréciable.

Il faut néanmoins souligner que toutes les études théoriques concernant la précision sont basées sur des suppositions simplifiées. Autrement dit, elles se réfèrent en principe à des modèles mathématiques simplifiés plutôt qu'à la réalité. Par manque d'une meilleure méthode et pour des raisons de facilité on traite habituellement les coordonnées cliché ou les coordonnées modèle comme des observations auxquelles on attribue au hasard des erreurs indépendantes. Ces suppositions fondamentales simplifiées ne correspondent pas du tout à la réalité. En principe, la question reste donc ouverte pour savoir jusqu'à quel point on peut considérer les résultats de ces études théoriques comme suffisamment réalistes. Est-ce que les erreurs des photographies réelles ressemblent aux présomptions théoriques? Est-ce que le modèle mathématique prédit suffisamment la réalité? Ce sont des questions qu'on ne peut pas résoudre uniquement sur un plan théorique.

La même divergence entre le modèle mathématique et la réalité apparaît dans les recherches théoriques qu'on a entreprises récemment considérant les erreurs systématiques des coordonnées photographiques. Ce problème se pose aussi pour les comparaisons théoriques des méthodes.

En conclusion, on peut constater que la valeur des modèles théoriques doit être expérimentée et vérifiée par des essais pratiques. Les tests expérimentaux des modèles théoriques sont d'une nécessité fondamentale en photogrammétrie de même qu'en tout domaine scientifique.

1.4. Dans l'aérottriangulation il existe encore un grand nombre d'autres problèmes relatifs à la précision dont on sait très peu de choses et sur lesquels on manque encore d'informations. J'en cite quelquesuns brièvement:

- la comparaison de clichés grand'angulaires et super-grand'angulaires dans l'aérottriangulation,
- la précision des blocs de 20 % par rapport aux blocs de 60 % de recouvrement latéral,

- la précision par points de liaison signalés par rapport aux points naturels ou marqués artificiellement, l'effet du nombre de points de liaison utilisés,
- l'influence de l'étalonnage de la caméra en laboratoire et de l'étalonnage en champ d'essai et des corrections par réseau sur la précision des bandes et des blocs compensés,
- aérottriangulation analytique par rapport aux procédés semi-analytiques avec formation de modèle et/ou bande par des instruments analogues.

Pour certaines de ces questions on dispose de quelques informations empiriques ou théoriques. Toutefois elles ne peuvent pas être considérées comme résolues.

1.5. Lorsque l'OEEPE a considéré la situation scientifique que je viens de décrire, la nécessité d'effectuer des tests empiriques étendus a paru évidente. De même il fut manifeste que des efforts considérables seraient nécessaires pour obtenir des résultats intéressants, efforts dus à l'extension des champs d'essai dans l'aérottriangulation.

A la suite de longues discussions et préparations durant les années 1967/68 le Comité-Directeur a décidé en automne 1968 de relever le défi et d'entreprendre une nouvelle série de tests. On a chargé la Commission AB sous la présidence du professeur Wiser d'une recherche empirique par tests contrôlés sur les questions de la précision dans la triangulation des bandes et des blocs.

Ces recherches devaient tout d'abord s'occuper de 3 groupes de problèmes:

- expérimenter par des méthodes statistiques la concordance ou la divergence des résultats empiriques par rapport aux modèles théoriques de précision (ou hypothèses de précision en termes statistiques) en considérant 3 groupes de paramètres: le recouvrement des clichés, la grandeur des blocs et la répartition des points de contrôle;
- déterminer par des essais les relations de la précision entre les différentes méthodes de l'aérottriangulation et comparer ces résultats à des théories si elles existent;
- donner des résultats empiriques pour quelques problèmes de précision difficiles à résoudre sur un plan théorique, tels que clichés grand'angulaires par rapport aux clichés super-grand'angulaires, points signalés par rapport aux points naturels ou marqués artificiellement, compensation planimétrique par rapport à la compensation spatiale, les différentes formules polynômiales, la photographie des réseaux et l'étalonnage en champs d'essai.

Cette liste des problèmes se rapporte en premier lieu à des recherches statistiques. Nous nous référons comme programme I du teste Oberschwaben. Il y existe une autre possibilité concernant l'étude des questions de la précision dans l'aérottriangulation. Il s'agit de la recherche de toutes les propriétés des erreurs de chaque photo et de ses mesures. Si l'on avait une connaissance complète de tous les éléments des erreurs d'une photo y compris leur corrélation avec les photos voisines, la précision de n'importe quelle configuration de photos (par exemple bandes, blocs) pourrait être dérivée par la méthode théorique de la propagation des erreurs et l'on pourrait s'y référer. Ce genre de recherche sera peut-être le dernier. Bien que les recherches concernant un seul cliché n'aient pas été poursuivies, le Comité Directeur de l'OEEPE a pourtant décidé d'utiliser pleinement le matériel du test en lançant le programme test Oberschwaben II. On avait l'intention de faire une étude plus détaillée des différents éléments causant les erreurs des modèles et

des bandes. On a utilisé 90 % de recouvrement longitudinal, des mesures répétées etc. afin d'obtenir des analyses plus détaillées des erreurs effectives et de leurs relations entre elles.

2. Le champ d'essai Oberschwaben

2.1. Au sein de la Commission AB de l'OEEPE on a élaboré les spécifications pour un champ d'essai et pour la photographie aérienne. Le choix de l'emplacement du champ a été forcément un compromis de conditions divergentes. On a appelé le champ Oberschwaben (Souabie du Sud). C'est le nom géographique et historique de la région entre le Danube et le Lac de Constance en Allemagne du Sud (voir tableau 1). Le champ d'essai délimité par des conditions différentes et couvrant 2 500 km² au total forme un rectangle de 62,5 km du nord au sud et de 40,0 km de l'est à l'ouest. Cette région vallonnée est située à 400 — 800 m au-dessus du niveau de la mer. Elle est couverte de moraines, de marécages, de bois et de terres fertiles datant de l'époque glaciaire et post-glaciaire.

2.2. 548 points ont été sélectionnés parmi les points trigonométriques de 1^e, 2^e et 3^e ordre du système national (voir tableau 2). On les a marqués à l'aide de repères afin de pouvoir les utiliser comme points de contrôle et comme points de comparaison. Pour 484 points la hauteur était également connue.

L'échelle a été fixée à 1 : 28 000 en compromis de spécifications contradictoires. D'une part l'échelle devait être assez grande pour pouvoir couvrir la région par plusieurs centaines de photos, d'autre part elle devait être suffisamment restreinte pour éviter que les erreurs des coordonnées terrestres des points trigonométriques et des points de hauteur n'aient d'influence sensible.

Par ailleurs tous les points de liaison standard (6 par modèle) ont été signalés au sol par 2 signaux chacun. De cette façon les résultats des essais de triangulation ne seront pas influencés par la possibilité d'un transfert imprécis de points dont les effets pourront être étudiés à part.

L'Institut für Angewandte Geodäsie (IfAG) à Francfort a accepté de placer les signaux et de se charger de tous les travaux préparatoires en coopération avec le Landesvermessungsamt Baden-Württemberg. Le rapport de M. Belzner donnera des détails supplémentaires concernant le champ d'essai et les travaux préliminaires.

2.3. Au printemps 1969 la Maison Häussermann, Waiblingen, a fait les prises de vue à l'aide de la caméra grand'angulaire Zeiss RMK A 15/23 et de la caméra super-grand'angulaire Zeiss RMK A 8,5/23. Comme pour les clichés grand'angulaires et super-grand'angulaires l'échelle 1 : 28 000 devait être obtenue, les hauteurs relatives de vol étaient de 4 284 m et de 2 380 m respectivement. A l'aide de chaque caméra un bloc de 15 bandes avec 60 % de recouvrement latéral a été effectué. Les bandes allant du nord au sud (ou inversement) ont 25 modèles chacune. Ainsi le champ d'essai est couvert de 390 photos par chaque type de photographique.

La signalisation des points de liaison a exigé des prises de vue visées, ce qui a entraîné quelques difficultés. Pendant tous les vols on a effectué des mesures au statoscope à l'aide du statoscope Zeiss S 2.

La photographie décrite jusqu'ici fait partie du programme test I. Pour le programme test II des photos grand'angulaires supplémentaires ont été prises de la partie est du chantier: bandes avec 90 % de recouvrement longitudinal et avec des photos à réseaux (Zeiss RMK AR 15/23).

Malheureusement les conditions météorologiques n'étaient pas bonnes au printemps 1969 et en raison des difficultés pour les prises de vue visées il a fallu une période de 2 1/2 mois pour effectuer les 4 jours de vol nécessaires pour couvrir le total du chantier. Ce long délai a causé des problèmes dus à la végétation et à la destruction des signaux.

En dehors des vols effectués pour le champ d'essai Oberschwaben quelques photos ont été prises, avec les mêmes caméras, du champ d'essai de calibration Rheidt, près de Bonn, mesurant 2 km x 2 km. Il n'était malheureusement pas possible de photographier ce champ d'essai immédiatement avant ou après les vols du champ d'essai Oberschwaben. Aussi la valeur des photos de Rheidt pour la recherche et la correction des erreurs systématiques est quelque peu limitée.

3. Mesures à l'aide du stéréocomparator

3.1. Après avoir développé les films on a tiré des épreuves sur papier et des diapositives. La préparation pour la prise des mesures a été faite par l'Institut für Angewandte Geodäsie (IfAG), Francfort. Chaque point a été marqué sur les épreuves et muni d'un numéro. De même on a établi des listes des coordonnées terrestres.

Le stéréocomparator a été utilisé pour toutes les mesures concernant le programme I. On a adopté le principe de ne mesurer les photos qu'une seule fois. On a divisé le travail en 4 parties et on les a données à 4 centres de photogrammétrie différents faisant partie des pays membres de l'OEEPE.

On a mesuré

- les 8 bandes grand'angulaires no. 1, 3, 5, 7, 9, 11, 13, 15 par l'Institut für Angewandte Geodäsie (IfAG), Francfort, à l'aide du stéréocomparator Zeiss PSK,
- les 7 bandes grand'angulaires no. 2, 4, 6, 8, 10, 12, 14 par l'Institut Photogramétrique de l'Université Technique de Vienne en coopération avec le Bundesamt für Eich- und Vermessungswesen, à l'aide du stéréocomparator Wild StK 1,
- les 8 bandes super-grand'angulaires no. 1, 3, 5, 7, 9, 11, 13, 15 par le Fotogrammetrische Dienst van het Cadaster, Den Haag, à l'aide du stéréocomparator Zeiss PSK,
- les 7 bandes super-grand'angulaires no. 2, 4, 6, 8, 10, 12, 14 par l'International Institute for Aerial Survey and Earth Sciences (ITC), Delft, à l'aide du stéréocomparator Wild StK 1.

De cette manière chacun des 4 centres a mesuré un bloc complet de 8 x 26 = 208 ou respectivement 7 x 26 = 182 photos (égal à 8 x 25 = 200 ou 7 x 25 = 175 modèles, respectivement) avec 20 % de recouvrement latéral.

Ces 4 blocs seront désignés par

- bloc Francfort,
- bloc Vienne,
- bloc La Haye,
- bloc Delft.

Les blocs Francfort et Vienne combinés forment naturellement un bloc grand'angle avec 60 % de recouvrement latéral de 390 photos ou 375 modèles. De même le bloc La Haye et le bloc Delft forment ensemble un bloc super-grand'angle avec 60 % de recouvrement latéral de 390 photos ou 375 modèles.

La préparation et la prise des mesures des photos du programme I ont duré de 1968 à 1970.

3.2. Après la réduction et la correction on a utilisé les mesures effectuées par le comparateur directement pour la méthode des gerbes de la triangulation par blocs.

Pour la méthode des modèles indépendants on a transformé les mesures du stéréocomparateur en modèles par orientation relative analytique (= stéréo-couples). Pour la méthode polynômiale on a formé des bandes à partir des modèles numériques prévisibles. Toutes les compensations réalisées plus tard par différentes méthodes ont pour base les mêmes mesures prises à l'aide du stéréocomparateur. L'IfAG, Francfort, s'est chargé des calculs analytiques à l'exception du bloc Vienne dont les modèles ont été calculés à l'Université Technique de Vienne.

4. Les compensations

4.1. Les coordonnées cliché et les coordonnées modèle obtenues comme cité ci-dessus ont servi d'input data pour les séries de compensation des bandes et des blocs du programme test I. Pour ce programme l'Institut Photogrammétrique de l'Université de Stuttgart a été le centre pilote. Il a effectué et il continue à effectuer les calculs de la plupart des compensations de bandes et de blocs du programme I. Ils consistent d'abord en un grand nombre de compensations de bandes en utilisant la méthode des modèles indépendants et 8 versions de compensation polynômiale, complétée récemment par la compensation de gerbes. Le programme total du test s'élève à près de 1800 compensations de bandes différentes. En même temps on a commencé la compensation de blocs se concentrant d'abord sur la méthode des modèles indépendants. Récemment la compensation de blocs a été étendue à la méthode des gerbes et depuis quelques semaines la méthode polynômiale (par l'IfAG, Francfort).

4.2. Au cours de la première journée et dans la matinée de la deuxième journée du symposium on présentera les résultats de la compensation des blocs et des bandes du programme test I. L'après-midi de la deuxième journée sera consacrée à la description du programme test II et à la présentation des résultats disponibles, élaborés sous la présidence du professeur Wiser de l'Université de Liège. Au cours de la 3^e journée on présentera des rapports concernant des recherches spéciales sur le réseau, l'étalonnage et les erreurs systématiques.

Les programmes test I et II ne sont toujours pas entièrement achevés. L'exploitation scientifique du matériel complet demandera encore plusieurs années.

Nous espérons pourtant que les résultats obtenus à ce stade intermédiaire des travaux ne manqueront pas d'intéresser les spécialistes de la triangulation aérienne. Quelquesuns de ces résultats obtenus sont en effet quelque peu surprenants. Ils ouvrent la possibilité de revoir quelques points de vue concernant la triangulation aérienne et ils auront une influence sur les recherches à venir.

4.3. Les représentations de ce symposium sont à considérer de 3 points de vue différents:

- une démonstration du haut degré de précision atteint en aérotriangulation,
- une évaluation des résultats empiriques par rapport aux théories courantes sur la précision de l'aérotriangulation,
- des conclusions concernant la recherche, le futur développement et le potentiel réel de la précision de l'aérotriangulation.

Nous proposons au cours de la discussion qui suivra à la fin du symposium de considérer et d'évaluer les résultats présentés conformément à ces points de vue. Nous espérons qu'il y aura suffisamment d'arguments qui permettront d'obtenir des conclusions sur la future recherche et le progrès dans l'aérotriangulation.

Pour conclure cette introduction j'ai l'honneur de remercier très chaleureusement tous les instituteurs et toutes les personnes d'ont l'effort appréciable a permis d'obtenir ces résultats du projet Gerschwahn de l'OEEPE.

Distribution p. 83.

DK 528.71:528.738.016.4(430-313.5)

The Planning, Establishing and Flying of the Test Field Oberschwaben

(with 2 figures and 1 map)

By H. Belzner, Frankfurt a. M.

RESUME see p. 46

1. Introduction

Controlled experimental research as is the task of the OEEPE requires, as a rule, special basic material such as aerial photographs, control points, check data, etc. Therefore the selection and establishing of a test field and its photographic coverage stand mostly at the beginning of each project performed by the OEEPE. The carrying out of this work belongs to the services to be rendered by the members of the Organization, according to the Agreement concerning the Formation of the OEEPE.

In the proceedings and publications of the OEEPE a series of test fields are listed which have been established by institutions of different member countries and used by the Commissions, in particular by Commission A, B and C, e. g.: Oberriet in Switzerland (1954) used by Commission C, Vorarlberg in Austria (1954/55) used by Commission B, Reichenbach in Southern Germany (1959) used by Commission B and C. Mentioned be also the Italian test field located in the neighbourhood of Florence which has been established and photographed for the first time in 1954 and once again in 1958 for the investigations performed by Commission A.

The multitude of problems and troubles connected with establishing a test field are usually not taken into consideration when numerous requirements are made in advance or certain deficiencies are pointed out afterwards.

2. Planning of the Test Field

Upon considerations as to the latest developments in the field of aerotriangulation, the OEEPE decided in 1966-67 to combine the former Commissions A and B and to prepare a new test program. Different program suggestions and corresponding conceptions for a test field have been discussed by the Steering Committee and Commission A/B. At the end of 1968 the decision fell in favour of the proposal elaborated by Prof. Ackermann. At the same time the Steering Committee accepted the offer of the German Group of the OEEPE to establish the test field.

The base for the planning of the test field have been the following requirements:

- Size and shape corresponding to a block of approximately 200 models
- Average differences in elevation and not too many wooded regions
- Favourable position as to the execution of the flights (short approach, no flight restrictions)

- At least 300 signalized points (1 or 2 points per model) with coordinates (position and elevation) of sufficient accuracy, to be used either as control points or as check points
- Signalized tie-points (6 points per model)
- Photographic coverage in the image scale of approximately 1 : 30 000 with wide-angle camera (15/23) and super wide-angle camera (8,5/23), pin-pointed photography with forward overlap of approximately 60 % (about 20 models per strip), strip distance for 60 % side overlap (for the formation of 2 undependant test blocks with 20 % side overlap).

The search for an area corresponding to these requirements has been concentrated already at an early stage of the project upon Southern Germany, precisely said upon the region of the state of Baden-Württemberg. One of the reasons for this has been the fact that there the net of the trigonometric points (trig points) of the national survey is the most favourable one with respect to accuracy and density. Another reason which should not be overlooked has probably been the circumstance that some of the gentlemen participating in the preparation originate from this region and are well familiar with the geographic and geodetic features.

Oberschwaben is the historically and topographically interesting south-eastern part of the State of Baden-Württemberg, located between the Danube in the north and the Lake of Constance in the south. In the east, along the river Iller, Oberschwaben borders at the Free State of Bavaria. In general, the terrain is slightly undulated, its elevation ranging mostly from 500 m to 700 m above mean sea level; only small parts of the region show extreme values of 410 m and 810 m respectively. For planning, the mean terrain level has been adopted with 650 m above mean sea level.

The detail planning for establishing the test field has first been carried out within the scope of a student's task at the photogrammetric institute of the Stuttgart University. The final dates have been elaborated by the Institut für Angewandte Geodäsie (IfAG) Frankfurt a. M. in cooperation with Prof. Ackermann.

In the following, some planning details are shortly described:

The requirement to dispose of at least 300 control and check points the coordinates of which are known, has been the very starting point. In order to reduce the field work to a reasonable extent, we had to strive for getting along with the points already existing. Baden-Württemberg disposes of a very good and dense first to fourth order trig point net with 1 or 2 points per square kilometer (in open terrain) and a planimetric accuracy of the trig points of 5 cm to 10 cm; the elevation of most of the points is also determined with an accuracy of approximately 20 cm. The monuments of the trig points are in good condition. All this information has been obtained from the State Survey Office Stuttgart, the valuable assistance of which during all phases of work merits special recognition.

The subsequent considerations as to the image scale yielded as limiting value a scale of 1 : 20 000 so that possible errors of the point coordinates with $< 5 \mu$ in the image can be neglected. Later on has been decided to perform the planning of the work at an image scale of 1 : 30 000. The requirement of executing the pin-pointed aerial photography with a forward and side overlap of 60 % entails that the camera stations form a square

grid with a mesh width of 2 760 m at an image scale of 1 : 30 000. The grid points had to be transferred to the terrain in order to establish the signalized tie-points, also required. It proved expedient to adapt these grids to the map grid and coordinate grid respectively of the Gauß-Krüger system.

From this, the following data have been obtained:

Image scale (for the mean terrain level 650 m above mean sea level): 1 : 28 000

Mesh width (at the same time strip distance and stereo base): 2 500 m

Strip axes: north-south on Gauß-Krüger grid lines.

The originally planned square form of the test field with an extension of approximately 55 km in both directions had to be discarded due to the regional conditions existing in Oberschwaben as to the topographic features and the homogeneity of the trig point net, yielding a width of 40 km and a length of 62.5 km i. e. an area of 2 500 square kilometers for 15 strips with 26 images each (see the enclosed map 1 : 200 000).

In the following, a compilation of the data resulting from the planning work is given:

Position and size:	western border: northing	3536.5 km	} in the Gauß-Krüger-System
	southern border: easting	5286.0 km	
	west/east extension	40 km	
	south/north extension	62.5 km	
	area: 2 500 square kilometers		
Ground elevation:	mean terrain level	650 m above mean sea level	
	lowest point	approx.	410 m above mean sea level
	highest point	approx.	810 m above mean sea level
Image scale	1 : 28 000 (nominally for the mean terrain level)		

Distance of camera stations and strip axes: 2 500 m

Number of flight strips:	15	} corresponding to a forward and side overlap of approx. 60 %
Number of images per strip:	26	

Furthermore has been decided that the size of the signals be at least 60 cm x 60 cm and that the permissible maximal deviation of the signalized tie-points from the point of the grid (theoretical) be 125 m.

The planning phase has been concluded with the provision of maps and other data for the performance of the field work and the photo flights, including:

- entering of the 2 500 m grid into the maps 1 : 25 000 and 1 : 50 000
- preselection of the trig points to be signalized; this was done by means of trig point indexes and trig lists provided by the State Survey Office Stuttgart.

A valuable help for the preparation and performance of the field work have been prints of aerial photographs 1 : 12 000, originating from a photographic coverage of the entire state in 1968. The selected trig points as well as the theoretical positions and tolerance circles for the tie-points have been transferred from the maps 1 : 25 000 to these prints.

Finally, forms and instructions for the field work have been prepared and a work plan, time plan, and a cost plan for the establishing and aerial photography of the test field Oberschwaben have been set up.

3. Establishing of the Test Field

Under the concept of "Establishing a Test Field" we understand the entirety of the field work such as the selection, marking, and signalization of all required points, the measurements necessary for the determination of the coordinates of these points, the watching and maintaining of the signals up to the termination of the photo flights, and finally the dismantling of the signals.

In the test field Oberschwaben exist two different types of points: Firstly, the control and check points the coordinates (position and elevation) of which are required for the investigations. As has already been pointed out at the beginning trigonometric points of the state survey have been used for this purpose. In the selection of the points care had to be exercised that a uniform density within the test field and a higher density at the borders of the test field was obtained and only clearly visible points were signalized. The monuments of the trig points in Oberschwaben are of different shape and size: there are old, large-sized stones protruding from the ground at different heights and there are also new stones the surface of which is level with the ground. The trig points have been signalized with white, square plates at a size of 80 cm x 80 cm. They have a central, 30 cm x 30 cm wide opening which permits their horizontal positioning on ground level with high stones, too. Top and sides of the stones have been painted with white, weatherproof dispersion paint. 544 of such stones have been signalized. The second type of points are tie-points which had to be marked by pegs and signalized at the 442 intersections of the 2 500 m grid. Coordinates for these points have not been required. The tie-points have been signalized by plates at a size of 60 cm x 60 cm. For security reasons, 2 plates have normally been placed at a distance of approximately 30 cm (corresponding to appr. 1 mm in the photograph) whereby a suitable place located within a tolerance circle with a radius of 125 m around the theoretical position has been chosen usually in north-south direction of the connecting line. The signal plates consisted of white, 1 mm thick Polystyrol; one plate costed about DM 2.25 and DM 1.25 respectively. As a rule, the plates have been fastened by long nails or wire needles; on inclined ground the plates have been fastened on pegs by means of nails.

In order to facilitate in certain cases the identification of the signalized control and check points in the photographs, identification helps at a size of 20 cm x 200 cm have been laid out in angular or line arrangement. This has proven to be necessary for approximately 100 points.

About 20 tie-points located in wooded regions have been signalized in a special manner, e. g. as elevated points.

Selection and signalization of the nearly 1 000 points have been accomplished from 10 March to 1 April 1969 by 6 motorized field crews consisting of the crew leader, one technician, and one driver, working also as rod man. Bad weather not only retarded the start of the work by one week but also impeded the work in the beginning. Two crews have been supplied by the Institut für Angewandte Geodäsie which has also been responsible for the entire preparation and supervision of the work. Topographic units of the German Federal Army had supplied four field crews, thus having a large share in the timely execution of the work. The working areas of the different crews have been fixed according to the different sheets of the Topographic Map 1 : 25 000. The control points as well as the tie-points have been signalized simultaneously. The control points have been located by means of trig lists and index maps 1 : 25 000. For the tie-points, the aerial photographs 1 : 12 000 have been the most reliable base as they showed the terrain true-to-nature whereas the maps 1 : 25 000 were partially obsolete. A point and signalization description and a positional sketch of each point have been entered in the forms already mentioned. The average output of signalization work amounted to 12 points per day and crew. Examples of signalized points are shown in fig. 1.

One to three motorized crews (1 crew leader, 1 driver/rod man) have inspected the signals until the termination of the photo flight. In addition, the signals have been supervised for three days by a low flying helicopter crossing the area along the north/south lines of the 2 500 m grid. Missing signal plates, in particular those of tie-points have been reported to the field crews. The signals of approximately 11 % of the points had to be repaired or replaced during the time in question. On an average, each crew checked 27 points per day.

The dismantling of the signals which took place from 29 May until 25 June 1969 has been handled the same way as has been the checking, reaching an average output of 25 points per day and crew. Approximately 27 % of the signals have been found to be missing, destroyed or seriously damaged, the main reason for this probably being the hay-harvest beginning in June.

The measurements for determining the control and check point coordinates have been limited to the measuring of 40 auxiliary points located in the very neighbourhood of trig points being unfit for use. This work has been carried out during signal checking. In addition to this, the centering elements of about 100 trig points still marked eccentrically by old Württemberg monuments have been ascertained for converting the coordinates onto the centre of the signal. The eccentricities ranged from 10 cm to 25 cm, the directions have been measured by means of a compass.

During signalization work a special test has been prepared, the so called Signal Test Baustetten. At the place of a tie-point 64 signal plates have been laid out in a grid form with 8 m grid distance on a plane meadow. The same plates were used as in the test field itself, but different shape has been given to the plates, namely horizontal, inclined and curved forms. Also dummies of high stones have been used.

The purpose of this test has been to find out whether or not the different kinds of signals in connection with the position of sun and camera influence the representation of the signals in the photographs as well as the measuring accuracy. The test area Baustetten is covered by more than 40 photographs of the regular flights.

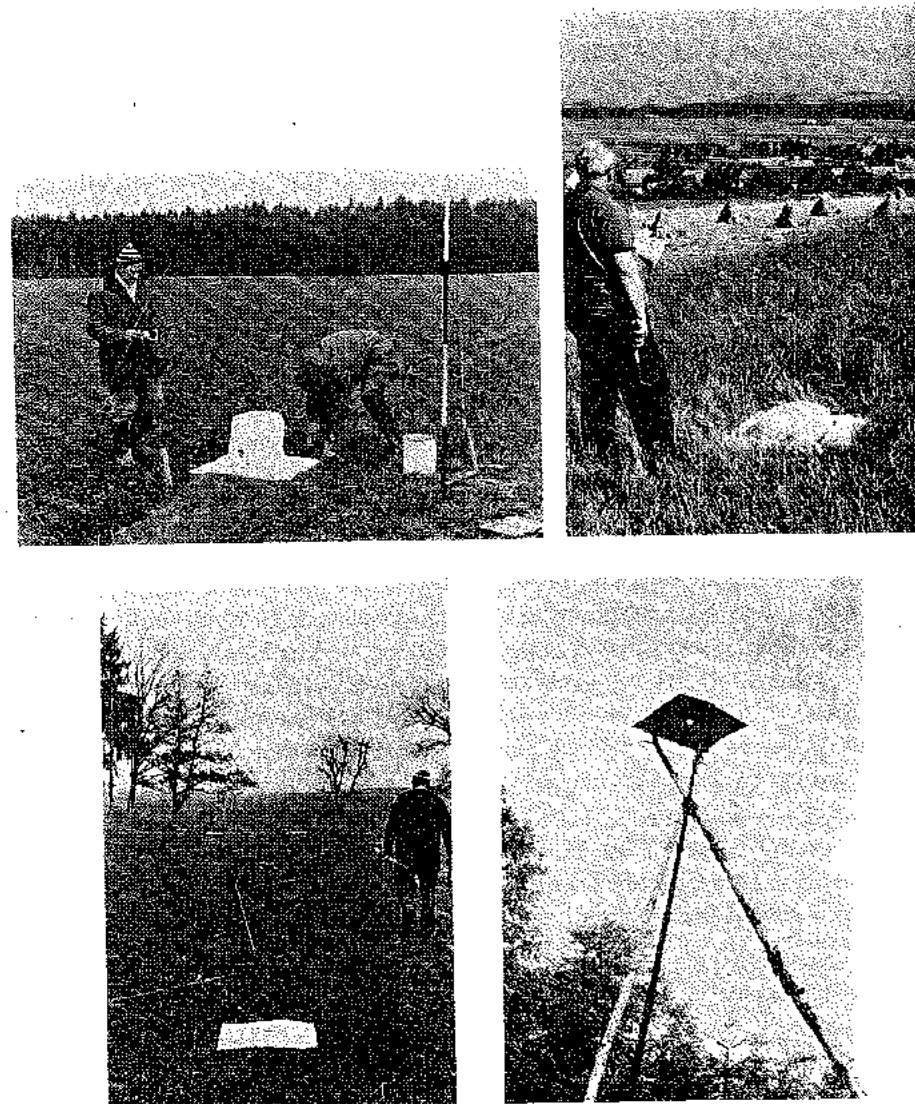


Figure 1

Signalized Points of the Test Field Oberschwaben

top: control and check points,
plate size 80 cm x 80 cm

bottom: tie-points, plate size 60 cm x 60 cm

left: standard type with 2 plates
at appr. 30 m distance

right: special type used in forest

4. Aerial Photography

For the carrying out of the photo flights the Executive Bureau of the OEEPE has issued a limited invitation to tender, based on a very detailed list of specifications set up by the TTC. The Engineering Office Häußermann, Waiblingen, has been awarded the contract and performed the flights with the photographic aircraft Aero Commander 560 F. Messrs. Carl Zeiss, Oberkochen, placed the following aerial cameras (with accessories) at the disposal of the OEEPE:

- Super wide-angle camera RMK A 8,5/23 with 125° S-Pleogon A
- Wide angle camera RMK A 15/23 with Pleogon A 2
- Reseau camera RMK AR 15/23 with Pleogon AR.

At an image scale of 1 : 28 000 have been covered:

- the entire test field with wide-angle and super wide-angle camera by pin-pointed photography (approximately 60 % forward and side overlap), 15 strips each, with 26 photographs per strip (a total of 780 photographs)
- a part of the test field with wide-angle and resseau camera, 5 strips each (strips Nr. 6, 8, 10, 12, and 14), with a forward overlap of 90 % and a side overlap of approximately 20 % (a total of approximately 1 000 photographs). This part has been intended for a test program suggested by Prof. Wiser.

During the photo flights the differences of flying height have been recorded by statoscope.

The flight altitudes amounted

with the wide-angle and resseau camera to	4 950 m above mean sea level
with the super wide-angle camera to	3 050 m above mean sea level.

The film material used has either been Agfa-Gevaert Aviphot Pan 30 PE or Kodak Plus-X Type 2401 (Estar). The following photographic data be mentioned:

Exposure time:	between 1/250 and 1/350 sec.
Aperture:	5,6 or 6,3
Filter:	for RMK 15/23 with and without resseau: yellow filter B
	for RMK 8,5/23: yellow filter with compensating coat B 33,
	i. e. transmittance in the centre 33 %.

An example of a wide-angle photo is shown in fig. 2.

The entire flight program including a few repeated flights has been performed during eight flying days which, unfortunately, had to be extended over a period of 2 months due to bad weather conditions. The flights have been carried out on:

8, 9, 26 April, 12, 13, 22, 27 May, and 10 June 1969.

The main difficulty encountered in the execution of the flights has been to meet the requirement of making pin-pointed photographs from such a high flight altitude and for relatively long strips, a requirement which probably has not been made before. For flight

navigation, adequately prepared sheets of the Topographic Map 1 : 50 000 and the Zeiss Navigation Telescope NT 1 have been used. The aircraft crew has acquitted itself of its task in a satisfactory manner. As has been revealed by computation results of the aerotriangulation, the mean deviation of the camera stations from the theoretical position amounts to approximately 100 m in flight direction as well as in transverse direction. At an aircraft operating speed of approximately 250 km per hour, a time error of 1 sec. corresponds with pin-pointed photography to a positional error of approximately 70 m in flight direction.

The photographic quality of the images has not been quite satisfactory. The final examination of the negative films carried out by the ITC and IfAG revealed that the exposure time has mostly been too short and the development of the films too high in contrasts. The super wide-angle films showed unexpectedly great differences in density between the centre and the corners. The measuring tests performed in the course of the examination revealed, however, that the signalized points could be identified and measured satisfactorily. As conclusion for future projects an even more careful adaption and supervision of the entire photographic process must be recommended, thereby utilizing all possibilities offered by scientific-technical photography.

It should also be mentioned that before and after the photo flights performed over the test field Oberschwaben, the test field Rheidt near Bonn has been photographed several times in order to check function and calibration of the three cameras.

5. Concluding Work

After completion of the photo flights and the field work and after delivery of the photo material (copying of prints and diapositives have been included in the contract), the IfAG has performed the following concluding work:

- Identification and marking of the signalized points in the paper prints. Thereby has been found out that approximately 90 % of the points could be identified with certainty, that only about 4,5 % could not be detected on the prints, and that the rest has been doubtful.
- Computations of the coordinates for control and check points signalized eccentrically or as auxiliary points, preparation of the coordinate lists.
- Selection of the photographs to be used for the investigations and preparation of photograph and point lists.
- Preparation of an index map 1 : 100 000 of the signalized points and flights strips (see the enclosed map, a reduction at scale 1 : 200 000 of this index map).
- Preparation of 82 photographic point sketches of points signalized irregularly.
- Preparation of a report explaining the signalization, numbering, etc. to the restitution centres.
- Distribution of photographs and accompanying data to the pilot and restitution centres.

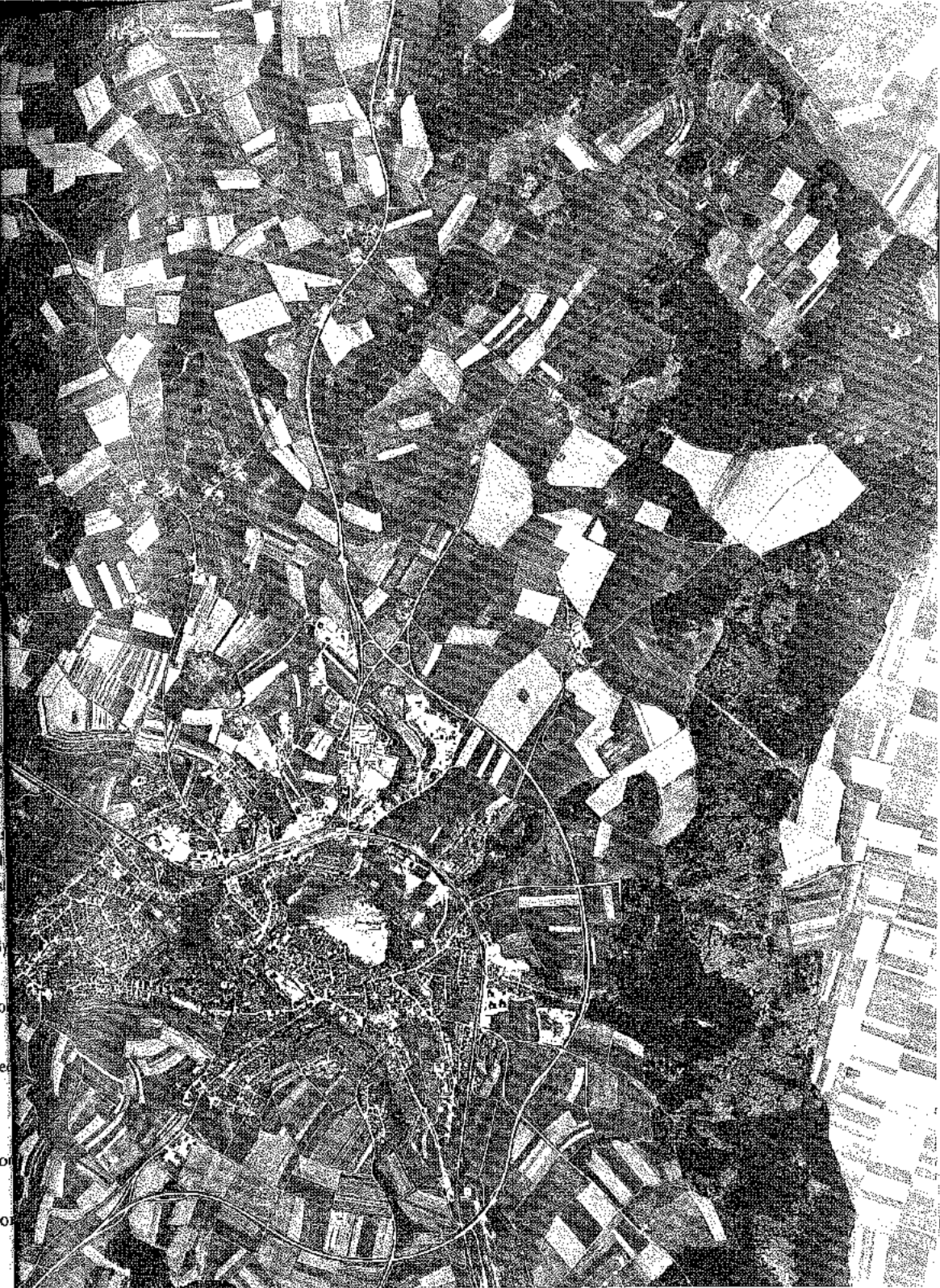
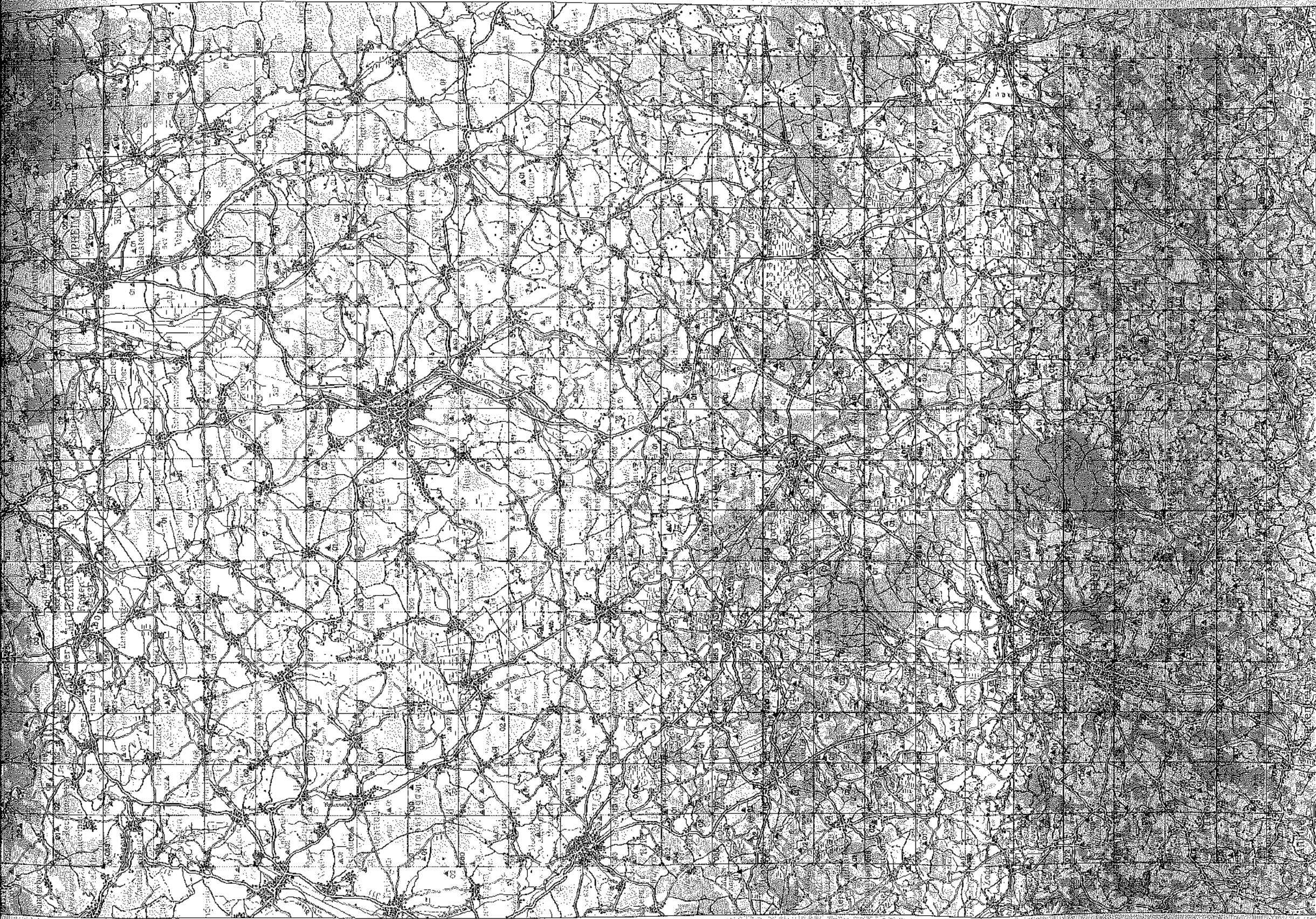


Figure 2

Sample of a Wide-angle Photo 1 : 28 000 Oberschwaben: Bad Waldsee

(Freigeg. u. Nr. 050-037 vom Reg.-Präs. Nord-Württemberg)



Legende

72
 Quadrillage (2,5 km x 2,5 km)
 avec numérotation des mailles

Point de comparaison signalés
 (station trigonométrique) avec
 numérotation abrégée

Intersection signalisée par deux
 points de liaison en position normale

Intersection signalisée par points de liaison
 en position particulière. Voir les explications
 dans la liste des points de liaison

Intersection signalisée par un point
 de comparaison au lieu des points de liaison

Institut für Angewandte Geodäsie
 Frankfurt a. M.

Base cartographique: Carte Générale Topographique 1/200 000.
 lire des feuilles CC 7918, CC 1926, CC 8718, CC 8726.
 Publication autorisée pour les feuilles CC 1926 et CC 8726
 par Baggerzonen Landesvermessungsamt München sous Nr. 1321/59

The final work also included the determination of expenditure of time and money. Some figures have already been mentioned in my report on the field work. Taking into consideration that this field work has been performed in approximately 400 man-days, on an average 0.4 day have been spent to signalize, check, and dismount each of the approximately 1 000 points. In the course of the work, a distance of approximately 37 000 km has been covered by car. The work done in the office before and after the execution of the field work required nearly the same time as did the field work i. e. 375 man-days or approximately 3 000 hours.

The overall costs for the test field Oberschwaben have been ascertained with DM 209 000,-, DM 123 000,- of this sum being personnel expenses (salaries, travel expenses, etc.). From the material expenses of about DM 86 000,-, approximately DM 66 000,- have been spent for the photo flights, including the preparation of paper prints and glass diapositives. This amount and the purchase costs of the signal plates (about DM 2 700) have been born by the OEEPE, leaving an amount of DM 140 000,- to be paid out of German budgets. In these calculations only the prime costs have been put to account. Surcharges for general costs are contained in the calculations no more than a compensation for the cameras furnished by Messrs. Zeiss on a loan basis free of charge, for the data supplied by the State Survey Office Baden-Württemberg, and for the helicopter flights performed by the German Federal Army to assist signal checking.

With the publication of these figures I should like to finish my report on the work carried out for and in the test field Oberschwaben. Thanks to the understanding cooperation of several institutions and individuals, this first phase of an extensive working program has been performed in approximately one and a half year. I do hope that the results of the investigations submitted to this symposium will convince all of you that the establishing of the test field Oberschwaben has been worth its while.

RÉSUMÉ:

En général les études expérimentales de l'OEEPE exigent l'installation et le survol photographique des champs d'essai particuliers. En 1968 la Commission A/B «Aérotriangulation» a établi son nouveau programme d'essai. Conformément à ce programme, en 1969 le groupe allemand de l'OEEPE a installé le champ d'essai d'Oberschwaben. On a choisi la région d'Oberschwaben, située au sud-est du pays de Bade-Wurtemberg, pour le premier lieu, parce que le canevas des points fixes de la triangulation est là d'une densité suffisante (1-2 points sur 1 km²) et d'une précision satisfaisante (précision planimétrique s'élevant à 5-10 cm) pour en obtenir les points de contrôle et les points de comparaison désirés et signalisés (1-2 points sur un modèle) sans levés coûteux.

La planification montra entre autres les données suivantes pour le champ d'essai:

Surface 2 500 km² (40 km x 62,5 km),

Echelle-image 1/28 000 (pour un niveau moyen du terrain de 650 m au-dessus du niveau de la mer),

Distance des stations de prise de vue et des axes de bandes 2 500 m (à un recouvrement longitudinal et latéral de 60 %) donc,

15 bandes de vol avec 26 photos chacune.

L'installation du champ d'essai, c'est-à-dire les travaux sur le terrain pour la sélection et la signalisation (y compris contrôle et démontage) des points ainsi que pour des mesures supplémentaires, commença en mars 1969 avec 6 brigades de topographes motorisées. On y a choisi 544 points de contrôle et de comparaison qui ont été signalisés à l'aide de plaques de matière plastique d'un format de 80 cm x 80 cm. En même temps on a créé 442 points de jonction (sans coordonnées) en forme d'un quadrillage, la distance des points étant 2 500 m. Pour chacun de ces points on a normalement créé 2 plaques de signaux d'un format de 60 cm x 60 cm avec une distance d'environ 300 m (environ 1 mm sur les photos). Dans cette première phase la prestation s'éleva à 12 points par journée par brigade. Il fallait contrôler les signaux jusqu'à la fin des vols photographiques et il fallait aussi les démonter (prestation chaquefois environ 26 points par journée et par brigade) (voir fig. 1).

Le vol photographique avec lequel le Bureau Exécutif a chargé une firme allemande s'effectua à l'aide de l'avion Aero Commander 560 F et des chambres de Zeiss RMK A 8,5/23 (chambre supergrand-angulaire), RMK A 15/23 (chambre grand-angulaire) et RMK AR 15/23 (chambre réseau).

Les prises de vue ont été exécutées à l'échelle-image de 1/28 000 (altitude de vol 3 050 m ou 4 950 m respectivement au-dessus du niveau de la mer):

- de tout le champ d'essai à l'aide des chambres grand-angulaire et supergrand-angulaire par des prises de vue visées (environ 60 % de recouvrement longitudinal et latéral, 15 bandes chacune avec 26 photographies (au total 780 photographies, voir fig. 2),
- d'une région partielle (les bandes Nos. 6, 8, 10, 12 et 14) à l'aide des chambres grand-angulaire et réseau, 90 % de recouvrement longitudinal et 20 % de recouvrement latéral (au total environ 1 000 photographies).

8 jours de vol étaient nécessaire (pour des raisons météorologiques entre les 8 avril et 10 juin 1969).

Les travaux de clôture étaient: Examen du matériel photographique, identification et piquage des points sur les copies-papier, calculs, préparation d'une carte générale (voir la carte annexée), listes et rapports explicatifs pour le centre pilote et les centres de restitution.

Les frais pour l'installation et les vols du champs d'essai d'Oberschwaben s'élevaient à 209 000 DM, dont environ 66 000 DM revenaient à l'ordre du vol photographique (y comprise la préparation des copies et des diapositives) payé par l'OEEPE.

Discussion see p. 83.

DK 528.735-187(430-313.5):528.14

Testblock Oberschwaben, Programme I Results of Strip Adjustments

(with 18 figures, 8 tables)

By B. Stark, Stuttgart

SUMMARY:

When summarizing the above results we can say that on the one hand many of the expectations concerning the behaviour of strip adjustments were answered but on the other hand really unexpected results were obtained. The latter need more detailed investigations — theoretical and experimental — in order to prove their significance.

Finally the most important results of the strip adjustment tests should be pointed out once more:

- rigorous adjustment methods give a very high accuracy
- the accuracy of strip adjustments significantly depends on the number of bridged models
- the accuracy decreases only slowly between a bridging distance of 2 and 8 models
- only for super-wide-angle photography the bundle method shows a higher accuracy than the method of independent models
- although in some cases polynomials can provide satisfactory results they are inferior to rigorous adjustment methods
- even in height super-wide-angle photography is less accurate than wide-angle photography (independent models)
- the available theoretical accuracy models differ more or less from the empirical conditions.

It may be expected that these findings are of interest for future investigations and that conclusions can be drawn for practical applications of strip triangulation and of block triangulation, too.

RESUME see p. 63

1. Introduction

Strip adjustments are a main subject in the historical development of spatial aerial triangulation. For a long time both terms were treated as equivalent, that means aerial triangulation problems were understood as strip triangulation problems. First of all this depends on the nature of a strip which is the direct reproduction of the process of taking aerial photographs. An additional reason is the fact that in the beginning aerial triangulation was understood as a method for which photogrammetric instruments could be used. That means the strips were formed with first-order stereo-plotters and then were transformed into the ground coordinate system by simple analytical or graphical methods.

The development of electronic computers gave the opportunity to use aerial triangulation to a larger extent than it happened up to then. But an economic application was only possible if the behaviour of the accuracy of strips was known well enough. F. Ackermann [1], [2] has published extensive theoretical investigations on this subject. But until now there haven't been any comprehensive empirical tests on aerial triangulation.

At present time strip triangulation is losing its importance as the multiple possibilities modern computers support the use of block adjustment methods which of course are more rigorous. Strips will only be applied when specially shaped objects have to be treated such as highways and rivers. But also in this connection strips have not to be considered separately because they can be conceived as special cases of blocks.

Anyway, the knowledge of the accuracy of strip adjustments should be brought to a more or less final stage. The OEBPE test field Oberschwaben has been prepared among other things for clearing up this question. The test programme I will determine experimentally the accuracy relations of strips and blocks. The present paper deals with the results of strip adjustments.

Three groups of problems, in first instance, have to be investigated:

- the dependency of the absolute accuracy of strips on the bridging distance
- the accuracy of different strip adjustment methods
- the accuracy relations of wide-angle and super-wide-angle photography.

The disposition of the paper is as follows. First of all the different strip adjustment methods applied for the Oberschwaben test are described. Then the results of strip adjustments are presented separately for each investigated method including a comparison of wide-angle and super-wide-angle photography. The next chapter will test the agreement or disagreement of empirical results with theoretical accuracy models. Finally a recapitulation will be given of the most important results and conclusions found in this study.

2. The Different Strip Adjustment Methods

Strip adjustments can be performed by different methods which are ranging from the simplest graphical interpolation methods to rigorous least squares adjustment procedures. Four of these techniques were chosen which are considered to be of the greatest practical importance:

- the method of independent models
- the interpolation method by second-degree polynomials
- the interpolation method by third-degree polynomials
- the bundle method.

The formulations of all four methods are briefly described before the results will be presented.

2.1. The Method of Independent Models

The strip adjustments with independent models were performed with the program STRIM of the Photogrammetric Institute of Stuttgart University [3]. Model coordinates computed by analytical relative orientation served as input data. The principle of the method is based on the performance of a seven-parameter transformation for each model. Every seven-parameters of all transformations of a strip have to be determined simultaneously. The defining elements are the horizontal and vertical ground control points as well as the tie points which are common to two adjacent models. The projection centres are treated as ordinary tie points with the same weight assumptions.

The adjustment problem is solved by the method of "indirect observations" with every seven transformation parameters and all ground coordinates of the tie points being unknown parameters. The ground coordinates of the control points are treated as error-free.

The following equation represents the transformation for a point i within a model j :

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ij} = -\lambda_j R_j \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} - \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i \quad (1)$$

with

- i = point number
- j = model number
- $[x \ y \ z]_{ij}^t$ = vector of model coordinates of point i measured in model j
- $[X \ Y \ Z]_i^t$ = vector of ground coordinates of point i (unknown parameters)
- $[V_x \ V_y \ V_z]_{ij}^t$ = vector of residual errors (corrections) to the transformed point i of model j
- λ_j = scale factor
- R_j = 3×3 orthogonal matrix (3 independent unknowns) orientation parameters of model j (7 unknowns)
- $[X_0 \ Y_0 \ Z_0]_j^t$ = shift

For the adjustment procedure equation (1) has to be linearized. We then get:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ij} = \begin{bmatrix} 0 & -z & y & -x \\ z & 0 & -x & -y \\ -y & x & 0 & -z \end{bmatrix}_{ij} \begin{bmatrix} da \\ db \\ dc \\ d\lambda \end{bmatrix}_j - \begin{bmatrix} dX_0 \\ dY_0 \\ dZ_0 \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} \quad (2)$$

da , db and dc are the increments of the three independent elements of the orthogonal matrix for which a special variant of the Rodriguez-Cayley-matrix was used.

The formation of the strip by joining all models together and the transformation into the ground control system is performed simultaneously. Therefore the method is rigorous with respect to the mathematical model of independent stereo-pairs.

2.2. The Interpolation Method by Polynomials

In principle strip triangulations with polynomials don't belong to the group of adjustment methods but represent an interpolation procedure. Least squares adjustment is only used to determine the parameters of the interpolating polynomial. The equation system for the adjustment is made up only by the coordinates of the control points. The remaining points are transformed after that by the polynomial formulae.

For the strip triangulation with polynomials all models have to be joined together by the technique of strip formation so that each terrain point is given only one set of coordinates x , y , z within a strip coordinate system. The strip formation is either performed by first-order stereo-plotters or by means of a special computer program.

Second-degree and third-degree polynomials were chosen for the test Oberschwaben which are relatively simple but in most cases provide satisfactory results. Both polynomial formulae are formed in four different ways with different interrelations between x , y and z . These four variants are:

a. Spatial polynomials

When spatial polynomials are used Δx , Δy and Δz are treated in common. That means 11 parameters (second-degree) or the 15 parameters (third-degree) respectively have to be determined simultaneously. The equations are:

second-degree	third-degree
$\Delta x = a_0 + a_1x - b_1y - c_1z + a_2x^2 - 2b_2xy - 2c_2xz$	$+ a_3x^3 - 3b_3x^2y - 3c_3x^2z$
$\Delta y = b_0 + a_1y + b_1x - d_1z + 2a_2xy + b_2x^2 - 2d_2xz$	$+ 3a_3x^2y + b_3x^3 - 3d_3x^2z$
$\Delta z = c_0 + a_1z + c_1x + d_1y + 2a_2xz + c_2x^2 + 2d_2xy$	$+ 3a_3x^2z + c_3x^3 + 3d_3x^2y$

b. Independent polynomials

Independent polynomials require one separate equation for Δx , Δy and Δz each. One equation includes five parameters (second-degree) or seven parameters (third-degree). The corresponding equations read:

second-degree	third-degree
$\Delta x = a_0 + a_1x + a_2y + a_3x^2 + a_4xy$	$+ a_5x^3 + a_6x^2y$
$\Delta y = b_0 + b_1x + b_2y + b_3x^2 + b_4xy$	$+ b_5x^3 + b_6x^2y$
$\Delta z = c_0 + c_1x + c_2y + c_3x^2 + c_4xy$	$+ c_5x^3 + c_6x^2y$

c. Polynomials for planimetry and elevation separately

In this case planimetry and elevation are adjusted by separate polynomials with five or seven parameters each for Δx and Δy in common and for Δz . The following equations are formed:

second-degree	third-degree
$\Delta x = a_0 + a_1x - b_1y + a_2x^2 - 2b_2xy$	$+ a_3x^3 - 3b_3x^2y$
$\Delta y = b_0 + a_1y + b_1x + 2a_2xy + b_2x^2$	$+ 3a_3x^2y + b_3x^3$
$\Delta z = c_0 + c_1x + c_2y + c_3x^2 + c_4xy$	$+ c_5x^3 + c_6x^2y$

(5)

d. Conformal polynomials for planimetric adjustment

This case is similar to case c. but the transformation for Δx and Δy is supposed to be conformal. Conformal functions are represented by the Cauchy-Riemann-differential-equations

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}; \quad \frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x} \quad (6)$$

with

$$U = x + \Delta x; \quad V = y + \Delta y \quad (7)$$

The transformation equations read as follows:

second-degree	third-degree
$\Delta x = a_0 + a_1x - b_1y + a_2(x^2 - y^2) - 2b_2xy$	$+ a_3(x^3 - 3xy^2) - b_3(3x^2y - y^3)$
$\Delta y = b_0 + a_1y + b_1x + 2a_2xy + b_2(x^2 - y^2)$	$+ a_3(3x^2y - y^3) + b_3(x^3 - 3xy^2)$

(8)

The Oberschwaben strips were formed analytically by the above mentioned program STRIM. In order to avoid overdetermination only the minimum number of control points was used. Therefore two control points x , y , z are required at the beginning of the strip and one control point z at the end of the strip.

A special computer program was developed at the Photogrammetric Institute of Stuttgart University to determine the parameters of the polynomials [6]. This program particularly complies with the requirements of the test Oberschwaben and is not prepared for general application.

2.3. The Bundle Method

Opposite to the other techniques the bundle method is not based on models but on single aerial photographs. In this respect it represents the most general procedure but demands high efficiency of the computer program and the computer itself. In the bundle method the observed plate coordinates of any image point are expressed as a function of the three parameters of the outer orientation, of the three coordinates of the projection centres and of the ground coordinates of the terrain point which is related to the image point. The known ground coordinates of a control point are treated as constants within the adjustment. All other ground coordinates are parameters to be determined by the adjustment procedure. Therefore the problem is solved by adjustment of "indirect observations".

The non-linear condition equations for a point i in a photo j may be written as:

$$x_{ij} + v_{x_{ij}} = c \frac{a_{1j}(X_i - X_{0j}) + a_{2j}(Y_i - Y_{0j}) + a_{3j}(Z_i - Z_{0j})}{a_{7j}(X_i - X_{0j}) + a_{8j}(Y_i - Y_{0j}) + a_{9j}(Z_i - Z_{0j})}$$

$$y_{ij} + v_{y_{ij}} = c \frac{a_{4j}(X_i - X_{0j}) + a_{5j}(Y_i - Y_{0j}) + a_{6j}(Z_i - Z_{0j})}{a_{7j}(X_i - X_{0j}) + a_{8j}(Y_i - Y_{0j}) + a_{9j}(Z_i - Z_{0j})}$$

with

- x_{ij}, y_{ij} = image coordinates of a point i in a photo j
- $v_{x_{ij}}, v_{y_{ij}}$ = residuals (corrections) to the image coordinates x_{ij}, y_{ij}
- c = calibrated focal length
- $a_{1j} \dots a_{9j}$ = orientation parameters of photo j
= elements of a 3×3 orthogonal matrix
- X_i, Y_i, Z_i = ground coordinates of point i
- X_{0j}, Y_{0j}, Z_{0j} = ground coordinates of the projection centre of photo j

After the linearization of equation (9) we get:

$$v_{ij} = K_{ij} dk_i + L_{ij} dt_j - f_{ij}$$

with

- v_{ij} = vector of the residuals to the image coordinates
- K_{ij} = coefficient matrix of the increments of the unknown ground coordinates (dk_i)
- dk_i = increments of the ground coordinates X_i, Y_i, Z_i
- L_{ij} = coefficient matrix of the orientation parameters (dt_j)
- dt_j = increments of the orientation parameters (3 independent elements of the orthogonal matrix and 3 coordinates of the projection centres)
- f_{ij} = constant vector

The Oberschwaben strips were adjusted with the program PAT-B of the Photogrammetric Institute of Stuttgart University [10]. This program is still in a stage of development; the strips of just two blocks could be computed until now. Therefore only the results of the Frankfurt block and of the Den Haag block are available.

3. Results

For the discussion of the results the following items are referred to:

- σ_0 : standard error of unit weight; it represents the accuracy of an observation with weight 1, e. g. of model or image coordinates
- μ : mean square value of a group of true coordinate residuals, e. g. of horizontal or vertical coordinates; it represents the absolute accuracy of a strip and is treated as a standard error.

Each strip was adjusted with six different control point distributions. These are distinguished by the bridging distance which on principle is expressed in base lengths or in the number of bridged models i . So the term $i = 4b$ means four models are bridged, or control points are arranged in intervals of four base lengths.

The control point distributions being investigated are:

$$i = 1b, 2b, 4b, 6b, 8b, 12.5b.$$

$i = 1b$ says that all available terrain points are used as control points. This variant primarily is of use for discovering gross errors in the measurements or in the ground control. The remaining versions are shown in figure 1.

Each section of control points consists at least of two points which should be located as narrow as possible at the edge of the strip. At the beginning and at the end of the strip normally 3 to 4 control points were chosen because there existed very unfavourable conditions. Therefore points had to be used which differed more or less from the ideal position.

The strip adjustments with independent models are treated in this paper most extensively. It is a rigorous method and the results are available in a complete form. Compared with the bundle method independent models have the advantage that they are applied more extensively all over the world.

3.1. Adjustment with Independent Models

3.1.1. Wide-angle Photography

The two wide-angle blocks consisting of the strips 1, 3, ... 15 and 2, 4, ... 14 were measured by the Institut für Angewandte Geodäsie (IfAG), Frankfurt, and by the Photogrammetric Institute of the Technical University of Vienna and they will be referred to as Frankfurt block and Wien block.

In order to find gross errors first of all a run was started with all control points ($i = 1b$). The result also allows a rough conclusion on the precision of the trigonometric system the control point coordinates are based on.

The computation of the Frankfurt block was started very early. That means provisional control point coordinates had to be used because a final list was not yet existing. Therefore a tedious process of eliminating and correcting points was necessary. Some strips required up to 6 runs with the version $i = 1b$ until all gross errors were eliminated. In contrary to this the first run of strip 9 yielded already satisfactory results.

With the use of the final coordinate list 14 points of a total of 550 control points remained incorrect which is equivalent to about 3%. 5 points could be corrected at once since their coordinates were punched wrong or their point numbers were permutated. The remaining 9 points were checked by the IfAG and the Landesvermessungsamt Baden-Württemberg, as a consequence of which six additional points could be corrected. There were found mistakes when computing eccentrically signalized points or there were wrong coordinates in the original catalogue. After all, 3 points remained incorrect and had to be eliminated. It turned out that after the photo flight their targets had been destroyed.

There were no gross errors at the coordinates of the tie points. This proves the extremely accurate measurement of the photos by the IfAG. As a consequence only 3 points, 0.2 % of a total of about 1400 measured points remained incorrect which seems to be an excellent result.

For the Frankfurt block the standard errors μ and the standard errors of unit weight of each strip as well as their mean square values are shown in table 1. The results are arranged in respect to the bridging distance.

First of all we can find a very high accuracy being obtained by the rigorous strip adjustment. One reason for this may be the use of signalized points but on the other hand the results show the effectivity of aerial triangulation. $\sigma_0 = 8.5 \mu\text{m}$ is by far better than the value of $16 \mu\text{m}$ assumed until now.

This applies also to the absolute accuracy which varies from $6 \mu\text{m}$ to $20 \mu\text{m}$ in planimetry and from $9 \mu\text{m}$ to $41 \mu\text{m}$ in elevation (about 0.06 ‰ to 0.27 ‰ of the flying height).

Compared with the Frankfurt block the Wien block is a little less accurate. Table 2 and figure 2 show the mean square values of the true residuals of all strips for the two blocks separately.

On principle the accuracy of strip adjustment significantly depends on the bridging distance. With an increasing number of bridged models the standard errors of x , y and z are increasing, too. This behaviour is more distinct for the Wien block than for the Frankfurt block. Indeed, the curve of accuracy is ascending very slowly between $i = 2b$ and $i = 8b$. The standard errors are increasing by the factor 1.2 (Frankfurt) and by the factor 2.0 (Wien) in x and y and by the factor 1.4 in z . From $i = 8b$ to $i = 12.5b$ the standard errors are increasing by the factor 1.4 in x and y and by the factor 1.8 in z .

The x - and y -coordinates of the Frankfurt block are almost equally accurate. The maximum difference is less than 10 %. In contrary the Wien block gives a less accurate y -coordinate than the x -coordinate (maximum difference about 30 %).

When considering the behaviour of each strip (table 1 and figures 3, 4, 5) there is sometimes a real disorder. In x and y the "band width" of the standard errors is quite narrow, especially between $i = 2b$ and $i = 8b$, but the behaviour of an individual strip is often very surprising. Anyway, the increasing trend of the curve is already visible. One can also find out that bad or good strips in principle remain bad or good for all control point distributions. Let's consider the y -coordinate of strip 11, for example, the accuracy of which is worse than that of all other strips. But the values of $14 \mu\text{m}$ to $20 \mu\text{m}$ are absolutely satisfactory. Strip 13 is in the beginning very accurate ($6 \mu\text{m}$ to $8 \mu\text{m}$) but becomes worse for $i = 12.5b$. The z -coordinates show the greatest variations of each strip. Strip 5 is a very curious example while the accuracy of strip 15 is very homogeneous.

As a conclusion of these examples we can say that the behaviour of a single strip normally is not significant for the general behaviour of a certain strip adjustment method. Only the mean square value of a greater number of strips allows a definite statement. This may be explained of course by the character of the strip deformations which can't be predicted precisely enough.

Another example may illustrate this. The true residual errors of all z -coordinates of strip 5 are presented for two control intervals in relation to the strip length. Figure 6 shows the result. For the case $i = 12.5b$ the strip deformations are very clear and behaving as expected. When 8 models are bridged there are nearly no systematic effects to be seen. The black dots refer to the western part and the light dots to the eastern part of the strip.

3.1.2. Super-wide-angle Photography

Another problem to be solved by the test Oberschwaben is the relation between wide-angle and super-wide-angle photography. For the same photo scale super-wide-angle photographs should be more accurate in elevation z than wide-angle photographs.

The super-wide-angle blocks of Oberschwaben were measured by two centres, that is the block consisting of the strips 1, 3, ... 15 by the Fotogrammetrische Dienst van het Cadaster, The Hague, and the block consisting of the strips 2, 4, ... 14 by the International Institute for Aerial Survey and Earth Sciences, formerly Delft. The blocks are referred to as Den Haag block and Delft block.

When computations were started with the Den Haag block there were found 60 gross errors in the data, that is equal to about 4 % of all measured points. 40 gross errors fell to control points which means a percentage of about 8 %. At 6 points the point numbers were permuted. All the remaining points had to be eliminated, a quite unsatisfactory result.

Within the Delft block another problem rose. Two models of strip 12 showed such a lot of gross errors also at the projection centres that they couldn't be used for the strip adjustment. The repeating of the comparator measurement and of the analytical relative orientation didn't succeed, so strip 12 was eliminated from all future strip adjustments. In this connection the Delft block consists of only 6 strips. The photo which is common to the two models was flown about 2 months later than the remaining photos of the strip. This may explain the failure of these models. The block adjustments of the Delft block could be performed without trouble including strip 12. Because of the connection of the mentioned models to two adjacent strips it was possible to eliminate the projection centres as tie points. In this case no gross errors occurred.

Table 3 and figure 7 show the results of the strip adjustments with independent models comparing Den Haag block and Delft block. The accuracy of the x -, y - and z -coordinates differs very much between the two blocks. This applies also to the control interval $i = 1b$ with all control points. The standard error of unit weight σ_0 varies from about $8 \mu\text{m}$ to $10 \mu\text{m}$. The standard errors of the x -coordinates vary for different control intervals from $10 \mu\text{m}$ to $27 \mu\text{m}$, those of the y -coordinates from $11 \mu\text{m}$ to $29 \mu\text{m}$, and those of the z -coordinates from $15 \mu\text{m}$ to $46 \mu\text{m}$.

The differences between the results of Den Haag and Delft block amount up to 60 % ($i = 8b$). The average difference still is about 30 % in x and z and about 25 % in y . σ_0 differs up to 20 %.

A significant difference between the Delft block and the Den Haag block was also shown by the relative orientation. While the Delft block yielded a mean y -parallax of about $p_y = 4 \mu\text{m}$ which agrees with Frankfurt and Wien blocks, the Den Haag block showed a mean y -parallax of about $p_y = 7 \mu\text{m}$. This means in principle that the differences must be caused by the measurement already.

Obviously the accuracy of the super-wide-angle strips depends on the number of bridged models, too. From $i = 2b$ to $i = 6b$ the curves are flat and the standard error is increasing by a multiplier of 1.3 (x, y, z) and from $i = 6b$ to $i = 12.5b$ by a multiplier of 1.6 (x, y) and 2.0 (z).

3.1.3. Comparison of Wide-angle and Super-wide-angle Photography

In order to compare the accuracy of wide-angle and super-wide-angle photography the mean square values of every two blocks are discussed. The results are shown in table 7 and in figure 8.

The most important conclusion is that super-wide-angle photography is inferior for control intervals and for all coordinates. The mean difference is about 30 % in planimetry and about 20 % in elevation, σ_0 differs only to a maximum of 5 %. In addition, y -coordinates are on principle less accurate than the x -coordinates. The difference is smaller for wide-angle than for super-wide-angle photography.

The relation between the accuracy and the bridging distance is practically the same for both kinds of photography. This means the curves are only shifted in parallel to a certain amount.

But it must be pointed out that the above results refer to the use of just one aerial camera for wide-angle and super-wide-angle respectively. Therefore the conclusions may not be valid in general. However, other experimental investigations have shown that super-wide-angle photography even in height is not more accurate than wide-angle photography (H.-K. Meier [9]).

3.2. Strip Adjustment with Polynomials

All Oberschwaben strips were adjusted with eight different versions of polynomials, there are four versions for second-degree and third-degree each.

Table 5 shows the results of all versions for the Frankfurt block. The four versions differ only very little especially for the z -coordinate. The difference in z is less than 0.2 % between version 1 and the other versions which of course are identical. Therefore it seems to be justified when using polynomial adjustments to treat planimetry and elevation separately.

Similar results are obtained from the super-wide-angle block (table 6). That's why in the future discussions only the results of the adjustment by spatial polynomials (version 1) are considered.

There is still one remark necessary to the third-degree polynomials when 12.5 models are bridged. In this case the z -polynomials theoretically are not determinable because of

three control point sections are used. But there are some practical results obtained because each section consists of at least three control points so that the adjustment problem is overdetermined. Anyway, these results are not used for further discussions.

Comparing the two kinds of polynomials we'll get the expected result that third-degree polynomials are more accurate than second-degree polynomials (see table 4). Figure 9 shows the diagrams for the wide-angle block including the method of independent models.

Using second-degree polynomials the accuracy of the y -coordinates is worse than that of the x - and z -coordinates. This is mainly caused by the Wien block where the difference between y and z is up to 30 %. Using third-degree polynomials the y -coordinate is more accurate than the x -coordinate and the z -coordinate is inferior to both of them.

The differences between second- and third-degree polynomials are almost constant from $i = 2b$ to $i = 8b$. They are equal to about 30 % in x and z and to about 70 % in y . Only when 12.5 models are bridged the differences are reduced to 5 % (x) and 50 % (y).

The same significance is shown when polynomials are compared with independent models. In this case the differences are no longer constant but are decreasing with an increasing bridging distance. Second-degree polynomials differ about 150 % in x, y and about 50 % in z from independent models at a control interval $i = 2b$. In the case of a bridging distance of 12.5 base lengths second-degree polynomials are inferior only by about 50 % in x, y while z is more accurate (about 10 %) than the independent model method.

In general, super-wide-angle photography shows the same results (figure 10). Nevertheless, there are some differences to be mentioned. In all cases the x -coordinates show the highest accuracy and the z -coordinate the lowest one. The distinctions between second- and third-degree polynomials amount to about 50 % in x , 25 % in y and 30 % in z when 2 to 8 models are bridged. Compared with the method of independent models the accuracy of second-degree polynomials is worse by about 150 % in x and 100 % in y and z ($i = 2b$). The differences are reduced to about 30 % in x, y and z for a bridging distance of 12.5 base lengths.

The relationship of the accuracy between second-degree and third-degree polynomials and between polynomials and independent models is practically the same for wide-angle and super-wide-angle strips. Therefore the differences being shown can be considered representative.

3.3. Strip Adjustment with Bundles

Strip adjustments with bundles are a rigorous method in respect to least squares and to the mathematical model. From a theoretical point of view bundle adjustments should be more accurate than the method of independent models because they directly use the measured plate coordinates as observations within the adjustment procedure. The following results may show if this is experimentally proved or not. Up to now only the results of the Frankfurt block and of the Den Haag block are available but they already allow some remarkable statements.

For the Frankfurt block table 7 shows the standard errors of each strip and the square values of all strips. In figure 11 and figure 12 the accuracy of the bundle method compared with the other methods. These figures are corresponding with table 5 and table 6.

The standard error of unit weight σ_0 is much smaller than σ_0 of the method of independent models ($\sigma_0 = 4.6 \mu\text{m}$ and $\sigma_0 = 6.7 \mu\text{m}$ compared with $\sigma_0 = 8.4 \mu\text{m}$ and $9.3 \mu\text{m}$). It now represents the standard error of an observed plate coordinate x or y . This means the standard errors of plate and model coordinates are in a ratio between 1 : 1 and 1 : 2. This agrees reasonably with the theoretical law of error propagation.

For wide-angle photography the absolute accuracy shows an unexpected behaviour (table 5 and figure 11). The average standard errors of planimetry are not at all smaller than those of the method of independent models. In addition the precision of the z -coordinate is significantly inferior for the bundle method. The average difference amounts to about 25 % and is nearly the same as for third-degree polynomials. When 12.5 models are bridged the height accuracy of the bundle method is even worse than that of second-degree polynomials.

The figures 13, 14, and 15 show the diagrams of the standard errors of each strip of the Frankfurt block. Compared with the figures 3, 4, and 5 of the method of independent models the behaviour of the x - and y -coordinates is not much different. Only the z -coordinates are still more confusing, for example strip 5 or strip 1.

The absolute accuracy of the super-wide-angle strips shows quite different characteristics (table 6 and figure 12). Up to a bridging distance of 6 base lengths the bundle method is about as accurate as the method of independent models. But for $i = 8 b$ and $i = 12.5 b$ the bundle method is more accurate — as expected. The increase of the accuracy is nearly 30 % in planimetry and about 45 % in height.

The reason for this behaviour of the bundle method can be the fact that it is influenced to a greater extent by systematic errors than the method of independent models. H. Bauer [4], H. Ebner [5], and K. Kubik [8] did some theoretical and practical investigations on this subject which may lead to this conclusion.

Of course the above results have to be proved by the strip adjustments with bundle method of the remaining two blocks Wien and Delft. It has to be mentioned that the Frankfurt block is the most accurate and the Den Haag block the least accurate of the three blocks. So it is possible that the mean accuracy of Frankfurt/Wien block and of Den Haag/Delft blocks will answer the theoretical expectations much better.

For the same reason it is not yet justified to compare the bundle results of wide-angle and super-wide-angle photography. In this respect also the results of the Wien block and Delft block have to be considered.

4. Comparison of Empirical Results with Theoretical Investigations

From the beginning of aerial triangulation a very unfavourable behaviour of bundle adjustments was discovered which was effected by the so-called strip deformation

error. To learn more about these deformations a suitable error propagation for strip triangulation was required. The most extended theory up to now is the Vermeir theory which is based on the principle of the double summation of errors.

In the following section the experimental results of the method of independent models only are compared with theoretical accuracy models. The Oberschwaben results obviously don't match with the Vermeir theory. That means the double summation of errors is superposed by other effects especially for short bridging distances. Therefore another theory should be used.

For horizontal strip adjustment the anblock theory may be of greater interest because it's also based on the method of independent models. It presumes a simplified stochastic model with all coordinates being treated as equally accurate and uncorrelated.

For vertical strip adjustment no similar theoretical investigations are available. In this case the Vermeir theory will be used.

In addition to these theories the method of simulation was applied to test the empirical results. A theoretically formed strip was derived in which the number and location of the control and tie points would come as close as possible to the actual values of the Oberschwaben strips. Then coordinates were assumed for all points. In order to get observations from these coordinates all the model points were superposed by random errors which were produced by electronic computation. After all, the strip was adjusted with the method of independent models using the program STRIM. All five control point distributions ($i = 2 b$ to $i = 12.5 b$) were computed. The whole process was repeated eight times in order to obtain significant results. The absolute accuracy was found in the same way as in the practical cases by comparing the true coordinates of a certain number of check points with their adjusted coordinates. With the method of simulation both, the wide-angle block and the super-wide-angle block were treated.

All results are shown in table 8 and in figure 16 and 17. The values are divided by the standard error of unit weight σ_0 in order to allow a direct comparison of all methods.

The results of the wide-angle block are most surprising because the theoretical accuracy of the anblock method is worse than the empirical one. The different number of tie points used in both cases (anblock: 4 and Oberschwaben: 6×2) may be one explanation. The different assumptions in the computation of the absolute accuracy could be another reason.

When the simulated and the empirical results are compared the simulation on principle shows a too optimistic accuracy. This of course is due to the assumption of random errors only. The difference of the z -coordinates amounts up to 55 %.

For wide-angle photography the values of the Vermeir theory are not shown in figure 16 because they are too bad and wouldn't fit into the diagram. They are up to three times as large as the empirical values.

Using super-wide-angle photography the theoretically derived accuracy is higher than the empirical one except the Vermeir theory in height. The difference between simulated and experimental results amounts up to 60 %, which is another symptom that the measurements of the super-wide-angle block must be less accurate than that of the wide-angle block.

The values based on the Vermeir theory were published by K. Kubik (see [7]). They are actually smaller than those H. G. Jerie has calculated. The comparison with the empirical values of the standard errors doesn't show too great differences. But the relation between wide-angle and super-wide-angle is really not realistic.

Finally the maximum standard errors of the anblock theory are compared with practical results. These are supposed to occur about in the middle between control point sections. In the Oberschwaben strips groups of check points were chosen which were located in that special areas, and the mean square values of their standard errors were computed.

Figure 18 demonstrates that in this respect only the results of the super-wide-angle block match with the anblock theory. The maximum standard errors of the wide-angle block are generally are by 50 % smaller than those of the theory. This is once more a sign for the outstanding accuracy of the Frankfurt block.

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RESUME

On a pris en considération les bandes des 4 blocs du test Oberschwaben pour des examens de précision des compensations par bandes.

On a examiné les trois questions suivantes:

- La dépendance de la précision absolue des bandes de la repartition des points de contrôle
- La précision de différentes méthodes de compensation par bandes
- Les relations de précision entre les prises de vue grand'angulaires et supergrand'angulaires.

On a utilisé quatre méthodes de compensation différentes:

- La méthode des modèles indépendants
- La méthode d'interpolation avec des polynômes de 2^e degré
- La méthode d'interpolation avec des polynômes du 3^e degré
- La méthode des faisceaux.

Les calculs ont été effectués à Stuttgart à l'aide de programmes de l'institut de Photogrammétrie de l'Université de Stuttgart en différenciant 6 versions de points de contrôle. On a reparté les points de contrôle de manière qu'il y ait une distance de 1, 2, 4, 6, 8 et 12,5 modèles. Le cas d'une distance d'un seul modèle signifie que tous les points de contrôle disponibles ont été utilisés.

Les deux données suivantes permettent d'apprécier la précision:

- σ_0 : l'erreur moyenne de l'unité de poids; elle donne la précision d'une observation de poids 1, c'est à dire de coordonnées de modèles ou de clichés
- μ : Valeur carrée moyenne d'un groupe d'erreurs vraies de coordonnées x , y et z ; elle donne la précision absolue d'une bande.

On obtient la valeur μ à l'aide de points de comparaison.

On a réservé la plus large place aux résultats des compensations par bandes avec modèles indépendants étant donné qu'il s'agit d'un procédé assez rigoureux d'une grande importance pratique. En ce qui concerne les compensations par faisceaux on ne dispose jusqu'à présent que des résultats des bandes du bloc grand'angulaire de Francfort et du bloc supergrand'angulaire de Den Haag.

Les résultats sont montrés par 7 tableaux selon de différents points de vue et par 14 illustrations. Les résultats empiriques sont comparés aux évaluations théoriques à l'aide de la méthode de simulation, de la théorie «Anblock» et de la théorie de Vermeil.

En conclusion on constate d'une part la réalisation de la plupart des estimations concernant la précision des compensations par bandes et d'autre part des résultats inattendus. Ces résultats restent à vérifier par d'autres examens théoriques et expérimentaux.

Voici les résultats les plus intéressants:

- Les procédés rigoureux de compensation donnent une précision très élevée.
- La précision des compensations par bandes dépend essentiellement de la distance des points de contrôle.
- La précision ne décroît que très lentement entre une distance de 2 et 8 modèles.
- Uniquement pour les prises de vue supergrand'angulaires la méthode des faisceaux donne une précision plus élevée que la méthode des modèles indépendants.
- Bien que les polynômes donnent dans quelques cas des résultats satisfaisants, ils sont cependant nettement inférieurs aux procédés rigoureux.
- Pour les modèles indépendants les prises de vue supergrand'angulaires sont même pour la hauteur moins précises que les prises de vue grand'angulaires.
- Entre les différents modèles théoriques et les résultats empiriques il y a des divergences plus ou moins prononcées.

OEEPE - OBERSCHWABEN

Control intervals for strip - adjustments

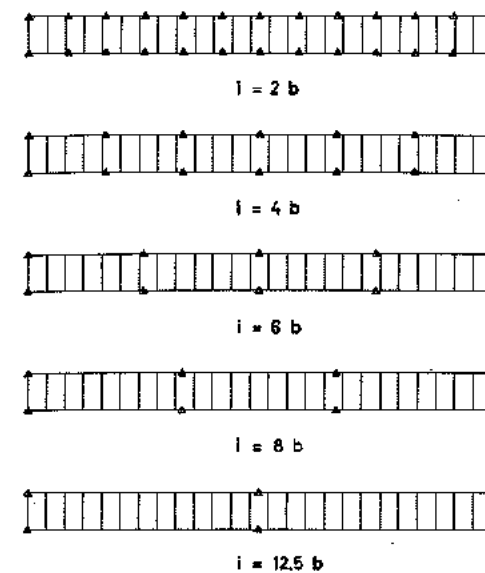


Figure 1

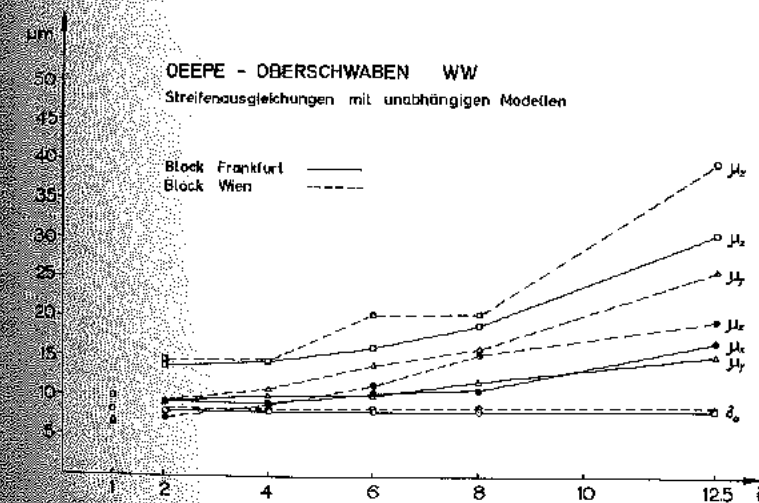


Figure 2

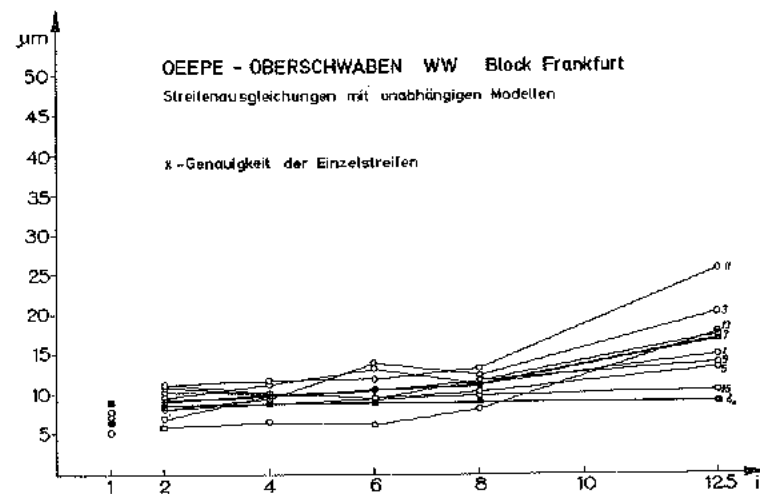


Figure 3

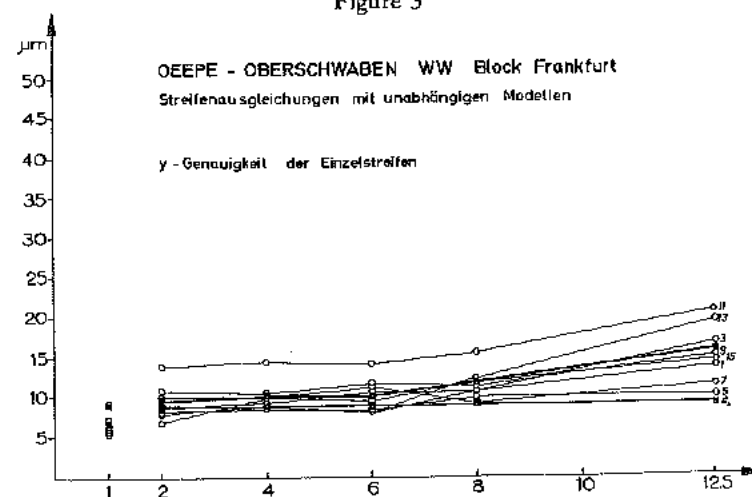


Figure 4

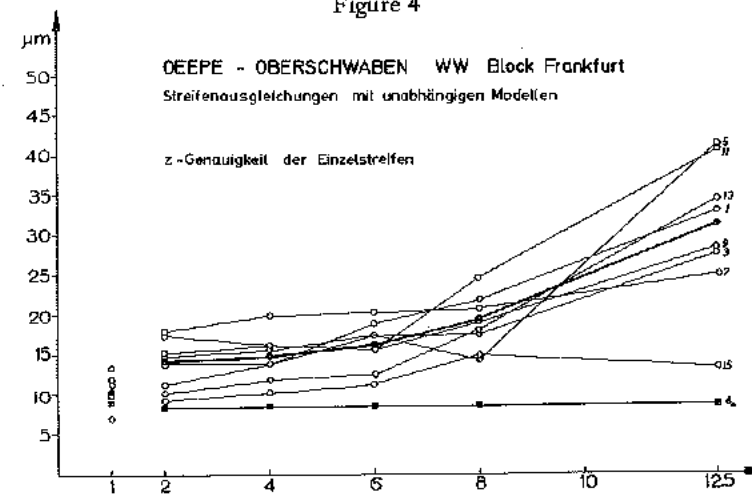
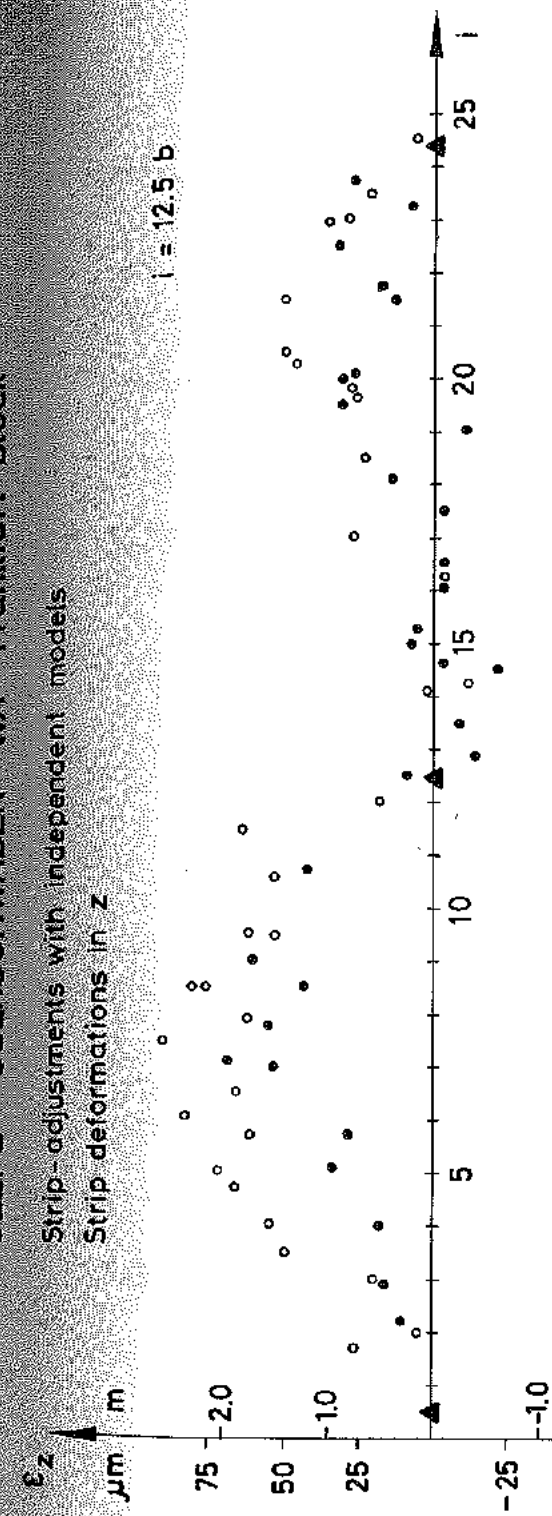


Figure 5

OEEPE - OBERSCHWABEN WA Frankfurt Block

Strip-adjustments with independent models

Strip deformations in z



$i = 8 \text{ b}$

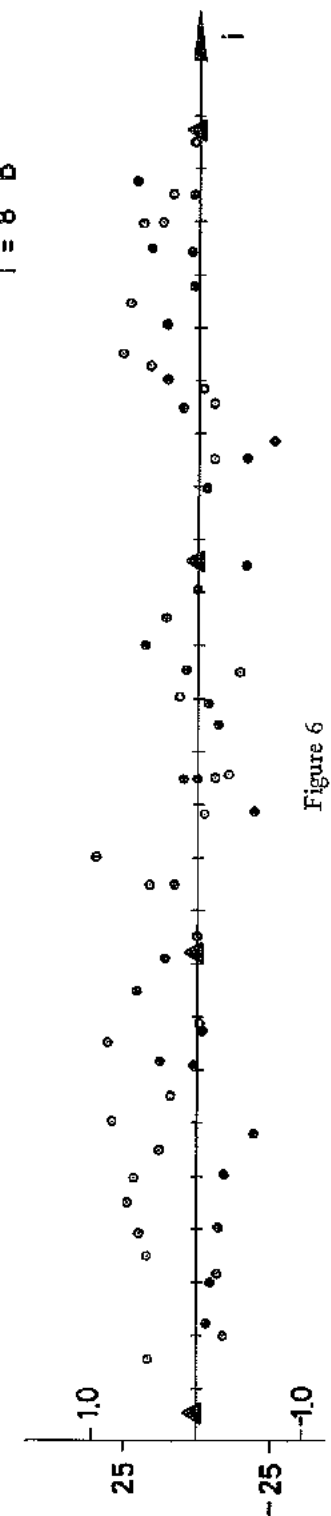


Figure 6

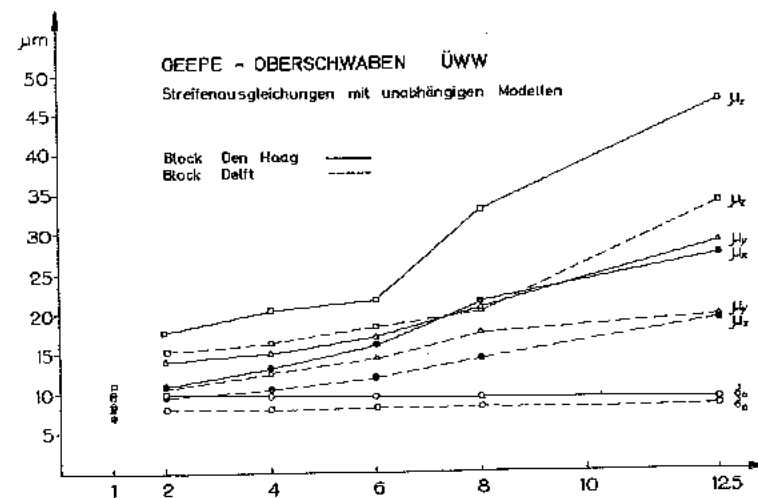


Figure 7

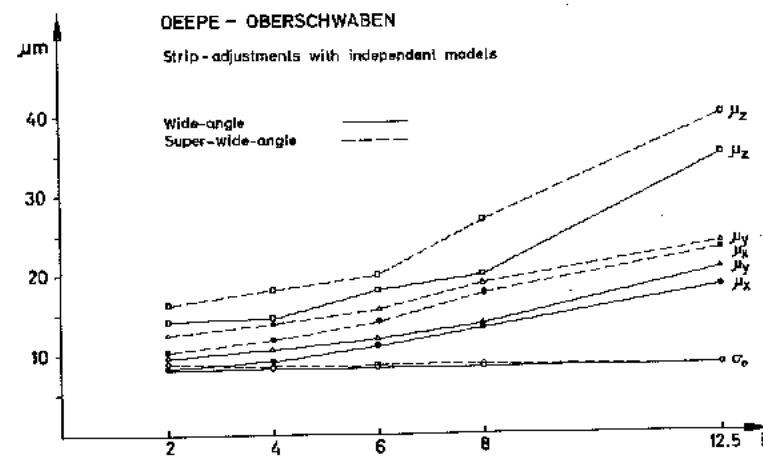


Figure 8

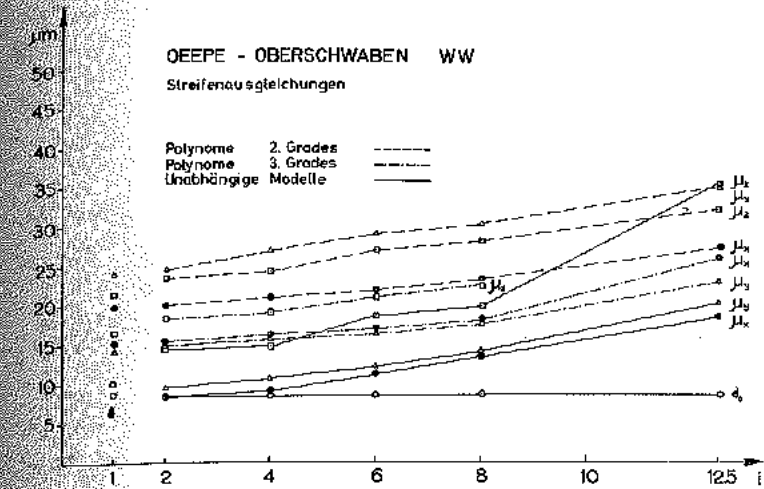


Figure 9

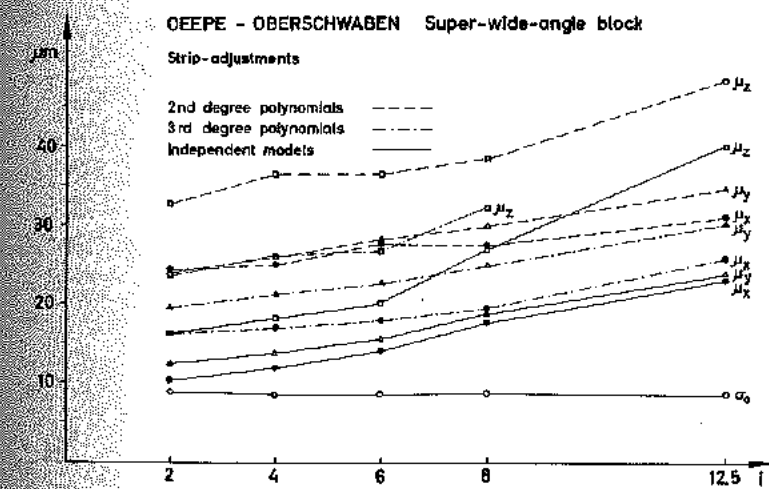


Figure 10

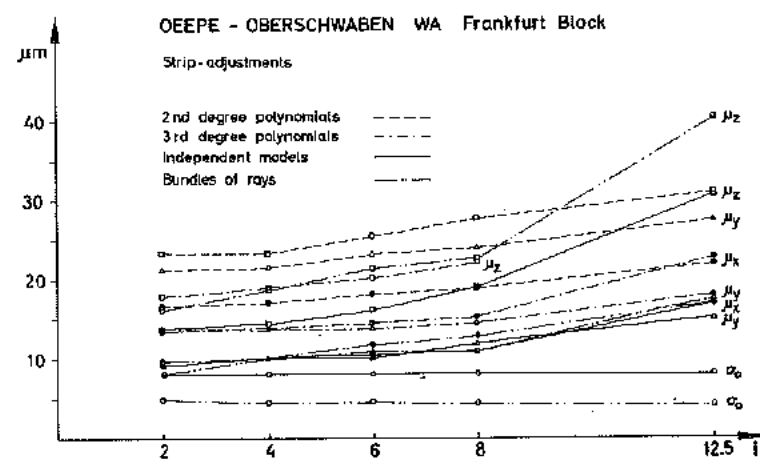


Figure 11

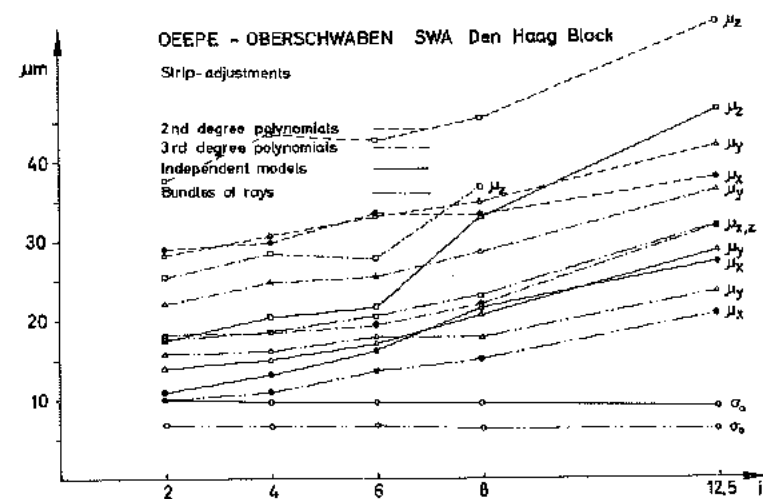


Figure 12

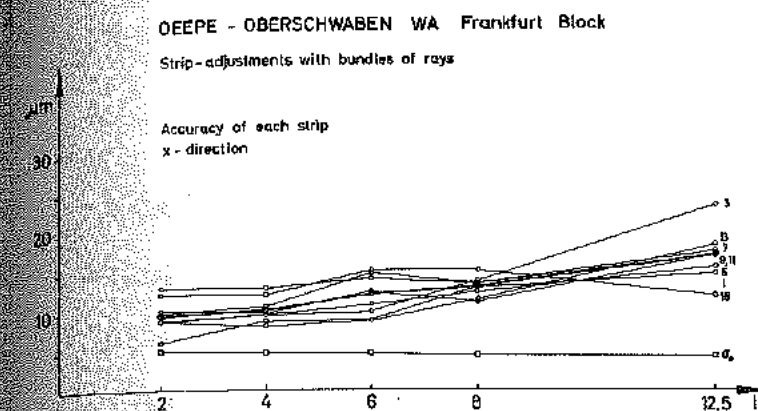


Figure 13

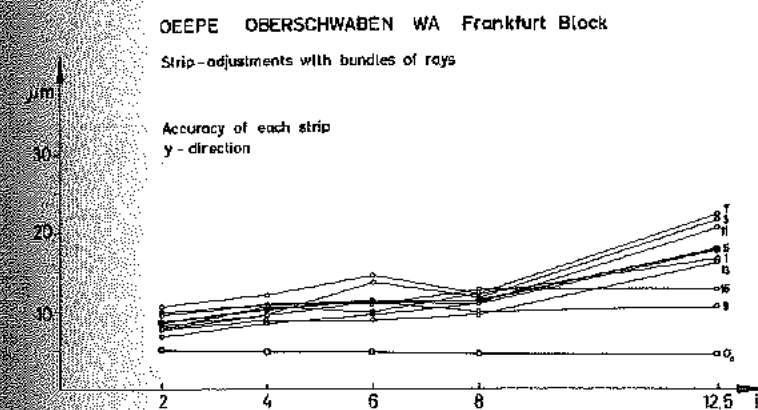


Figure 14

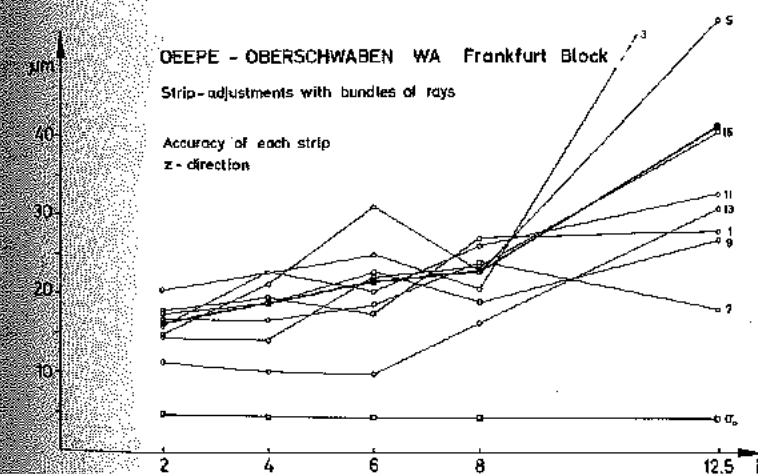
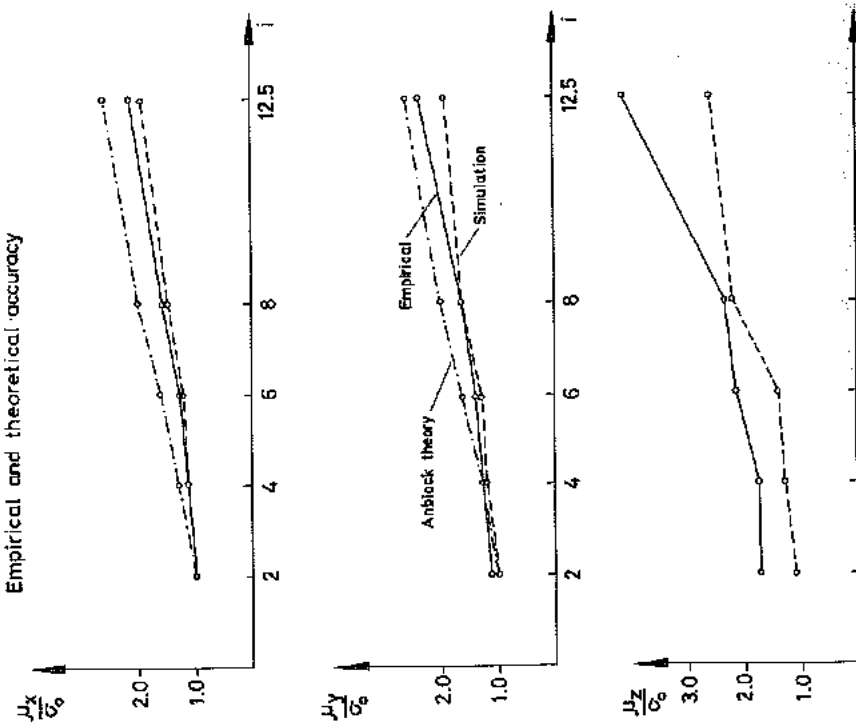


Figure 15

OEEPE - OBERSCHWABEN

Wide-angle blocks

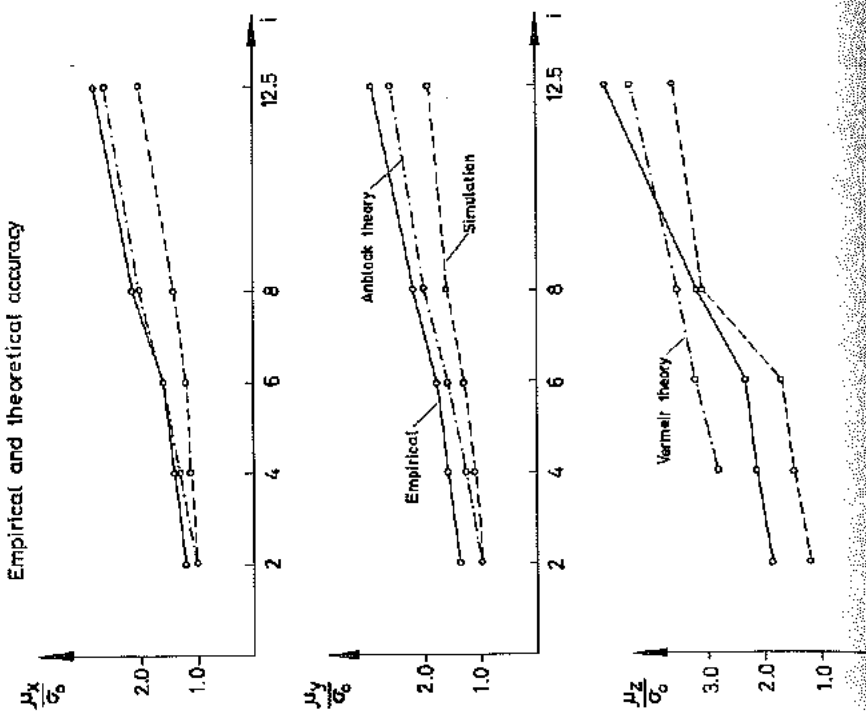
Strip-adjustment with independent models
Empirical and theoretical accuracy



OEEPE - OBERSCHWABEN

Super-wide-angle blocks

Strip-adjustment with independent models
Empirical and theoretical accuracy



OEEPE - OBERSCHWABEN

Frankfurt block - Den Haag block

Strip-adjustment with independent models

Empirical and theoretical values of the maximum standard errors in planimetry x,y

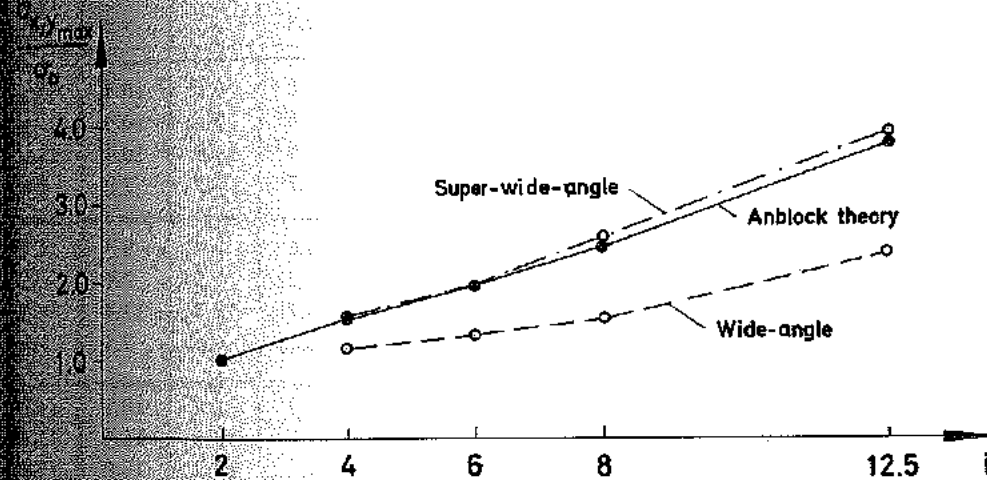


Figure 18

Table 1 — OEBPE Oberschwaben — Frankfurt block

(Wide-angle, 1 : 28 000, 8 strips with 25 models each)

Absolute accuracy of strip-adjustment with independent models for different intervals

Data in μm , referring to the photo scale

Control interval i (base lengths)	strip 1	strip 3	strip 5	strip 7	strip 9	strip 11	strip 13	strip 15
$i = 1$								
μ_x	6.4	7.0	6.4	6.4	6.8	7.9	5.3	7.1
μ_y	5.9	6.9	5.5	6.5	7.4	9.3	5.7	6.8
μ_z	9.2	12.0	10.4	13.2	11.3	10.2	7.1	7.2
σ_0	8.8	10.2	8.7	9.0	9.4	10.3	6.8	7.7
$i = 2$								
μ_x	9.6	8.2	8.6	7.1	11.1	11.4	6.1	10.4
μ_y	7.9	8.9	8.2	7.1	11.1	13.9	7.9	10.4
μ_z	11.8	15.0	13.9	17.9	17.5	15.4	10.4	9.3
σ_0	9.0	10.1	8.6	8.3	8.7	9.8	6.7	7.2
$i = 4$								
μ_x	11.4	9.6	8.9	10.0	10.4	11.8	6.8	10.4
μ_y	8.9	9.3	8.6	10.0	10.7	14.6	8.6	10.0
μ_z	13.9	15.7	13.9	20.0	16.4	16.4	12.1	10.4
σ_0	8.9	10.1	8.6	8.0	8.7	9.7	6.7	7.2
$i = 6$								
μ_x	13.2	13.9	9.6	10.7	9.6	11.8	6.4	9.6
μ_y	8.2	10.7	8.2	11.4	11.8	14.3	8.2	9.6
μ_z	18.9	17.5	17.5	20.4	15.7	15.7	12.5	11.4
σ_0	8.9	10.0	8.5	7.9	8.8	10.1	6.6	7.4
$i = 8$								
μ_x	11.4	12.5	10.4	11.8	11.8	13.2	8.2	10.0
μ_y	10.7	10.7	10.0	9.3	11.4	15.7	12.5	12.5
μ_z	21.8	17.5	14.3	20.7	18.9	24.6	18.2	15.0
σ_0	9.0	9.9	8.6	8.0	8.7	9.8	6.6	7.3
$i = 12.5$								
μ_x	15.0	20.4	13.2	17.5	13.9	25.7	17.1	10.4
μ_y	13.9	16.8	10.0	11.4	15.0	20.7	19.3	14.3
μ_z	32.5	27.5	41.4	25.0	28.2	40.7	34.3	13.2
σ_0	8.9	9.9	8.5	7.9	8.7	9.7	6.6	7.4

 $i = 1$ means all control points are used.

OEBPE Oberschwaben — Wide-angle block

Absolute accuracy of different strip-adjustment methods for different control intervals

Data in μm , referring to the photo scale 1 : 28 000

Control interval i (base lengths)	Frankfurt block (8 strips) bundles	indep. models	2 nd deg. polynom.	3 rd deg. polynom.	Wien block (7 strips) indep. models	2 nd deg. polynom.	3 rd deg. polynom.
$i = 1$							
μ_x	—	6.6	16.6	13.1	6.4	22.6	16.7
μ_y	—	6.7	19.9	12.6	6.8	28.1	15.8
μ_z	—	10.1	21.0	16.4	10.0	21.7	16.2
σ_0	—	8.8	—	—	8.9	—	—
$i = 2$							
μ_x	9.7	9.1	16.8	13.8	7.5	22.8	17.1
μ_y	8.4	9.5	21.4	14.0	9.6	27.4	15.4
μ_z	16.2	14.0	23.1	17.9	14.7	23.9	18.6
σ_0	5.1	8.4	—	—	8.6	—	—
$i = 4$							
μ_x	10.1	10.0	17.5	14.3	8.9	23.9	17.8
μ_y	9.9	10.2	21.6	14.0	11.1	31.2	17.4
μ_z	18.8	14.7	24.4	19.0	14.7	24.4	19.1
σ_0	4.6	8.4	—	—	8.6	—	—
$i = 6$							
μ_x	12.0	10.5	18.5	14.7	11.4	24.6	18.5
μ_y	11.1	10.2	23.1	13.9	13.9	34.2	18.4
μ_z	21.7	16.2	25.6	20.4	20.4	27.4	21.3
σ_0	4.6	8.4	—	—	8.6	—	—
$i = 8$							
μ_x	12.9	11.2	18.9	15.3	15.4	27.2	20.3
μ_y	11.2	11.6	23.9	14.7	16.1	35.9	20.3
μ_z	22.8	19.0	27.8	22.2	20.4	27.5	22.6
σ_0	4.4	8.4	—	—	8.6	—	—
$i = 12.5$							
μ_x	17.1	16.8	22.0	22.6	19.7	31.9	28.9
μ_y	17.5	15.4	27.7	17.8	26.0	41.5	27.0
μ_z	41.1	30.9	30.8	—	39.3	32.5	—
σ_0	4.3	8.4	—	—	8.6	—	—

 $i = 1$ means all control points are used.

Table 3 — OEEPE Oberschwaben — Super-wide-angle block
Absolute accuracy of different strip-adjustment methods for different control intervals
Data in μm , referring to the photo scale 1 : 28 000

Control interval i (base lengths)		Den Haag block (8 strips)				Delft block (6 strips)			
		bundles	indep. models	2 nd deg. polynom.	3 rd deg. polynom.	indep. models	2 nd deg. polynom.	3 rd deg. polynom.	
$i = 1$	μ_x	—	8.4	27.8	16.2	7.1	17.8	13.1	13.1
	μ_y	—	9.9	28.6	22.4	8.2	19.1	16.5	16.5
	μ_z	—	11.1	34.0	22.1	9.6	24.5	19.1	19.1
	σ_0	—	10.2	—	—	8.7	—	—	—
$i = 2$	μ_x	9.9	11.1	29.0	17.9	9.6	18.4	13.1	13.1
	μ_y	15.7	13.9	28.4	22.2	11.1	18.9	16.5	16.5
	μ_z	17.6	17.5	37.8	25.2	15.4	26.6	22.1	22.1
	σ_0	7.1	9.6	—	—	8.1	—	—	—
$i = 4$	μ_x	11.0	13.2	29.8	18.5	10.7	19.2	13.1	13.1
	μ_y	16.1	15.0	30.7	24.6	12.8	20.2	16.5	16.5
	μ_z	18.5	20.4	43.5	28.4	16.4	27.7	22.1	22.1
	σ_0	6.7	9.4	—	—	7.9	—	—	—
$i = 6$	μ_x	13.6	16.1	33.2	19.5	12.1	20.6	13.1	13.1
	μ_y	17.6	17.1	33.1	25.3	14.3	22.1	16.5	16.5
	μ_z	20.3	21.8	42.8	27.8	18.2	28.6	22.1	22.1
	σ_0	6.6	9.3	—	—	7.9	—	—	—
$i = 8$	μ_x	15.1	21.4	33.0	21.6	14.3	20.9	13.1	13.1
	μ_y	17.9	20.7	34.6	28.5	17.2	24.1	16.5	16.5
	μ_z	23.1	32.9	45.4	36.7	20.4	29.6	22.1	22.1
	σ_0	6.5	9.2	—	—	7.9	—	—	—
$i = 12.5$	μ_x	20.5	27.1	37.8	31.7	18.9	22.6	13.1	13.1
	μ_y	23.1	28.6	41.6	36.3	19.3	26.0	16.5	16.5
	μ_z	31.7	46.4	57.1	—	33.9	37.2	22.1	22.1
	σ_0	6.4	9.1	—	—	7.8	—	—	—

$i = 1$ means all control points are used.

Table 4 — OEEPE Oberschwaben
(Wide-angle and super-wide-angle, 1 : 28 000)
Absolute accuracy of different strip-adjustment methods for different control intervals
Data in μm , referring to the photo scale

Control interval i (base lengths)		Wide-angle block			Super-wide-angle block		
		indep. models	2 nd deg. polynom.	3 rd deg. polynom.	indep. models	2 nd deg. polynom.	3 rd deg. polynom.
$i = 1$	μ_x	6.5	19.6	14.9	7.8	23.3	15.0
	μ_y	6.7	24.1	14.2	9.2	24.3	18.5
	μ_z	10.1	21.3	16.3	10.4	29.6	21.0
	σ_0	8.8	—	—	9.5	—	—
$i = 2$	μ_x	8.4	19.8	15.4	10.4	24.3	16.3
	μ_y	9.5	24.4	14.7	12.6	24.1	19.6
	μ_z	14.3	23.5	18.2	16.5	32.7	24.0
	σ_0	8.5	—	—	8.9	—	—
$i = 4$	μ_x	9.5	20.7	16.0	12.0	25.1	17.0
	μ_y	10.6	26.5	15.7	13.9	26.0	21.3
	μ_z	14.7	24.4	19.0	18.5	36.5	26.0
	σ_0	8.5	—	—	8.7	—	—
$i = 6$	μ_x	10.9	21.6	16.6	14.2	27.6	17.9
	μ_y	12.1	28.8	16.2	15.8	28.1	22.6
	μ_z	18.3	26.5	20.8	20.1	36.4	26.6
	σ_0	8.5	—	—	8.6	—	—
$i = 8$	μ_x	13.3	23.1	17.8	18.2	27.3	19.2
	μ_y	13.9	30.3	17.5	19.0	29.8	25.0
	μ_z	19.7	27.7	22.4	27.4	38.3	32.2
	σ_0	8.5	—	—	8.6	—	—
$i = 12.5$	μ_x	18.2	27.1	25.7	23.4	31.1	25.6
	μ_y	20.1	34.8	22.6	24.4	34.7	30.1
	μ_z	35.1	31.6	—	40.6	48.2	—
	σ_0	8.5	—	—	8.5	—	—

$i = 1$ means all control points are used.

Table 5 — OEEPE Oberschwaben — Frankfurt block

(Wide-angle, 1 : 28 000, 8 strips with 25 models each)

Absolute accuracy of different strip-adjustment methods for different control intervals

Data in μm , referring to the photo scale

Control interval i (base lengths)	Adjustment with											
	bundles of rays	indep. models	2 nd degree polynomials				3 rd degree polynomials					
			(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)		
$i = 1$	μ_x	—	6.6	16.6	15.3	16.7	16.7	13.1	12.0	13.3	13.3	13.3
	μ_y	—	6.7	19.9	18.7	19.4	19.3	12.6	10.5	11.9	11.9	11.9
	μ_z	—	10.1	21.0	21.0	21.0	21.0	16.4	16.4	16.4	16.4	16.4
	σ_0	—	8.8	—	—	—	—	—	—	—	—	—
$i = 2$	μ_x	9.7	9.1	16.8	16.0	16.9	16.9	13.8	12.8	14.0	13.9	13.9
	μ_y	8.4	9.5	21.4	20.2	20.9	20.6	14.0	12.2	13.3	12.7	12.7
	μ_z	16.2	14.0	23.1	23.0	23.0	23.0	17.9	17.8	17.8	17.8	17.8
	σ_0	5.1	8.4	—	—	—	—	—	—	—	—	—
$i = 4$	μ_x	10.1	10.0	17.5	16.9	17.7	17.7	14.3	13.8	14.4	14.4	14.4
	μ_y	9.9	10.2	21.6	20.5	21.0	20.8	14.0	12.7	13.4	12.7	12.7
	μ_z	18.8	14.7	24.4	24.2	24.2	24.2	19.0	19.0	19.0	19.0	19.0
	σ_0	4.6	8.4	—	—	—	—	—	—	—	—	—
$i = 6$	μ_x	12.0	10.5	18.5	17.9	18.6	18.6	14.7	14.3	14.9	14.9	14.9
	μ_y	11.1	10.2	23.1	21.8	22.6	22.6	13.9	12.9	13.3	13.3	13.3
	μ_z	21.7	16.2	25.6	25.5	25.5	25.5	20.4	20.4	20.4	20.4	20.4
	σ_0	4.6	8.4	—	—	—	—	—	—	—	—	—
$i = 8$	μ_x	12.9	11.2	18.9	18.5	19.0	19.1	15.3	16.5	15.5	15.5	15.5
	μ_y	11.2	11.6	23.9	23.1	23.3	23.1	14.7	14.3	14.1	13.7	13.7
	μ_z	22.8	19.0	27.8	27.7	27.7	27.7	22.2	22.2	22.2	22.2	22.2
	σ_0	4.4	8.4	—	—	—	—	—	—	—	—	—
$i = 12.5$	μ_x	17.1	16.8	22.0	22.5	22.2	22.2	22.6	31.6	22.4	22.6	22.6
	μ_y	17.5	15.4	27.7	27.3	27.2	27.0	17.8	20.1	17.0	16.8	16.8
	μ_z	41.1	30.9	30.8	30.8	30.8	30.8	—	—	—	—	—
	σ_0	4.3	8.4	—	—	—	—	—	—	—	—	—

 $i = 1$ means all control points are used.

- (1) x, y, z in common
 (2) x, y, z separately
 (3) $x, y - z$, planimetry and elevation separately
 (4) same as (3), x, y conformal.

Table 6 — OEEPE Oberschwaben — Den Haag block

(Super-wide-angle, 1 : 28 000, 8 strips with 25 models each)

Absolute accuracy of different strip-adjustment methods for different control intervals

Data in μm , referring to the photo scale

Control interval i (base lengths)	Adjustment with											
	bundles of rays	indep. models	2 nd degree polynomials				3 rd degree polynomials					
			(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)		
$i = 1$	μ_x	—	8.4	27.8	27.0	27.8	27.6	16.2	15.3	16.2	16.0	16.0
	μ_y	—	9.9	28.6	26.9	28.3	28.2	22.4	20.0	21.9	21.9	21.9
	μ_z	—	11.1	34.0	33.9	33.9	33.9	22.1	22.3	22.3	22.3	22.3
	σ_0	—	10.2	—	—	—	—	—	—	—	—	—
$i = 2$	μ_x	9.9	11.1	29.0	28.5	28.9	28.6	17.9	17.6	17.9	17.6	17.6
	μ_y	15.7	13.9	28.4	26.9	28.1	28.0	22.2	20.5	21.7	21.6	21.6
	μ_z	17.6	17.5	37.8	37.8	37.8	37.8	25.2	25.3	25.3	25.3	25.3
	σ_0	7.1	9.6	—	—	—	—	—	—	—	—	—
$i = 4$	μ_x	11.0	13.2	29.8	29.4	29.7	29.6	18.5	18.2	18.5	18.4	18.4
	μ_y	16.1	15.0	30.7	29.2	30.3	30.3	24.6	23.5	24.0	23.9	23.9
	μ_z	18.5	20.4	43.5	43.4	43.4	43.4	28.4	28.5	28.5	28.5	28.5
	σ_0	6.7	9.4	—	—	—	—	—	—	—	—	—
$i = 6$	μ_x	13.6	16.1	32.2	32.5	32.1	32.0	19.5	19.5	19.5	19.4	19.4
	μ_y	17.6	17.1	33.1	33.1	32.7	32.7	25.3	25.7	24.9	24.8	24.8
	μ_z	20.3	21.8	42.8	42.7	42.7	42.7	27.8	27.9	27.9	27.9	27.9
	σ_0	6.6	9.3	—	—	—	—	—	—	—	—	—
$i = 8$	μ_x	15.1	21.4	33.0	32.6	32.8	32.8	21.6	22.2	21.7	21.7	21.7
	μ_y	17.9	20.7	34.6	35.1	34.3	34.3	28.5	31.5	28.2	28.2	28.2
	μ_z	23.1	32.9	45.4	45.2	45.2	45.2	36.7	36.6	36.6	36.6	36.6
	σ_0	6.5	9.2	—	—	—	—	—	—	—	—	—
$i = 12.5$	μ_x	20.5	27.1	37.8	38.2	37.6	37.6	31.7	29.4	31.4	30.4	30.4
	μ_y	23.1	28.6	41.6	43.1	41.5	41.5	36.3	52.1	35.5	35.1	35.1
	μ_z	31.7	46.4	57.1	56.9	56.9	56.9	—	—	—	—	—
	σ_0	6.4	9.1	—	—	—	—	—	—	—	—	—

 $i = 1$ means all control points are used.

- (1) x, y, z in common
 (2) x, y, z separately
 (3) $x, y - z$, planimetry and elevation separately
 (4) same as (3), x, y conformal.

Table 7 — OEEPE Oberschwaben — Frankfurt block

(Wide-angle, 1 : 28 000, 8 strips with 25 models each)

Absolute accuracy of strip-adjustment with bundles of rays for different control intervals

Data in μm , referring to the photo scale

Control interval i (base lengths)	strip 1	strip 3	strip 5	strip 7	strip 9	strip 11	strip 13	strip 15	mean value
$i = 2$									
μ_x	8.6	8.6	8.6	9.6	10.0	12.8	6.1	12.1	9.5
μ_y	10.4	7.5	6.8	7.5	7.9	9.3	7.5	9.6	8.3
μ_z	17.9	16.1	15.0	16.8	17.5	20.7	11.4	14.6	15.5
σ_0	5.2	5.5	5.1	5.0	5.2	5.7	4.4	4.7	5.1
$i = 4$									
μ_x	7.9	9.3	9.3	10.4	9.6	12.8	8.6	12.1	10.0
μ_y	11.8	9.3	8.2	10.4	9.3	10.7	8.6	10.7	9.6
μ_z	19.6	22.8	21.4	16.8	18.9	23.2	10.4	14.3	18.0
σ_0	4.6	4.9	4.7	4.6	4.9	4.9	4.1	4.3	4.6
$i = 6$									
μ_x	8.6	9.6	10.7	14.6	12.5	13.9	8.6	15.0	11.2
μ_y	14.3	13.2	9.3	9.6	11.1	11.1	8.6	10.7	11.2
μ_z	17.5	25.0	31.1	18.6	22.8	20.4	10.0	22.2	20.5
σ_0	4.6	4.7	4.5	4.1	4.8	5.4	4.0	4.3	4.6
$i = 8$									
μ_x	12.8	13.6	12.5	13.2	11.1	13.2	11.4	15.0	12.8
μ_y	12.1	11.4	10.7	12.1	9.6	10.7	9.3	12.5	11.1
μ_z	27.1	21.8	22.8	23.9	18.9	26.0	16.4	23.6	22.5
σ_0	4.4	4.4	4.4	4.1	4.6	4.8	4.0	4.1	4.4
$i = 12.5$									
μ_x	14.6	23.2	15.4	17.5	17.1	17.1	18.2	11.8	17.2
μ_y	16.4	21.4	17.5	22.2	10.4	20.4	16.1	12.5	17.2
μ_z	27.8	70.7	54.4	17.8	26.8	32.5	30.7	40.4	31.1
σ_0	4.4	4.3	4.4	4.0	4.6	4.7	4.0	4.2	4.4

OEEPE Oberschwaben

(Wide-angle and super-wide-angle, 1 : 28 000)

Theoretical and empirical accuracy of strip-adjustment with independent models for different control intervals

Values of $\frac{\mu}{\sigma_0}$

Control interval i (base lengths)	Wide-angle block				Super-wide-angle block			
	empirical	simulation	anblock theory	Vermeir theory	empirical	simulation	anblock theory	Vermeir theory
$i = 2$								
μ_x/σ_0	1.0	1.0	1.0	—	1.2	1.0	1.0	—
μ_y/σ_0	1.1	1.0	1.0	—	1.4	1.0	1.0	—
μ_z/σ_0	1.7	1.1	—	—	1.9	1.2	—	—
$i = 4$								
μ_x/σ_0	1.1	1.1	1.3	—	1.4	1.1	1.3	—
μ_y/σ_0	1.2	1.2	1.3	—	1.6	1.1	1.3	—
μ_z/σ_0	1.7	1.3	—	4.9	2.1	1.5	—	2.8
$i = 6$								
μ_x/σ_0	1.3	1.2	1.6	—	1.6	1.2	1.6	—
μ_y/σ_0	1.4	1.3	1.6	—	1.8	1.3	1.6	—
μ_z/σ_0	2.2	1.4	—	5.6	2.3	1.7	—	3.2
$i = 8$								
μ_x/σ_0	1.6	1.5	2.0	—	2.1	1.4	2.0	—
μ_y/σ_0	1.6	1.6	2.0	—	2.2	1.6	2.0	—
μ_z/σ_0	2.3	2.2	—	6.0	3.2	3.1	—	3.5
$i = 12.5$								
μ_x/σ_0	2.1	1.9	2.6	—	2.8	2.0	2.6	—
μ_y/σ_0	2.4	1.9	2.6	—	2.9	1.9	2.6	—
μ_z/σ_0	4.1	2.6	—	7.4	4.8	3.6	—	4.3

Discussion about the Subjects 1-3 under the Chairmanship of Prof. H. G. Jerie

Chairman: Is there any discussion on the scope of the Oberschwaben programme as outlined by *Ackermann*? As this does not seem to be the case, are there any questions on *Belzner's* paper dealing with the preparation and flying? This seems to have been clear enough so let us proceed to the discussion on *Stark's* paper on the strip adjustment results.

Wahlhausl: Since the reliability of polynomial strip adjustments depends upon the length of the strip, I would suggest that the accuracy investigations of the polynomial adjustments should have been confined to strips of 10-15 models in length.

Ackermann: The test was not set up in order to thoroughly investigate the polynomial method. The results stand on their own merit. Of course, many more cases could and should be computed.

Bauer: What is the absolute accuracy of the control points used in the test field and how was this established?

Belzner: The State Survey Office of Baden-Württemberg claim a planimetry accuracy of 5-10 cm and a height accuracy of 20 cm for the trigonometric network. We have not checked this.

Bauer: The 550 control points available in an area of 2500 km² give a fairly low density of one point per 5 km².

Belzner: Only readily identifiable trigonometrical points were selected and signalised.

Ackermann: When the test was set up, we did not want the photogrammetric results to be disturbed by an eventual inaccuracy of the ground control. However, the originally expected accuracy of 20 μ m (0.6 m on the ground) has been superceded. Nevertheless the relative accuracy of the ground control is certainly sufficient, being in the neighbourhood of 10 cm which is only 3 μ m at photo scale.

Kupfer: There is a distinct difference in these results with those of *Meier* on the comparison of WA and SWA photography.

Stark: *Meier* used measurements with the Planimat. He obtained about equal accuracies for WA and SWA photography.

Proctor: Are the tests to be extended using natural or pricked tie points as opposed to the sole use of signalised tie points in these investigations?

Ackermann: Ordinary methods of point transfer are included in the investigations. Preliminary results indicate, however, that there is only a slight deterioration of about 10 % in the final accuracy.

Konecny: I would like to ask Mr. *Belzner* whether the misidentification of tie points was due to the small size of signal used, namely 60 cm?

Belzner: In spite of their small size, most of the points were identifiable since we used double points.

Ackermann: The philosophy behind the signalisation was due to the rather poor experience with previous OEEPE test fields. Now, since we were dealing with such a large number of measurements, we did not have time to investigate doubtful points and merely rejected them.

Verlaine: Am I correct in inferring from Prof. *Ackermann's* introduction that he considers the lack of improvement in using a bundle adjustment to be due to the systematic errors in the strip?

Ackermann: I think this is indeed due to systematic image errors which the bundle adjustment is not as capable of absorbing as the independent model adjustment.

Baetsle: I come to the same conclusion. The stochastic model must be correct if one wants good results.

Ackermann: When *Stark* says that rigorous methods give good results this does not exclude the possibility of obtaining better results with an improved method.

Baetsle: Or by using more complicated models with more unknowns?

Ackermann: Although the stochastic model is simplified, the bundle adjustment itself is fairly rigorous.

Baetsle: Perhaps we have to define the term "rigorous"?

Chairman: A possible definition is: "the best we know at present".

Ackermann: "Rigorous" means either sufficiently rigorous, generally accepted as rigorous or as rigorous as possible, for the time being. The real problem is in choosing significant, realistic, and workable mathematical models.

Gotthardt: I find it difficult to relate the interpolation formulae in *Stark's* paper with the results presented in table 5.

Stark: The 2nd and 3rd degree polynomials are tabulated separately.

Leberl: I have a question on the presentation of the results in the form of mean square values of the coordinate residuals. Could not the correlation between the residuals at different points partly explain the difference found between the results of the bundle and model adjustments?

Ackermann: This may be so.

Amer: Is there an explanation for the significant reduction in height accuracy in both the model and bundle adjustment when the bridging distance increases from 8 to 12.5 models?

Stark: I cannot offer an explanation.

Ackermann: Considering the theoretical expectation and the sample size I see no special necessity for an explanation.

Fürstner: I would like to ask Dr. *Stark* two questions. Firstly, what was the y-parallax accuracy in the Vienna block and secondly, what does σ_0 in a bundle adjustment imply, compared to y-parallaxes?

Stark: The y-parallax accuracy of the Vienna block was also about 4 μm . As to your second question, σ_0 in a bundle adjustment represents the standard error of a single plate coordinate, either x or y.

Chairman: Unfortunately our time is up and so after thanking the three authors for their presentation of their papers and you, the audience, for participating in the discussion, I hereby close this session.

TP 528.735.2-187(430-313.5):528.14

Testblock Oberschwaben, Program I

Results of Block-Adjustment by Independent Models

(with 27 figures, 17 tables)

By H. Ackermann, Stuttgart

RESUME see p. 104

1. Introduction

1.1. This paper deals with the results of the block-adjustments by the method of independent models, as part of the Oberschwaben test program I of statistical investigations into the accuracy of blocks.

The general description of the Oberschwaben program will not be repeated any more. It is assumed that the participants of the symposium are informed, at this stage, about the scope of the tests, the test field, the photography, and about the stereocomparator measurements up to the analytical computation of independent models. It is also assumed that the participants are familiar with the principles of block-adjustment by the method of independent models.

1.2. A few essentials are briefly recalled. With regard to photography and photo-coverage we have (see [1], [2])

- wide-angle photography, $f = 153$ mm, photo scale $1 : 28,000$, relative flying height $h = 4284$ m, pin-point exposures;

- 15 strips of 25 models each, total 375 models, lateral overlap $q = 61\%$, longitudinal overlap $p = 61\%$, base-length $b = 2.500$ km;

- the total block of 375 models is subdivided in 2 blocks of $q = 22\%$ lateral overlap each, with, respectively, 8 or 7 strips: uneven strip numbers, $8 \times 25 = 200$ models, block Frankfurt; even strip numbers, $7 \times 25 = 175$ models, block Vienna;

- super-wide-angle photography, $f = 85$ mm, photo-scale $1 : 28,000$, relative flying height $h = 2380$ m; pin-point exposures;

- 15 strips of 25 models each; total 375 models, lateral overlap $q = 61\%$, longitudinal overlap $p = 61\%$, base-length $b = 2.500$ km;

- the total block of 375 models is subdivided in 2 blocks of $q = 22\%$ lateral overlap each, with, respectively, 8 or 7 strips: uneven strip numbers, $8 \times 25 = 200$ models, block The Hague; even strip numbers, $7 \times 25 = 175$ models, block Delft.

The points in the terrain were targetted:

- 548 trigonometric points, evenly distributed, to be used as planimetric control- and check-points; of 484 of them also the heights are given;

— 438 tie-points, targetted by double signals, 30 m apart;

thus, most of the models contained $6 \times 2 = 12$ targetted tie-points each, located in 6 standard positions.

All the measurements were done by stereocomparator, in 4 different centres (Frankfurt, Vienna, The Hague, Delft).

The comparator coordinates were

- transformed into the plate coordinate systems by plane similarity transformations (4-parameter-transformations) and
- corrected for radial lens distortion, refraction, and earth curvature.

The corrected plate coordinates were processed to model coordinates, by computer independent models through analytical relative orientation, utilizing all measured points (rather than a selection of 6 points only). Each of the resulting models consists of

- 2 perspective centres (air stations)
- an average of about 15 model points.

The recordings of a model contain the numbers of the points, and their x -, y -, z -coordinates, referring to the independent rectangular model-coordinate-system. These data, completed by constants and control data, formed the input-data for the block-adjustments by the method of independent models.

1.3. The pilot centre Stuttgart (Photogrammetric Institute of Stuttgart University (and still continues) the computations of the block-adjustments by applying its computer program PAT-M-43 for block-adjustment by the method of independent models, see [1]. The Control Data CDC 6600 computer of the University Computing Centre has been used for the quite extended series of computations.

The block-adjustment of independent models consists, in principle, of simultaneous 3-dimensional similarity transformations (7-parameter-transformations) of all models, taking all tie-points (including air stations) and control-points into account. The computer program PAT-M-43 achieves the 3-dimensional adjustment by iteration between planimetric and height adjustment, and by relineralisation and updating the transformations in between the iterations. Usually 2–3 iteration steps were sufficient.

For the various block-adjustments the input data were weighted as follows:

- coordinates of model points
 x, y weight 1; z weight 1
- coordinates of perspective centres, according to previous investigations,
 x, y weight 0.25; z weight 1
- control coordinates
 X, Y weight ∞ ; Z weight ∞

coordinates of check-points

x, y weight 0; z weight 0

check-points are formally treated as control-points with weight 0, thus they do not influence the adjustment; their error statistics are treated separately).

The printout of the adjustments gives error statistics, coordinates of (transformed and/or) adjusted points, residual errors at tie- and control-points. Due to the separation of planimetric and height adjustment within PAT-M-43 standard errors of unit weight are obtained separately for planimetry (σ_{0P}) and heights (σ_{0H}). The weight units relate to the precision of model coordinates, separately for planimetry and for heights.

Other features of the computer program, such as optimisation of band width and direct solution of normal equations are here of no special concern. It may be mentioned, however, that the computing times, for block-adjustments with independent models, with CDC 6600, for 3 iterations, amounted to about 1.5 sec (system time) per model.

At the beginning of the investigations also a preliminary version of the computer program PAT-M-4 was used for the planimetric block-adjustment by independent models which is also known as the Anblock-adjustment.

1.4. Before the actual series of block-adjustments started the data errors had to be filtered out. For this purpose the 0-versions of the block-adjustments were computed first in which all given points were introduced as control-points. All model points were discarded of which at least one of the residual coordinate errors exceeded the tolerance of $2 \cdot \sigma_0$. With the statistical aim of the investigation in mind no special efforts were made to save discarded points by clearing their particular source of error.

2. Notes on the Adjustments and on Notation

The investigation into the accuracy of the method of independent models is, in first instance, concerned with the separate or combined influence of 4 different groups of parameters on the planimetric and height accuracy of adjusted blocks:

- type of photography (wide-angle / super-wide-angle)
- lateral overlap ($q = 20\% / q = 60\%$)
- block-size
- pattern of control-points.

Before presenting the results, first a review is given of the notations used, and of the classification and the code-symbols of the various cases.

2.1. For an easy review the results of the various block-adjustments are, in this paper, expressed and summarized by the numerical values of a few essential terms only. The symbols used are:

σ_{0P} standard error of unit weight of the planimetric part of adjustment; expressed in units of cm and of μm with reference to the terrain and to the photo-scale, respectively

- σ_{0H} standard error of unit weight of the height adjustment; in cm and μm
- μ_x, μ_y root mean square values of the "true" x- and y-errors of the check-points, in cm and μm
- $\mu_{xy} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$ = root mean square value of the "true" planimetric coordinate errors of the check-points; in cm and μm ; is considered representative for absolute planimetric accuracy of the adjusted block
- μ_z root mean square value of the "true" height errors of the check-points, in cm and μm ; is considered representative for the absolute height accuracy of the adjusted block
- $\mu_{xy}/\sigma_{0P}, \mu_z/\sigma_{0H}$ absolute planimetric and height accuracy of the adjusted block, in cm of σ_0 , respectively; such terms are suited for theoretical evaluation.

2.2. For the investigation into the relation of accuracy and block-size, for various arrangements of control-points, the blocks are subdivided in subblocks. The total number of adjustments having not been completed, as yet, here only the subdivisions of the wide-angle block Frankfurt are represented. Fig. 1 shows the subdivisions used to distinguish 4 different sizes:

- $2 \times 4 = 8$ models, 12 subblocks no. 1-12
- $4 \times 8 = 32$ models, 6 subblocks no. I-VI
- $8 \times 17 = 136$ models, 2 subblocks A, B, partly overlapping
- $8 \times 25 = 200$ models, the block Frankfurt.

The subblocks are square shaped, or close to it.

2.3. Following theoretical knowledge (see [4]) the arrangements of planimetric control-points consider only distribution of control along the perimeter of a block. The various arrangements are distinguished only by the average distance of control-points along the perimeter. The distances are denoted by i and expressed in multiples of the block length. Apart from the version 0 which uses all known terrain points as control-points, distinguish 5 different versions of planimetric perimeter control. The versions 1-5 have average distances between control-points of 2, 4, 6, 8 base-lengths, respectively, see fig. 2. Version 5 is the case of 4 planimetric control-points only, located in the corners of a block.

In principle, those versions are maintained for different sizes of blocks. They are also used to describe appropriately the arrangement of planimetric control for blocks with 60 % lateral overlap or with an additional frame of models around the controlled area, see fig. 2. With blocks of 60 % lateral overlap it is preferable not to consider the corner areas which have no double coverage.

Note: The results concerning control version 5 (4 corners) of rectangular blocks $8 \times 25 = 200$ models will be plotted in the diagrams with respect to the average interval between control of $i = 21$ ($21 \approx$ quadratic mean of 16 and 25).

2.4. Contrary to planimetric control the height control-points have to be arranged in the traditional way in chains across the block, for $q = 20$ % lateral overlap, see [5]. Thus, the different versions can be clearly distinguished by the "bridging distance" i , which is the distance between chains of height control-points, expressed in base-lengths. Fig. 3 shows the control arrangements of versions 1 to 5, with bridging distances of $i = 2, 4, 6, 8, 12.5$ base-lengths, respectively. It also shows that along the perimeter the distance between control-points is only $i/2$, which is denoted by $i = 4(2), 6(3)$ etc. Those additional height control-points avoid poor height accuracy at the open edges of a block.

For blocks with 60 % lateral overlap the chains of height control-points can be dissolved to form a grid pattern of more or less square shaped meshes. Such an arrangement of height control is described, in fig. 4, by $i = 4/4, i = 6/5$ and $i = 8/7$ and is referred to as versions 3*, 4*, and 5*, respectively.

2.5. In the following chapters the accuracy results of block-adjustments by the method of independent models will be presented, as far as available up to now.

It is proposed to distinguish clearly between 3 different points of view, when evaluating the results:

The first task of this paper is the presentation of empirical results as such. It is the order of magnitude of the various accuracy values which is of special interest to the practitioner as well as to the theoretician. In the same sense some empirical relationships between certain parameters will be stated.

While there will be hardly doubts or arguments about the empirical results the real question will be the agreement with theory, or by how far the results can be considered representative.

It has been stressed several times that the final purpose of the Oberschwaben investigations is statistical testing of accuracy hypotheses, or, in other words, comparison of empirical results with theoretical prediction. Agreement would mean confirmation of the theory and justify its application also outside the tested range.

Comparison with theory will, in this paper, only be dealt with to some extent, for several reasons. Firstly, the preliminary results should be completed by the adjustments still to come, before far reaching conclusions can be drawn. Secondly, theoretical estimates are not available for all accuracy parameters, but rather for some of them only. And thirdly, there is not only one, generally accepted theory on the accuracy of block triangulation. Indeed, the results of the tests will motivate research into more refined theories.

The comparison of empirical results with (simplified) theoretical estimates will show agreement in some respect and disagreement in some others. There will be sufficient evidence that the disagreements are mainly caused by systematic and/or correlated errors (which also opens the question why there are agreements in other cases).

At the end of this paper a few remarks will be made, concerning the evidence of systematic image errors. A complete discussion, however, and the explanation of results should consider the combined results of the other presentations as well. Therefore conclusions can be drawn only at the end of the symposium.

2.6. The evaluation of the results has to consider that the test Oberschwaben corresponds in many respects to ordinary practical working conditions of aerial triangulation. In some other respects, however, special (ideal) test conditions are valid:

- special conditions: signalized points only, pin-point exposures; errors of terrestrial control are supposed to be negligible.
- standard or poor conditions: flying spread over 2 1/2 months (growing vegetation, signals destroyed), reflying due to gaps, large tilts; standard image quality of wide-angle photographs, poor image quality of super-wide-angle photographs (underexposed); stereocomparator measurements according to standard routine; radial symmetry; corrections of plate coordinates; standard method (Schut) for analytical relative orientation.

3. Preliminary Investigations

3.1. Independence of Planimetry and Heights

Theory predicts that the planimetric accuracy of an adjusted block is practically independent from the height accuracy, and vice versa, see [5]. The independence is theoretically valid for flat terrain.

The results of tables 1 and 2 confirm empirically, for the method of independent models, that indeed plan- and height accuracy have no mutual influence. For a given arrangement of planimetric control-points the planimetric accuracy of the adjusted block is not affected by the arrangement of height control. Similarly, for a given arrangement of height control-points the height accuracy of the adjusted block proves not to be dependent on whether the block is well or is poorly controlled in planimetry. The empirical confirmation extends to rather extreme cases, beyond control distributions which are likely to be met in practice. It also extends to both wide-angle- and super-wide-angle-blocks of independent models.

As a result, the following discussions can entirely separate considerations of planimetric accuracy from those of height accuracy.

3.2. Planimetric Block-Adjustment

During the early parts of the investigations the wide-angle models, in particular of the block Frankfurt, were used for a number of planimetric block-adjustments with the program PAT-M-4. The planimetric block-adjustment by independent models is also known as the "Anblock"-method. It deals with planimetric coordinates only and asks previous levelling of the independent models.

There is no point in comparing the empirical results of the PAT-M-4 adjustment with the planimetric part of corresponding PAT-M-43 adjustments. Here in particular both results would be exactly identical, as the planimetric iteration step within PAT-M-43 is, in fact, an Anblock-adjustment.

Although these preliminary adjustments have not been continued into the main series of investigations their results are briefly compiled in tables 3 and 4. They refer to partly different control-points than the later adjustments. They also refer to a slightly different mode of filtering out gross data errors, with less rigid rules than adopted later on. And finally, some of the subblocks were selected different from fig. 1. Thus, by comparing the results of the preliminary planimetric adjustments with the corresponding ones of tables 9, 10, 16, and 17 they can serve as interesting examples for the degree of variation to be expected from somewhat differing treatment of the same material.

Of course, the results of the planimetric block-adjustments are of independent interest too. Therefore, they are summarized in the tables 3 and 4, as well as graphically represented in the figures 5 and 6.

The results of the separate planimetric block-adjustments are in general and good agreement with the planimetric results of the 3-dimensional adjustment with independent models, see fig. 7, 25 and tables 6, 10, 16, 17. Therefore, they will not be discussed here.

It is only the case of a bordered block (see fig. 2) with an additional frame of models around the actually controlled area which needs some comments here. The understanding is, that the block-adjustment would utilize all models for the computation, whilst the resulting accuracy is considered of the inner parts of the block only and referred to as σ_{0p} . It has been theoretically predicted that, for otherwise equal circumstances, the additional frame improves the accuracy within the controlled area, see [4]. In practice, this property is relied upon. Instead of densifying planimetric perimeter control it is often expedient to expand the photo-coverage somewhat beyond the area of immediate interest.

The test-results from the bordered block show, as theoretically expected, a clear improvement for the case of 20 % lateral overlap. However, the block with 60 % lateral overlap displays the effect not very clearly.

3.3. Standard Errors of Unit Weight

In table 5 the standard errors of unit weight are compiled as obtained from the various block-adjustments to be treated in chapters 4 and 5.

Reviewing table 5 the results impress first by a remarkable consistency and also, perhaps, by rather small magnitudes. Until 2 years ago the values would have been considered exceptionally small. Recently similar or better values have been obtained more often (with signalized points!).

Of special interest is the ratio between the σ_0 -values for wide-angle and super-wide-angle. As expected, the planimetric values (σ_{0p}) for s. w. a. are consistently larger by about

30 %. This ratio is almost maintained also with blocks of 60 % side overlap. However, the precision values of the z -coordinates of model points, as indicated by σ_{0H} , are somewhat unexpected. The values of σ_{0H} range for wide-angle ($q = 20\%$) between $7.5\ \mu\text{m}$ and $9.6\ \mu\text{m}$ or 0.05‰ h and 0.06‰ h , which is considered very good. As expected, the σ_{0H} values for super-wide-angle are smaller, but the difference amounts only to about 13 % or $1\ \mu\text{m}$. Contrary to σ_{0P} the values of σ_{0H} are slightly increased to about $9\ \mu\text{m}$ by blocks of 60 % side-overlap and show no difference any more between w. a. and s. w. For a given block the values σ_{0P} and to a lesser degree also σ_{0H} are slightly influenced by the distribution of ground control. Fewer ground control reduce the values of σ_{0P} and σ_{0H} . If the theoretical assumptions of independent random errors alone would be valid the values of σ_0 would not be affected at all by variation of block-size, control, or overlap. If the σ_0 values are affected it must be due to either tensions in the terrestrial system of ground control and/or systematic or correlated errors in the photogrammetric data. The obviously small effects would seem to indicate the presence of small non-random effects only. Nevertheless, theoretical considerations indicate the effects as highly significant.

Even accepting the somewhat unexpected values for super-wide-angle the standard error of unit weight, as obtained from block-adjustments with independent models, indicate a high and consistent precision of the independent models, with some evidence of systematic error effects.

4. Variation of Control for Given Block-Size

4.1. The main series of block-adjustments with independent models is presented in this chapter. It deals with the effect of different arrangements of planimetric and height control-points on plan- and height-accuracy, respectively, of adjusted blocks. Here, the block-size is kept fixed; the adjustments refer always to the maximum available size (200 models and 175 models, respectively, for $q = 20\%$ side overlap; 375 models for $q = 60\%$ overlap) of the rectangular blocks Oberschwaben.

The variation of control goes through the versions 1 through 5, increasing gradually the distances i between planimetric perimeter control and the bridging distances i between chains or grid of height control. The results are listed in tables 6 through 15:

- table 6: wide-angle block Frankfurt, $q = 20\%$
- table 7: wide-angle block Vienna, $q = 20\%$
- table 8: super-wide-angle block The Hague, $q = 20\%$
- table 9: super-wide-angle block Delft, $q = 20\%$
- table 10: wide-angle block Oberschwaben, $q = 60\%$
- table 11: super-wide-angle block Oberschwaben, $q = 60\%$
- tables 12, 13: summaries of tables 6–9, $q = 20\%$
- tables 14, 15: summaries of tables 10, 11, $q = 60\%$

The results will be classified and reviewed according to various points of view.

4.2. Planimetric Accuracy, $q = 20\%$

The planimetric results of the adjustments of the 4 different blocks of 20 % side overlap are displayed graphically in figures 7 and 8.

Referring first to wide-angle (fig. 7) the overall results, expressed in μm , must be considered very satisfactory from a practitioner's point of view. Nevertheless a few facts which were not quite expected, can be pointed out:

The average accuracies of planimetric coordinates (μ_{xy}) of the 2 blocks Frankfurt and Vienna differ considerably, perhaps more than might have been expected. However, the differences are not indicated by the values of σ_{0P} .

Within each of the blocks the accuracies μ_x and μ_y of the x - and y -coordinates differ also considerably. It is particularly noticeable that the ratios between μ_x and μ_y are reversed between the blocks Frankfurt and Vienna.

There is no obvious explanation for such facts. It may be related to the strips running mostly north-south in one case and mostly south-north in the other (Frankfurt: 5 strips north-south, 3 strips south-north; Vienna: 5 strips south-north, 2 strips north-south), see fig. 9. Most likely the effect is just accidental, i. e. hardly statistically significant (will have to be checked by statistical tests).

As far as the general relationship between planimetric accuracy μ_{xy} and distance i of the planimetric control-points on the perimeter is concerned fig. 7 suggests a linear relationship, whilst theory predicts a parabola $\mu_{xy} \sim i^e$, $e > 1$.

The results of the super-wide-angle blocks The Hague and Delft (fig. 8) can, in first instance, be commented along the same lines as for the wide-angle blocks. Also here the 2 blocks differ considerably, although the values of σ_{0P} are practically equal. Also the differences between μ_x and μ_y are marked, although of lesser magnitude and of consistent sign. It is to be mentioned that all 8 strips of the block The Hague run north-south, whilst all 7 strips of the block Delft run south-north, see fig. 9.

The seemingly linear relationship between μ_{xy} and i of fig. 7 shows up in fig. 8 for super-wide-angle only up to $i = 8$. The case of 4 corner points does here not well agree with the extrapolation of the straight line, and even less with the theoretically expected parabola. This case carries little weight, however, having a large confidence interval.

4.3. Planimetric Accuracy, $q = 60\%$

The planimetric results of the adjustments of the total wide-angle block and of the total super-wide-angle block, each with 375 models and with 60 % side overlap, are displayed in tables 10, 11, and 14, and in fig. 10. As far as planimetric control is concerned the perimeter versions 1–5 were used, slightly modified to enclose the area of double overlap only, excluding the margins on 2 sides (see fig. 2).

The planimetric results, as presented in fig. 10 show some features which have already been met with blocks of 20 % side overlap. Here the super-wide-angle block has marked differences between μ_x and μ_y , whilst the wide-angle block does not show such effect.

The overall dependence of μ_{xy} on i is monotonously increasing. Whilst up to $i = 8$ straight lines would fit the curves very well the standard errors of the case of 4 corners are considerably smaller than linear extrapolation would suggest. But again this case carries the least significance.

The most important, and highly surprising, result is the empirical comparison of the 2 cases of overlap, see fig. 11. The planimetric accuracy of 60 % side overlap is only moderately improved compared with equivalent control versions of blocks with 20 % side overlap. This is quite contrary to the general expectation and also contrary to theoretical results which claim an improvement approximately by a factor 1.5 (33 %), see [5]. The empirical improvements are for

- wide-angle: 18 % up to $i = 8$, 41 % for version 5
- super-wide-angle: 16 % up to $i = 8$, 9 % for version 5.

4.4. Height Accuracy, $q = 20$ %

The results of the height accuracies of the adjusted blocks, with 20 % side overlap, are collected in tables 6–9, and 13 and displayed in fig. 12 and 13.

The first statement must be, that the obtained height accuracy is excellent, in particular of the wide-angle blocks; for instance, $\mu_z = 16 \mu\text{m} = 0.1\text{‰} h$ when bridging 6 models. The wide-angle blocks Frankfurt and Vienna show very good mutual agreement up to a bridging distance of 8 models. Only with the long bridging distance of 12.5 models (3 chains) the 2 blocks diverge somewhat, but rather not significantly. The dependency, for wide-angle, of height accuracy μ_z on bridging distance i is less strong as might, perhaps, have been expected. The ratio of $23 \mu\text{m} = 0.15\text{‰} h$ for $i = 12.5$ to $13 \mu\text{m} = 0.09\text{‰} h$ for $i = 2$ is only 1.8.

The very favourable and consistent results of the wide-angle blocks are not to the same degree shown by the super-wide-angle results (fig. 13, table 13). The 2 blocks The Hague and Delft diverge stronger, with the exception of short bridging distances. Also the dependence of height accuracy on bridging distance is somewhat more pronounced. Again, the ratios of σ_{0H} and μ_z of the 2 blocks are reversed.

4.5. Height Accuracy, $q = 60$ %

The height accuracies (μ_z) concerning the blocks with 60 % side overlap are summarized in fig. 14 and in tables 10, 11, and 15.

With height control arranged conventionally in chains across the block the height accuracy is, also for blocks with 60 % side overlap, a function of bridging distance i . Fig. 14 shows, however, that the errors increase rather rapidly with increasing bridging distance. This holds for both wide-angle and super-wide-angle blocks. When compared with the height accuracies of blocks of 20 % side overlap, for the same bridging distances, the results are startling. For short bridging distances ($i = 2$, $i = 4$) the blocks of 60 % lateral overlap have moderately increased height accuracy, compared with blocks of 20 % lateral overlap. The improvement amounts to only 14 % for wide-angle and 11 % for

super-wide-angle blocks. Such little improvement is even less than for planimetry, and it is definitely less than theory predicts, see [5], and what would generally have been expected. Most unexpected, however, are the empirical results for bridging distances larger than $i = 4$. Here the height accuracies of blocks of 60 % side overlap do not even reach, for comparable bridging distances, the results of blocks of 20 % side overlap. Again, this holds for both wide-angle and super-wide-angle blocks.

Blocks with 60 % side overlap allow, in principle, relaxation of the chains of height control-points until the control-points form a grid. Three such versions have been computed. The results give again occasion for some warnings, as the grid pattern of height control-points seems not to meet theoretical expectation. The results differ considerably from the comparable ones obtained with the chain pattern of height control-points. The height accuracies deteriorate from the chain arrangement to the grid arrangement of height control-points. Fig. 14 shows that the height errors increase rapidly if the mesh-size of the grid pattern is increased. With $i = 8/7$ the values for z happen to be $49 \mu\text{m}$ for wide-angle and $64 \mu\text{m}$ for super-wide-angle which is a factor of 2.1 and 1.8, respectively, as compared to the equivalent bridging distance of $i = 8(4)$ with chains.

The height accuracies as obtained with blocks of 60 % side overlap are most unexpected, and in a way highly irregular. They can only be explained by very great tensions in the combined blocks, perhaps due to systematic image errors which go with the flight direction.

4.6. Comparison of Wide-angle and Super-wide-angle

The previous presentation has touched repeatedly the comparison between wide-angle and super-wide-angle results. Such a comparison is of great practical importance, in particular as there is not much known about it, neither by experiment nor by theory. Fig. 11, and 15–19 and tables 12 and 13 summarize the empirical results of the Oberschwaben tests with independent models. Reviewing them it seems that we are faced with some startling results.

It has always been expected, that the planimetric accuracy of super-wide-angle blocks is not quite up to the planimetric accuracy of wide-angle blocks. This is directly confirmed by fig. 15. The ratio of the standard errors of unit weight is 1.3, that of the planimetric accuracy of adjusted coordinates (μ_{xy}) is 1.2, or rather 1.27 up to $i = 8$. The difference in planimetric accuracy shows up in the standard errors of unit weight. Hence the ratios μ_{xy}/σ_{0P} are practically equal for both wide-angle and super-wide-angle, see fig. 16. This also holds approximately for blocks with 60 % lateral overlap. Only the (weak) case of 4 control-points in the corners of the block does not entirely fit into that rule.

The great surprise is the comparison of the height accuracies of wide-angle and super-wide-angle blocks. First, the standard error of unit weight (σ_{0H}) for super-wide-angle is, in terms of microns in the negative scale, by 13 % smaller than for wide-angle. In terms of permille of the flying height the ratio for the standard errors is, however, $0.05\text{‰} h$ (wide-angle) : $0.08\text{‰} h$ (super-wide-angle) or 1 : 1.6. Whilst such a ratio might have been expected, it must be considered a surprise that the absolute height errors (μ_z), expressed

in terms of microns in the negative scale are, in all cases which have been computed, larger for super-wide-angle than for wide-angle. The average difference amounts to 21 % for blocks with 20 % lateral overlap. Because of the reversed effect from the respective ratios of σ_{0H} , the difference in absolute height accuracy is even more marked (1.4 : 1) on the basis of the ratios μ_z/σ_{0H} (fig. 19). And finally, in terms of per mille of the flying height, the average ratio of the absolute height accuracy of the super-wide-angle blocks against that of the wide-angle blocks is as high as 2.2 : 1. This is definitely more than would have been expected. This picture is generally confirmed and even aggravated by the results of block-adjustments with 60 % lateral overlap.

If the empirical results can be taken as generally representative, which is questionable, it would mean that the general belief into the superior height accuracy of super-wide-angle photography will have to be modified drastically, at least as far as block-triangulation by independent models is concerned.

4.7. Comparison with Theory

Some of the results obtained up to here have been commented as surprising, others as being in agreement with expectation. Such statements have, in fact, been comparisons of empirical results with theoretical expectation, without specifying the theory very much.

For a number of parameters the theoretical equivalent is obvious or is known from theoretical investigations. Examples are:

- constant values for σ_0 (standard errors of unit weight; σ_{0P} , σ_{0H})
- equal accuracy results of the wide-angle blocks Frankfurt and Vienna on the one hand and of the super-wide-angle blocks The Hague and Delft on the other
- almost equal accuracies μ_x and μ_y for any block-adjustments.

It has been shown, that the experimental values for σ_0 had little variation. Concerning the second and the third item, however, the empirical findings disagreed in quite a number of cases quite considerably from theoretical expectation, although the differences might not be significant. It is intended to apply proper statistical tests, during the final stage of the Oberschwaben investigation, in order to measure the agreement of experiment with theory according to statistical rules.

There is a second group of parameters for which a theory is not available, not worked out, or only formulated in rather loose terms. The most important example is the comparison between wide-angle and super-wide-angle. With regard to whatever theory may be used for a comparison, the empirical relationship for heights is rather unexpected.

The third group of accuracy relations is concerned with the dependence of accuracy on bridging distance i for heights and on distances i of planimetric perimeter control-points, for given block-size. For such relationships theoretical values have been worked out, see [5]. The theoretical values for planimetric accuracy as function of i are not yet published. Therefore they are not considered here, at this moment. However, the theoretical relationship for height accuracy as function of bridging distance i , for block-adjustment with independent models. It is considered in fig. 20 and 21.

Before commenting on the figures it should first be mentioned that agreement between experiment and theory can, in this context, be stated on 3 different levels:

- Complete agreement would be the case, when the numerical values of μ_z would be predictable. This would imply also the theoretical knowledge of σ_{0H} .
- As theory usually treats only the ratio μ_z/σ_{0H} this is the most suitable parameter for comparison. Agreement with theory would, in this case, leave open the actual numerical values of σ_{0H} , and consequently of μ_z .

It could be the case that the ratio μ_z/σ_{0H} for a certain bridging distance (for instance $i = 2$) does not agree with theory, but that from there on the influence of bridging distance is in agreement with theory. Thus some degree of predictability would be maintained, e. g. by the test parameter

$$\frac{\mu_z^{(1)}}{\sigma_{0H}} : \frac{\mu_z^{(2)}}{\sigma_{0H}} = \frac{\mu_z^{(1)}}{\mu_z^{(2)}} \quad (i, 2 \text{ indexing bridging distance}).$$

If we look now at fig. 20 we can state satisfactory agreement between theory and experimental results for the wide-angle blocks, on the basis of the test parameter μ_z/σ_{0H} .

The opposite conclusion is to be drawn from fig. 21 concerning super-wide-angle results. The empirical height accuracy, as expressed in terms of μ_z/σ_{0H} , is really not in agreement with theory. If, however, the test-parameter $\mu_z^{(1)}/\mu_z^{(2)}$ is applied, the empirical curve shows somewhat closer agreement with the theoretical curve. It means that the influence of altering the bridging distance as compared with the height accuracy for a certain bridging distance (here for example $i = 2$) is to a reasonable degree as predicted by theory.

5. Accuracy as Function of Block-Size

The investigations of chapter 4 referred to the influence of different arrangements of control-points on the accuracy of adjusted blocks of independent models, for given (maximum) block-size. It is of equal interest to study in the same way the dependence of accuracy on block-size, for given arrangements of control-points. This investigation has been completed, at this moment, only for the wide-angle block Frankfurt. The area was subdivided according to fig. 1, and the same patterns of control distribution were utilized as described in sections 2.3 and 2.4.

The results are summarized in tables 16, 17 and in fig. 22–27. They will be discussed separately for planimetry and for height.

5.1. Planimetric Accuracy as Function of Block-Size

When studying the dependence of accuracy on block-size the arrangement of control-points is kept fixed. There are 2 cases to be distinguished:

- $i = \text{constant}$, bridging distance constant, or, for planimetric perimeter control, intervals between control-points constant. Here the total number of control-points is increasing with block-size

- number of control-points constant for blocks of different size, for example 4, 8, 16 etc. This case applies only to planimetry. Here the intervals i increase with block-size.

The results of the first case, $i = \text{constant}$, are represented in fig. 22, for planimetry. The figure also shows the theoretical relationship according to [5]. It refers to the planimetric accuracy of adjusted tie-points with 4 tie-points per model, for the case $i = 2$ which is also known as the case of "dense perimeter control" and which is more and more applied in practice. The theoretical relationship predicts that with dense perimeter control the planimetric accuracy of adjusted blocks is almost independent of block-size, and that it is close to values of σ_{0p} . From the graphs in fig. 22 it can be concluded, that the empirical results confirm the theoretical trend. However, close agreement is only reached after adapting a factor 1.4. With this modification the case of dense perimeter control, which is of great practical importance, is sufficiently confirmed to be relied upon in practice.

Perhaps to a slightly lesser degree the weak dependence of planimetric accuracy on block-size can be considered confirmed also for relaxed perimeter control ($i = 4$, $i = 6$, $i = 8$).

Fig. 23 and 24 refer to the case where the number of control-points is kept constant while block-size is increasing. 3 curves are plotted, concerning, respectively, 4, 8 and 16 planimetric perimeter control-points. Fig. 24 shows in addition the theoretical expectations, according to [4] and [5]. It needs no sophisticated statistical test to realize that the empirical results are, for those cases, not at all in agreement with theory. It is only for dense perimeter control ($i = 2$) that the experimental results agree well with theory. But from there on, with increasing values for i the actual errors increase much more rapidly than theory would predict, reaching disagreements by a factor 2 and more. Although the disagreement is considerable, it is nevertheless quite systematic, which can also be seen from the representation of fig. 25. The various results seem to be quite consistent among each other. It could be concluded that quite distinct effects must be acting which are not taken into account by the theory which considers random errors only.

5.2. Height Accuracy as Function of Block-Size

The evaluation of height accuracy, for given arrangements of height control is given in fig. 26 and 27 as function of block-size, for different bridging distances, and as function of bridging distance, for different block-sizes.

Theory predicts, that for given bridging distance i the height accuracy μ_z should be practically independent of block size, see [5]. Fig. 26 confirms that this rule is effective to an astonishing degree and thus can evidently be relied upon in practice.

The second remarkable result is the slow increase of the height errors with increasing bridging distance. Fig. 27 shows the comparison with theory. Generally speaking the agreement of the test results with theory is quite satisfactory, up to $i = 8$. Combining all results in fig. 27 an empirical average curve is suggested the slope of which is somewhat less than of the theoretical curve. However, when considering fig. 20, it seems that the smaller slope is a peculiarity of the Frankfurt block which will be rectified by the results of the Vienna block. Thus it is quite likely that the height accuracy as function of bridging distance i , for different block-sizes, will agree very well with theory, when the results of the Frankfurt block will be combined with those of the Vienna block.

6. Summary and Conclusions

6.1. Before closing this presentation it may be recalled that the first intention was to show the empirical results of block-adjustments according to the method of independent models. The presentation of the test-results should be clearly separated from theoretical considerations, in order to permit everybody to draw his own conclusions.

Stage 2 of the investigation is a comparison of the empirical results with theory, with the aim to confirm or to disprove the applicability of theory to predict results of practical blocks. At the same time statements are asked about relationships which are not or not very well covered by theory.

As might have been expected from a general point of view the results of the independent model block-adjustments confirm some aspects of theory and disagree with others. Thus the total picture seems confusing to some extent. It is therefore attempted to summarize the essential results and to draw some conclusions. It should be remembered, however, that some of the results still await completion by additional adjustments to come and are considered preliminary.

6.2. Reviewing the results of the block-adjustments by the method of independent models it is of great practical importance to point first to the high level of accuracy which has been reached quite consistently. For example:

- standard errors of unit weight between 5.9 and 9.0 μm for planimetry, and between 6.5 and 9.6 μm for heights
- planimetric coordinate accuracies of blocks of 200 (175) models with dense perimeter control of 12.1 (11.9) μm for wide-angle, and of 15.4 (14.0) μm for super-wide-angle
- height accuracies of blocks of 200 (175) models with bridging distance of 6 models of 15.9 (16.1) $\mu\text{m} = 0.1 \text{ } \sigma_{00} h$ for wide-angle, and of 18.4 (22.6) μm for super-wide-angle; transferred to units of the terrain it is 15 km bridging distance and height accuracy of 45 (45) cm for wide-angle and 52 (63) cm for super-wide-angle.

Thus the high accuracy capability of aerial triangulation has been demonstrated.

6.3. There are a number of results which are not quite in agreement with theory or which do not meet general expectation:

- When wide-angle and super-wide-angle results are compared it is not only the planimetric accuracy of s. w. a. which is poorer by about 20 %, as was more or less expected, but it is also the height accuracy which is poorer by about the same percentage. It seems that some reorientation of thoughts concerning the qualities of super-wide-angle photography can be advised.
- The use of 60 % side overlap instead of 20 % did not improve the accuracy as much as expected, neither in planimetry nor in heights. Also the grid pattern of height control does not meet expectations. Thus in general some caution seems to be justified against relying too much on the theoretical merits of 60 % side overlap.

- In addition the tests revealed a number of other features which also do not very well agree with theoretical expectation in general or with theory proper. Conclusions to be drawn will depend, however, on the results of statistical tests of significance to be applied. Examples are: The accuracy of x- and y-coordinates is very often quite different; also different blocks flown with the same camera differ more than might have been expected.

Thus indeed quite some results of the tests were not expected at all or do not entirely agree with theory. Nevertheless, it must be mentioned that often the deviations from theory are not erratic but are quite consistent. Therefore they show still some internal interrelations which do agree with theory.

6.4. A number of accuracy features of the test results are in good agreement with theory. Fortunately these are features which are of great practical importance:

- The experimental results confirm that with dense perimeter control the planimetric accuracy is, with about 12 μm , very good in absolute terms, in addition it is also almost independent of block-size as theory predicts; of course the empirical confirmation of the theory can only be claimed up to blocks of 200 models.
- The empirical height accuracy turns out to be independent of block size, for a given bridging distance; secondly the empirical height accuracy as function of bridging distance is in good agreement with theory.

From a practical point of view those two cases, which refer to wide-angle photography are of great importance. Dense perimeter control for planimetry and chains of height control are not only the most favourable arrangements of control-points, they are also the ones on which practice relies very much. It is therefore a most satisfactory result beyond all possibly debatable details and all unexplained features that the Oberschwaben tests with the independent model method confirm the reliability and applicability of the theoretical relations between accuracy and those arrangements of control which are most widely used in practice.

6.5. Following the scientific logic of the Oberschwaben investigation it would now be necessary to go into a thorough analysis of the causes for agreement and disagreement of the empirical results with theory. It has been stated earlier that this paper does not attempt to extend to an explanation of the results.

Generally speaking the situation is rather clear. If empirical and theoretical results do not agree it is always the theory which is not the appropriate description of what has actually happened. If such disagreements are sufficiently consistent it is the theory which must be refined.

With regard to the accuracy hypotheses of aerial triangulation it has always been known that they are more or less simplified. In particular, only independent random errors have been taken into account. The obvious conclusion is that the Oberschwaben material contains error effects which cannot be sufficiently described by the concept of independent random variates. Thus the explanation of the accuracy behaviour of the empirical

results must refer to systematic and/or correlated image errors, and consequently also to systematic and/or highly correlated errors of model coordinates, and perhaps also errors in the trigonometric points will have to be considered.

There is evidence enough that indeed there are systematic errors prevailing in the photographic material. Vector diagrams of residual errors, which can only be mentioned here, support the hypothesis of systematic errors. Also a preliminary analysis of the test results presented here agrees in many points with such a hypothesis. The conclusion will be that adjustment methods with a more refined mathematical model will have to be applied. By doing so, first a better agreement between theory and tests is expected and, secondly, considerably increased accuracies ought to be obtained.

It remains to be seen whether a more detailed analysis of the results and a more sophisticated theory of the accuracy of aerial triangulation will be able to explain all the results obtained here and to reach full agreement between the refined theory and the empirical results. It is not the least result of the Oberschwaben investigation to raise new problems and to put new emphasis into the research on aerial triangulation, from which more advanced methods of block triangulation will emerge.

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RÉSUMÉ:

L'étude présente les résultats des compensations du bloc Oberschwaben du test l'OEEPE selon la méthode des modèles indépendants.

On étudie la précision planimétrique et altimétrique en relation avec la grandeur des blocs et avec la répartition des points de contrôle.

Les deux questions suivantes sont au centre de l'intérêt:

- Variation de la répartition des points de contrôle pour une grandeur de bloc donnée
- Variation de la grandeur du bloc pour des répartitions de points de contrôle données

L'étude s'étend en outre à la comparaison de prises de vue grand'angulaires et de prises de vue supergrand'angulaires ainsi qu'à la comparaison de $q = 20\%$ et de $q = 60\%$ de recouvrement latéral.

Pour les compensations des blocs on a utilisé le matériel du programme I du bloc de test Oberschwaben:

- grand'angulaire; bloc Francfort, $8 \times 25 = 200$ modèles, $q = 20\%$
 bloc Vienne, $7 \times 25 = 175$ modèles, $q = 20\%$
 bloc total, $15 \times 25 = 375$ modèles, $q = 60\%$
- supergrand'angulaire; bloc Den Haag, $8 \times 25 = 200$ modèles, $q = 20\%$
 bloc Delft, $7 \times 25 = 175$ modèles, $q = 20\%$
 bloc total, $15 \times 25 = 375$ modèles, $q = 60\%$

Voici les résultats les plus significatifs:

- On peut qualifier de très bonnes les précisions planimétriques et altimétriques obtenues, surtout en ce qui concerne les prises de vue grand'angulaires.
- Les expectatives théoriques de précision qui sont d'une grande importance pour l'application pratique se sont réalisées dans une large mesure: le bord du bloc étant couvert d'un grand nombre de points de contrôle, la précision planimétrique dépend à peine de la grandeur du bloc; de même la précision altimétrique est indépendante de la grandeur du bloc et elle est uniquement définie par la distance des chaînes des points de contrôle altimétriques.
- Par ailleurs, des divergences considérables apparaissent cependant entre les résultats effectifs et les expectatives théoriques. Ceci concerne avant tout les répartitions des points de contrôle moins dense et les blocs de 60% de recouvrement latéral.
- La précision altimétrique des prises de vue supergrand'angulaires a été médiocre par rapport aux prises de vue grand'angulaires; c'est également un résultat inattendu.
- L'existence d'erreurs systématiques (ou hautement corrélées) d'image et de modèle fournit l'explication de la divergence entre la théorie et les résultats empiriques. Il y a de fortes indices pour cela.

Les développements ultérieurs auront pour tâche de corriger à l'aide de méthodes mathématiques améliorées les erreurs systématiques pendant la compensation des blocs soit par modèles indépendants soit par faisceaux. On en attend une augmentation considérable de la précision et une plus grande conformité entre la théorie (qui prend en considération les erreurs accidentelles seulement) et les résultats empiriques.

Table 1

OEEPE - OBERSCHWABEN
Independence of planimetry and heights
Block Frankfurt (w. a.) - Independent models

Control plan	height	σ_{0P}		μ_x		μ_y		μ_{xy}		μ_z		$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
		cm	μm	cm	μm	cm	μm	cm	μm	cm	μm		
perimeter $i = 2$	13 chains $i = 2$	19.4	6.9	23.1	8.2	41.8	14.9	33.8	12.1	36.2	12.9	1.74	1.54
perimeter $i = 2$	3 chains $i = 12.5(6)$	19.4	6.9	23.1	8.2	41.8	14.9	33.8	12.1	49.3	17.6	1.74	2.12
4 corners	3 chains $i = 12.5(6)$	16.8	6.0	112.7	40.2	139.8	49.9	127.0	45.3	53.1	19.0	7.56	2.27
4 corners	13 chains $i = 2$	16.9	6.0	112.4	40.1	139.7	49.9	126.8	45.3	36.2	12.9	7.50	1.53

Table 2

OEEPE — OBERSCHWABEN
Independence of planimetry and heights
Block The Hague (s. w. a.) — Independent models

Control plan	height	σ_{0P}		μ_x		μ_y		μ_{xy}		μ_z		$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
		cm	μm	cm	μm	cm	μm	cm	μm	cm	μm		
perimeter $i = 2$	7 chains $i = 4$	27.4	9.8	38	13	54	19	47	16	46	16	1.72	1.78
perimeter $i = 2$	perimeter 14 points*)	27.4	9.8	38	14	54	20	47	17	110	39	1.72	4.31
4 corners	7 chains $i = 4$	24.4	8.7	99	35	173	62	141	50	46	17	5.78	1.80
4 corners	perimeter 14 points*)	24.4	8.7	99	35	172	62	140	50	117	42	5.74	4.64

*) very poor height control, system almost singular

Table 3

OEEPE — OBERSCHWABEN
Independent models — wide-angle
Preliminary planimetric adjustments

Planimetric Control Version	$q = 20\%$ (block Frankfurt)			$q = 60\%$ (Frankfurt and Vienna) ¹⁾		
	σ_{0P} cm μm	μ_{xy} cm μm	$\frac{\mu_{xy}}{\sigma_{0P}}$	σ_{0P} cm μm	μ_{xy} cm μm	$\frac{\mu_{xy}}{\sigma_{0P}}$
0 all CP	21 7.5	(15) (5.4)	(0.7)	19 6.8	(16) (5.8)	(0.85)
1 perimeter $i = 2$	20 7.1	35 12.5	1.8	18 6.4	35 12.5	2.0
2 perimeter $i = 4$	19 6.8	49 17.5	2.6	18 6.3	48 17.1	2.7
4 perimeter $i = 8$	18 6.5	71 25.4	3.9	17 6.1	60 21.4	3.5
5 4 corners $i = 16/25$	17 6.2	113 40.3	6.5	17 6.1	74 26.4	4.3

with additional frame of models around controlled area

	σ_{0P}	μ_{xy}	$\mu_{xy}^{i, 2)}$	$\frac{\mu_{xy}^{i, 2)}}{\sigma_{0P}}$	σ_{0P}	μ_{xy}	$\mu_{xy}^{i, 2)}$	$\frac{\mu_{xy}^{i, 2)}}{\sigma_{0P}}$
2 $i = 4$	19 6.7	52 18.6	33 11.8	1.8	18 6.4	36 12.9	30 10.6	1.7
3 $i = 6$	18 6.6	58 20.6	36 12.9	2.0	17 6.1	47 16.8	42 15.0	2.4
4 $i = 12$ (6 CP)	18 6.4	79 28.1	55 19.6	3.1	17 6.1	73 26.1	68 24.3	4.0
5 $i = 12/21$ (4 CP)	17 6.2	102 36.4	61 21.8	3.5	17 6.1	69 24.6	63 22.5	3.7

¹⁾ control on the perimeter; contrary to fig. 2

²⁾ check-points within the controlled area only (μ^i)

Table 4
OEEPE - OBERSCHWABEN
Block Frankfurt (w. a.) - Independent models
Subblocks - Preliminary planimetric adjustment

subblock	planimetric control (perimeter)	σ_{0P} cm μm	μ_{xy} cm μm	$\frac{\mu_{xy}}{\sigma_{0P}}$
1 subblock 8 x 16 = 128 models	$i = 2$	20 7.1	36 12.8	1.8
	$i = 16$ (4 corners)	18 6.4	88 31.5	4.8
2 subblocks 6 x 12 = 72 models	$i = 2$	20 7.2	30 10.8	1.5
	$i = 4$	19 6.9	41 14.7	2.1
	$i = 6$	19 6.8	48 17.2	2.5
	$i = 12$ (4 corners)	18 6.4	88 31.5	4.8
6 subblocks 4 x 8 = 32 models	$i = 2$	19 6.9	28 9.9	1.5
	$i = 4$	18 6.6	35 12.3	1.9
	$i = 8$ (4 corners)	18 6.3	44 15.6	2.5
24 subblocks 2 x 4 = 8 models	$i = 2$	18 6.6	28 10.0	1.5
	$i = 4$ (4 corners)	18 6.4	34 12.0	1.9

Table 5
OEEPE - OBERSCHWABEN - Independent models
Comparison of standard errors of unit weight (σ_{0P} , σ_{0H})

blocks			control - version					
			0	1 $i = 2$	2 $i = 4$	3 $i = 6$	4 $i = 8$	5 4 corners
σ_{0P} [μm]	wide-angle	Frankfurt	7.2	6.9	6.7	6.6	6.3	6.0
		Vienna	6.9	6.5	6.3	6.2	6.1	5.9
		$q = 60\%$	6.8	6.4	6.3	6.2	6.1	6.0
	super-wide-angle	The Hague	9.0	8.7	8.3	8.1	8.0	7.6
		Delft	8.8	8.7	8.5	8.4	8.3	8.1
		$q = 60\%$	8.5	8.2	8.0	7.9	7.8	7.6
	wide-angle Frankfurt	A, B		7.0	6.7		6.2	6.0
		I - VI		6.7	6.5		6.0	
		1 - 12		6.6	6.2			
		Anblock		7.1	6.8		6.5	6.2
σ_{0H} [μm]	wide-angle	Frankfurt	9.2	8.4	8.3	8.4	8.3	8.4
		Vienna	8.5	7.8	7.7	7.6	7.6	7.5
		$q = 60\%$	9.6	9.2	9.1	9.0	9.0	8.7
	super-wide-angle	The Hague	8.8	7.7	7.6	7.5	7.5	7.5
		Delft	7.8	6.8	6.7	6.5	6.5	6.5
		$q = 60\%$	9.7	9.2	9.2	9.0	8.9	8.7
	wide-angle Frankfurt	A, B		8.3	8.2		8.2	8.3
		I - VI		8.1	8.3		8.3	
		1 - 12		8.5	8.5			

Table 6

OEEPE - OBERSCHWABEN

Block Frankfurt (wide-angle) - Independent models

Version	Control plan	Control height	σ_{0P} cm μm	σ_{0H} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
0	all CP (510)	all CP (452)	20.3 7.2	25.8 9.2	(17.2) (6.1)	(19.3) (6.9)	(18.3) (6.5)	(27.4) (9.8)	(0.90)	(1.06)
1	perimeter $i = 2$	13 chains $i = 2$	19.4 6.9	23.5 8.4	23.2 8.3	41.8 14.9	33.8 12.1	36.2 12.9	1.74	1.54
2	perimeter $i = 4$	7 chains $i = 4(2)$	18.7 6.7	23.3 8.3	36.8 13.1	56.8 20.3	47.9 17.1	38.7 13.8	2.56	1.66
3	perimeter $i = 6$	5 chains $i = 6(3)$	18.4 6.6	23.4 8.4	50.5 18.0	64.4 23.0	57.9 20.7	44.6 15.9	3.15	1.91
4	perimeter $i = 8$	4 chains $i = 8(4)$	17.6 6.3	23.4 8.4	63.0 22.5	79.4 28.4	71.7 25.6	45.1 16.1	4.07	1.93
5	4 corners $i = 12,5(6)$	3 chains $i = 12,5(6)$	16.8 6.0	23.4 8.4	112.7 40.5	139.8 49.9	127.0 45.3	53.1 19.0	7.56	2.27

OEEPE - OBERSCHWABEN

Block Vienna (wide-angle) - Independent models

Version	Control plan	Control height	σ_{0P} cm μm	σ_{0H} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
0	all CP (450)	all CP (404)	19.2 6.9	23.8 8.5	(16.0) (5.7)	(18.3) (6.5)	(17.2) (6.1)	(25.0) (8.9)	(0.90)	(1.05)
1	perimeter $i = 2$	13 chains $i = 2$	18.1 6.5	21.7 7.8	37.4 13.4	28.4 10.1	33.2 11.9	37.9 13.5	1.83	1.75
2	perimeter $i = 4$	7 chains $i = 4(2)$	17.6 6.3	21.5 7.7	45.2 16.1	34.5 12.3	40.2 14.4	42.2 15.1	2.28	1.96
3	perimeter $i = 6$	5 chains $i = 6(3)$	17.4 6.2	21.2 7.6	54.1 19.3	36.9 13.2	46.3 16.5	45.1 16.1	2.66	2.13
4	perimeter $i = 8$	4 chains $i = 8(4)$	17.1 6.1	21.2 7.6	47.8 17.1	47.5 17.0	47.6 17.0	54.2 19.4	2.79	2.56
5	4 corners $i = 12,5(6)$	3 chains $i = 12,5(6)$	16.5 5.9	21.1 7.5	133.2 47.6	54.6 19.5	101.8 36.4	76.1 27.2	6.17	3.61

Table 8

OEEPE — OBERSCHWABEN

Block The Hague (super-wide-angle) — Independent models

Version	Control plan	Control height	σ_{OP} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
0	all CP (492)	all CP (432)	25.1 9.0	(21.0) (7.5)	(23.8) (8.5)	(22.4) (8.0)	(27.5) (9.8)	(0.89)	(1.12)
1	perimeter $i = 2$	13 chains $i = 2$	24.3 8.7	35.2 12.6	49.8 17.8	43.1 15.4	42.3 15.1	1.77	1.97
2	perimeter $i = 4$	7 chains $i = 4(2)$	23.3 8.3	47.5 17.0	71.6 25.6	60.8 21.7	45.1 16.1	2.61	2.13
3	perimeter $i = 6$	5 chains $i = 6(3)$	22.7 8.1	51.9 18.5	87.2 31.1	71.7 25.6	51.6 18.4	3.16	2.45
4	perimeter $i = 8$	4 chains $i = 8(4)$	22.3 8.0	81.0 28.9	91.4 32.6	86.3 30.8	53.0 18.9	3.87	2.54
5	4 corner	3 chains $i = 12.5(6)$	21.4 7.6	100.6 35.9	151.3 54.0	128.5 45.9	64.1 22.9	6.00	3.07

Table 9

OEEPE — OBERSCHWABEN

Block Delft (super-wide-angle) — Independent models

Version	Control plan	Control height	σ_{OP} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
0	all CP (441)	all CP (398)	24.6 8.8	(18.9) (6.7)	(22.3) (8.0)	(20.7) (7.4)	(25.3) (9.0)	(0.84)	(1.16)
1	perimeter $i = 2$	13 chains $i = 2$	24.5 8.7	32.4 11.6	44.8 16.0	39.1 14.0	41.3 14.7	1.60	2.16
2	perimeter $i = 4$	7 chains $i = 4(2)$	23.9 8.5	43.4 15.5	58.0 20.7	51.2 18.3	43.8 15.6	2.14	2.32
3	perimeter $i = 6$	5 chains $i = 6(3)$	23.5 8.4	49.7 17.7	70.4 25.1	60.9 21.7	63.4 22.6	2.60	3.46
4	perimeter $i = 8$	4 chains $i = 8(4)$	23.2 8.3	66.6 23.8	76.4 27.3	71.7 25.6	79.1 28.2	3.09	4.32
5	4 corners	3 chains $i = 12.5(6)$	22.6 8.1	92.7 33.1	93.2 33.3	93.0 33.2	91.9 32.8	4.11	5.05

OEBPE — OBERSCHWABEN

Table 10

Wide-angle block, $q = 60\%$ — Independent models

Version	Control plan	Control height	σ_{OP} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OP}}$
0 a 0 ¹⁾	all CP (528) all CP (466)	all CP (470) all CP (417)	19.1 19.0 6.8	27.3 27.0 9.6	(16.8) (16.7) (6.0)	(18.8) (18.9) (6.7)	(17.8) (17.8) (6.4)	(0.94)	(1.03)
1	perimeter $i = 2$	13 chains $i = 2$	18.0 6.4	25.7 9.2	29.6 10.6	25.1 9.0	27.4 9.8	1.52	1.27
2	perimeter $i = 4$	7 chains $i = 4(2)$	17.7 6.3	25.5 9.1	40.6 14.5	32.1 11.5	36.6 13.1	2.07	1.41
3	perimeter $i = 6$	5 chains $i = 6(3)$	17.5 6.2	25.3 9.0	46.5 16.6	38.8 13.9	42.8 15.3	2.45	2.00
4	perimeter $i = 8$	4 chains $i = 8(4)$	17.2 6.1	25.2 9.0	50.4 18.0	50.1 17.9	50.3 18.0	2.92	2.60
5	4 corners	3 chains $i = 12.5(6)$	16.8 6.0	24.8 8.9	69.5 24.8	68.3 24.4	68.9 24.6	4.10	6.17
3*	perimeter $i = 6$	grid $i = 4/4$	17.5 6.2	25.2 9.0	46.5 16.6	38.9 13.9	42.9 15.3	2.45	1.78
4*	perimeter $i = 8$	grid $i = 6/5$	17.2 6.1	25.2 9.0	50.5 18.0	50.1 17.9	50.3 18.0	2.92	2.25
5*	4 corners	grid $i = 8/7$	16.8 6.0	24.5 8.7	69.3 24.4	68.4 24.4	68.3 24.6	4.07	6.17

OEBPE — OBERSCHWABEN

Table 11

Super-wide-angle block, $q = 60\%$ — Independent models

Version	Control plan	Control height	σ_{OP} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_z cm μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OP}}$
0	all CP (455)	all CP (407)	23.8 8.5	27.2 9.7	(20.1) (7.2)	(23.3) (8.3)	(21.8) (7.8)	(0.92)	(1.05)
1	perimeter $i = 2$	13 chains $i = 2$	22.9 8.2	25.9 9.2	28.8 10.3	38.9 13.9	34.2 12.2	1.49	1.44
2	perimeter $i = 4$	7 chains $i = 4(2)$	22.5 8.0	25.8 9.2	39.9 14.2	53.6 19.1	47.2 16.9	2.10	1.53
3	perimeter $i = 6$	5 chains $i = 6(3)$	22.2 7.9	25.4 9.1	50.2 17.9	62.4 22.3	56.6 20.2	2.55	2.80
4	perimeter $i = 8$	4 chains $i = 8(4)$	21.9 7.8	25.3 9.0	57.9 20.7	76.1 27.2	67.6 24.1	3.09	3.94
5	4 corners	3 chains $i = 12.5(6)$	21.3 7.6	24.6 8.8	75.1 26.8	123.1 44.0	102.0 36.4	4.79	9.02
3*	perimeter $i = 6$	grid $i = 4/4$	22.2 7.9	25.3 9.0	50.2 17.9	62.4 22.3	56.6 20.2	2.55	2.07
4*	perimeter $i = 8$	grid $i = 6/5$	21.9 7.8	25.0 8.9	57.8 20.6	76.2 27.2	67.6 24.1	3.09	3.22
5*	4 corners	grid $i = 8/7$	21.3 7.6	24.5 8.7	75.1 26.8	122.7 43.8	101.7 36.3	4.77	7.35

Table 12

OERPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Planimetric accuracy

				control versions					
blocks				0	1 $i=2$	2 $i=4$	3 $i=6$	4 $i=8$	5 $i=10$
μ_{xy}	wide-angle	Frankfurt	cm	(18.3)	33.8	47.9	57.9	71.7	127.0
			μm	(6.5)	12.1	17.1	20.7	25.6	45.1
		Vienna	cm	(17.2)	33.2	40.2	46.3	47.6	101.8
			μm	(6.1)	11.9	14.4	16.5	17.0	36.4
	mean		cm	(17.8)	33.5	44.2	52.4	60.9	116.0
			μm	(6.3)	12.0	15.8	18.7	21.8	41.0
	super-wide-angle	The Hague	cm	(22.4)	43.1	60.8	71.7	86.3	128.0
			μm	(8.0)	15.4	21.7	25.6	30.8	45.1
		Delft	cm	(20.7)	39.1	51.2	60.9	71.7	93.1
			μm	(7.4)	14.0	18.3	21.7	25.6	38.2
	mean		cm	(21.6)	41.1	56.2	66.5	79.3	112.2
			μm	(7.7)	14.7	20.1	23.8	28.3	40.1
$\frac{\mu_{xy}}{\sigma_{0P}}$	wide-angle	Frankfurt		(0.90)	1.74	2.56	3.15	4.07	7.8
		Vienna		(0.90)	1.83	2.28	2.66	2.79	6.2
		mean		(0.90)	1.79	2.42	2.92	3.49	6.9
	super-wide-angle	The Hague		(0.89)	1.77	2.61	3.16	3.87	8.0
		Delft		(0.84)	1.60	2.14	2.60	3.09	4.1
		mean		(0.87)	1.69	2.39	2.89	3.50	5.2

Table 13

OERPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Height accuracy

				control version					
block				0	1 <i>i</i> = 2	2 <i>i</i> = 4(2)	3 <i>i</i> = 6(3)	4 <i>i</i> = 8(4)	5 <i>i</i> = 12.5(6)
μ_z	wide-angle	Frankfurt	cm	(27.4)	36.2	38.7	44.6	45.1	53.1
			μm	(9.8)	12.9	13.8	15.9	16.1	19.0
	Vienna	cm	(25.0)	37.9	42.2	45.1	54.2	76.1	
		μm	(8.9)	13.5	15.1	16.1	19.4	27.2	
	mean	cm	(26.2)	37.1	40.5	44.9	49.9	65.6	
		μm	(9.4)	13.2	14.5	16.0	17.8	23.4	
			$\%_{00} h$	(0.06)	0.09	0.09	0.10	0.12	0.15
	super-wide-angle	The Hague	cm	(27.5)	42.3	45.1	51.6	53.0	64.1
			μm	(9.8)	15.1	16.1	18.4	18.9	22.9
	Delft	cm	(25.3)	41.3	43.8	63.4	79.1	91.9	
		μm	(9.0)	14.7	15.6	22.6	28.2	32.8	
	mean	cm	(26.4)	41.8	44.5	57.8	67.3	79.2	
		μm	(9.4)	14.9	15.9	20.6	24.0	28.3	
			$\%_{00} h$	(0.11)	0.18	0.19	0.24	0.28	0.33
$\frac{\mu_z}{\sigma_{0H}}$	wide-angle	Frankfurt	(1.06)	1.54	1.66	1.91	1.93	2.27	
		Vienna	(1.05)	1.75	1.96	2.13	2.56	3.61	
		mean	(1.06)	1.65	1.82	2.02	2.27	3.02	
	super-wide-angle	The Hague	(1.12)	1.97	2.13	2.45	2.54	3.07	
		Delft	(1.16)	2.16	2.32	3.46	4.32	5.05	
		mean	(1.14)	2.07	2.23	3.00	3.54	4.18	

Table 12

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Planimetric accuracy

	blocks			control versions				
				0	1 <i>i</i> = 2	2 <i>i</i> = 4	3 <i>i</i> = 6	4 <i>i</i> = 8
μ_{xy}	wide-angle	Frankfurt	cm	(18.3)	33.8	47.9	57.9	71.7
			μm	(6.5)	12.1	17.1	20.7	25.6
		Vienna	cm	(17.2)	33.2	40.2	46.3	47.6
			μm	(6.1)	11.9	14.4	16.5	17.0
		mean	cm	(17.8)	33.5	44.2	52.4	60.9
			μm	(6.3)	12.0	15.8	18.7	21.8
	super-wide-angle	The Hague	cm	(22.4)	43.1	60.8	71.7	86.3
			μm	(8.0)	15.4	21.7	25.6	30.8
		Delft	cm	(20.7)	39.1	51.2	60.9	71.7
			μm	(7.4)	14.0	18.3	21.7	25.6
		mean	cm	(21.6)	41.1	56.2	66.5	79.3
			μm	(7.7)	14.7	20.1	23.8	28.3
$\frac{\mu_{xy}}{\sigma_{0P}}$	wide-angle	Frankfurt		(0.90)	1.74	2.56	3.15	4.07
		Vienna		(0.90)	1.83	2.28	2.66	2.79
		mean		(0.90)	1.79	2.42	2.92	3.49
	super-wide-angle	The Hague		(0.89)	1.77	2.61	3.16	3.87
		Delft		(0.84)	1.60	2.14	2.60	3.09
		mean		(0.87)	1.69	2.39	2.89	3.50

Table 13

OBERSCHWABEN

Independent models, $q = 20\%$ - Height accuracy

block			control version					
			0	1 <i>i</i> = 2	2 <i>i</i> = 4(2)	3 <i>i</i> = 6(3)	4 <i>i</i> = 8(4)	5 <i>i</i> = 12.5(6)
wide-angle	Frankfurt	cm	(27.4)	36.2	38.7	44.6	45.1	53.1
		μm	(9.8)	12.9	13.8	15.9	16.1	19.0
	Vienna	cm	(25.0)	37.9	42.2	45.1	54.2	76.1
		μm	(8.9)	13.5	15.1	16.1	19.4	27.2
	mean	cm	(26.2)	37.1	40.5	44.9	49.9	65.6
		μm	(9.4)	13.2	14.5	16.0	17.8	23.4
		°/100 <i>h</i>	(0.06)	0.09	0.09	0.10	0.12	0.15
super-wide-angle	The Hague	cm	(27.5)	42.3	45.1	51.6	53.0	64.1
		μm	(9.8)	15.1	16.1	18.4	18.9	22.9
	Delft	cm	(25.3)	41.3	43.8	63.4	79.1	91.9
		μm	(9.0)	14.7	15.6	22.6	28.2	32.8
	mean	cm	(26.4)	41.8	44.5	57.8	67.3	79.2
		μm	(9.4)	14.9	15.9	20.6	24.0	28.3
		°/100 <i>h</i>	(0.11)	0.18	0.19	0.24	0.28	0.33
wide-angle	Frankfurt		(1.06)	1.54	1.66	1.91	1.93	2.27
	Vienna		(1.05)	1.75	1.96	2.13	2.56	3.61
	mean		(1.06)	1.65	1.82	2.02	2.27	3.02
super-wide-angle	The Hague		(1.12)	1.97	2.13	2.45	2.54	3.07
	Delft		(1.16)	2.16	2.32	3.46	4.32	5.05
	mean		(1.14)	2.07	2.23	3.00	3.54	4.18

OEEPE — OBERSCHWABEN

Table 14

Independent models, $q \approx 60\%$, 375 models — Planimetric accuracy

control version ¹⁾	wide-angle			super-wide-angle		
	μ_x cm μm	μ_y cm μm	μ_{xy} cm μm	μ_x cm μm	μ_y cm μm	μ_{xy}/σ_{0p} cm μm
0 wide-angle all contr. pts. ($i=1$)	(16.7) (6.0)	(18.9) (6.7)	(17.8) (6.4)	(20.1) (7.2)	(23.3) (8.3)	(0.92) (7.8)
1 perimeter, $i=2$	29.6 10.6	25.1 9.0	27.4 9.8	28.8 10.3	38.9 13.9	1.49 12.2
2 perimeter, $i=4$	40.6 14.5	32.1 11.5	36.6 13.1	39.9 14.2	53.6 19.1	2.10 16.9
3 perimeter, $i=6$	46.5 16.6	38.8 13.9	42.8 15.3	50.2 17.9	62.4 22.3	2.55 20.2
4 perimeter, $i=8$	50.4 18.0	50.1 17.9	50.3 18.0	57.9 20.7	76.1 27.2	3.09 24.1
5 4 corners $i=14/25$	69.5 24.8	68.3 24.4	68.9 24.6	75.1 26.8	123.1 44.0	4.79 36.4

¹⁾ double overlap areas only

OEEPE — OBERSCHWABEN

Table 15

Independent models, $q \approx 60\%$, 375 models — Height accuracy

	control version:	i =	chains						grid			
			0	1	2	3	4	5	3*	4*	5*	
			0	1	2	3	4	5	4/4	6/5	8/7	

¹⁾ double overlap areas only

Table 16

OEEPE — OBERSCHWABEN

Block Frankfurt (w. a.) — Independent models
Subblocks A, B (8 x 17 = 136 models)

Control		σ_{0P}	σ_{0H}	μ_x	μ_y	μ_{xy}	μ_z	$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
plan	height	cm μm	cm μm	cm μm	cm μm	cm μm	cm μm		
4 corners (per. $i = 16$)	9 chains $i = 2$	16.8 6.0	23.3 8.3	73.7 26.3	120.6 43.1	99.9 35.7	35.5 12.7	5.93	1.52
perimeter $i = 8$	5 chains $i = 4(2)$	17.4 6.2	23.1 8.2	64.7 23.1	71.4 25.5	68.1 24.3	37.3 13.3	3.91	1.61
perimeter $i = 4$	3 chains $i = 8$	18.7 6.7	23.0 8.2	37.5 13.4	55.4 19.8	47.3 16.9	54.4 19.4	2.53	2.36
perimeter $i = 2$	3 chains $i = 8(4)$	19.5 7.0	23.0 8.2	24.1 8.6	40.5 14.5	33.3 11.9	54.1 19.3	1.71	2.35

Table 17

OEEPE — OBERSCHWABEN

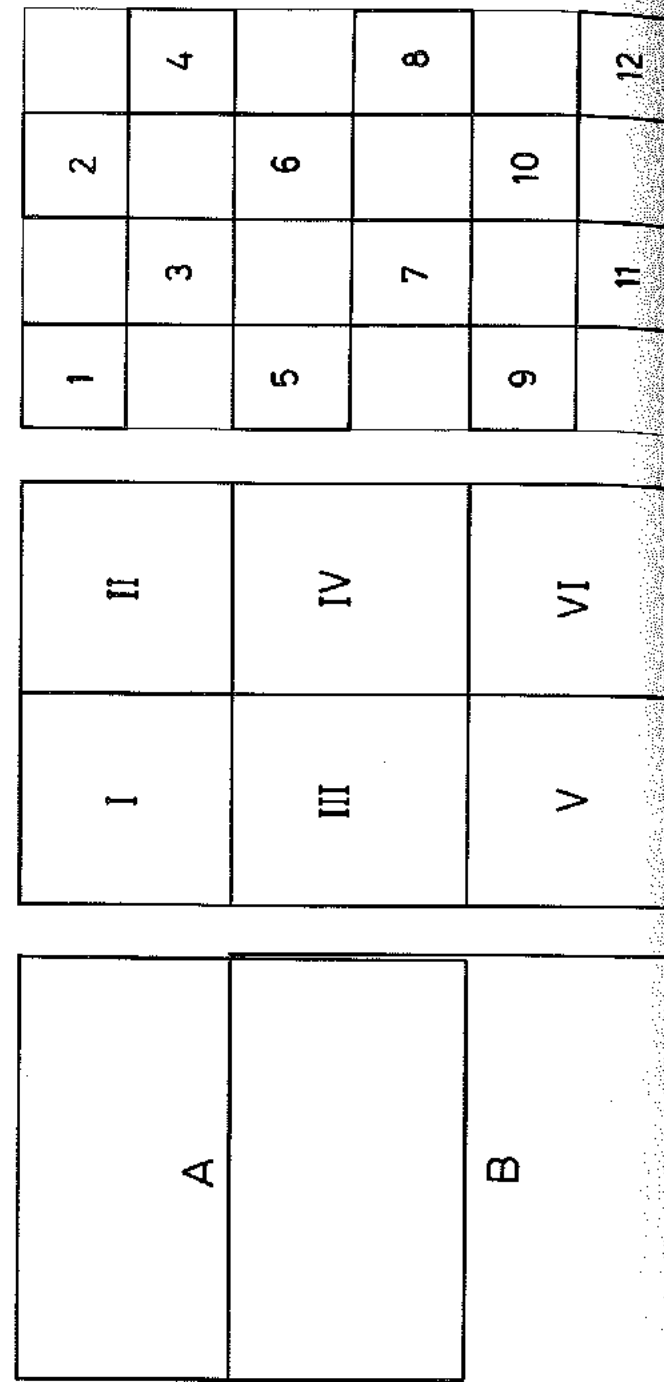
Block Frankfurt (w. a.) — Independent models
Subblocks I — VI (4 x 8 = 32 models) and 1 — 12 (2 x 4 = 8 models)

Control		σ_{0P}	σ_{0H}	μ_x	μ_y	μ_{xy}	μ_z	$\frac{\mu_{xy}}{\sigma_{0P}}$	$\frac{\mu_z}{\sigma_{0H}}$
plan	height	cm μm	cm μm	cm μm	cm μm	cm μm	cm μm		
I — VI									
4 corners (per. $i = 8$)	5 chains $i = 2$	16.8 6.0	23.4 8.4	43.9 15.7	44.4 15.9	44.2 15.8	36.8 13.1	2.63	1.57
perimeter $i = 4$	3 chains $i = 4(2)$	18.1 6.5	23.2 8.3	29.1 10.4	34.6 12.4	32.0 11.4	38.6 13.8	1.77	1.66
perimeter $i = 2$	3 chains $i = 4$	18.9 6.7	23.2 8.3	21.2 7.6	28.2 10.1	24.9 8.9	39.6 14.1	1.32	1.71
4 corners (per. $i = 4$)	3 chains $i = 2$	17.4 6.2	23.8 8.5	24.1 8.6	33.7 12.0	29.3 10.5	40.2 14.4	1.68	1.69
perimeter $i = 2$	3 chains $i = 2$	18.4 6.6	23.8 8.5	19.6 7.0	26.2 9.4	23.1 8.2	40.2 14.4	1.26	1.69

OEEPE - OBERSCHWABEN

schematic subdivision of 200 model block

subblocks A, B: 8 x 17 = 136 models
 I...VI: 4 x 8 = 32 models
 1...12: 2 x 4 = 8 models



schematic arrangement of perimeter control - Planimetry

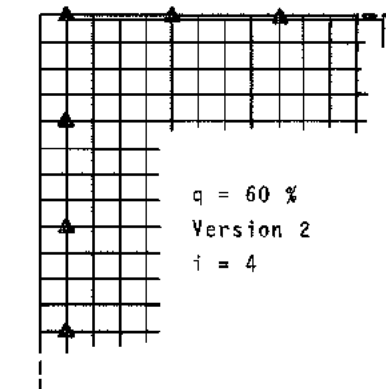
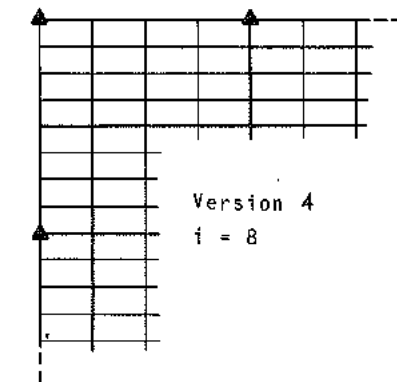
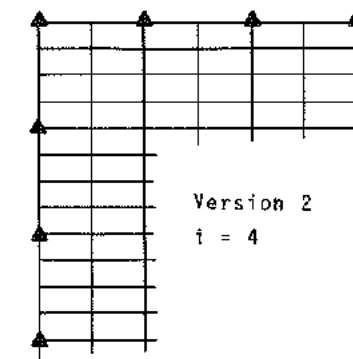
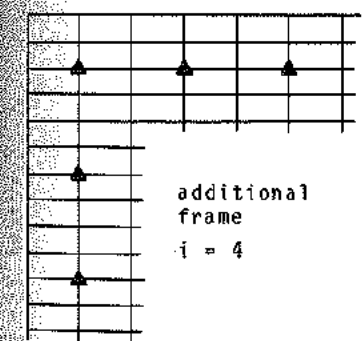
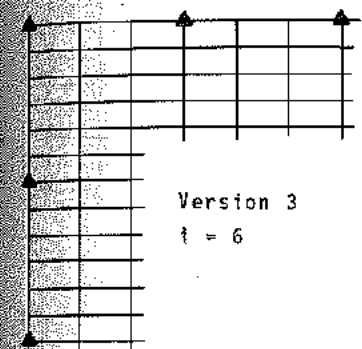
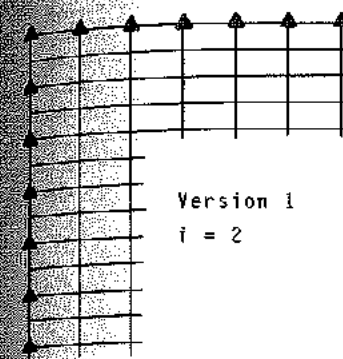


Figure 2

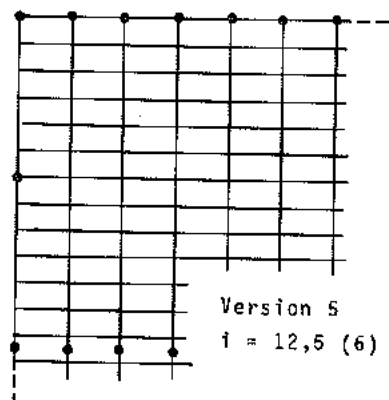
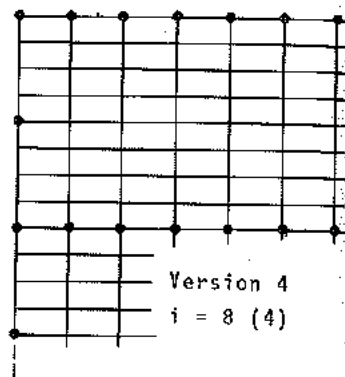
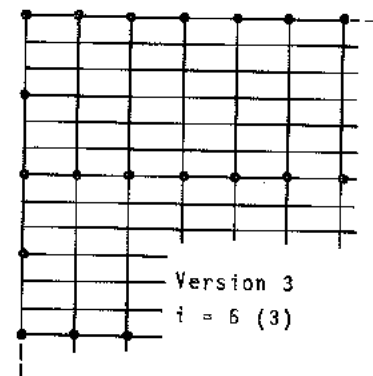
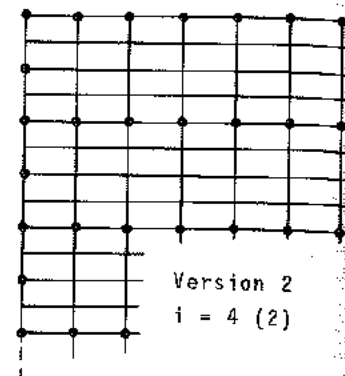
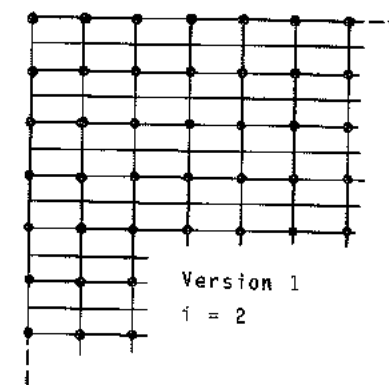


Figure 3

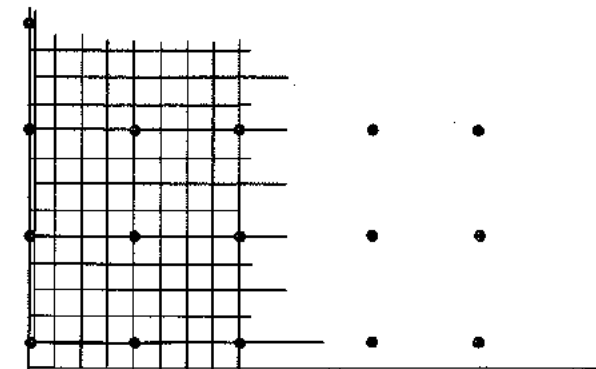
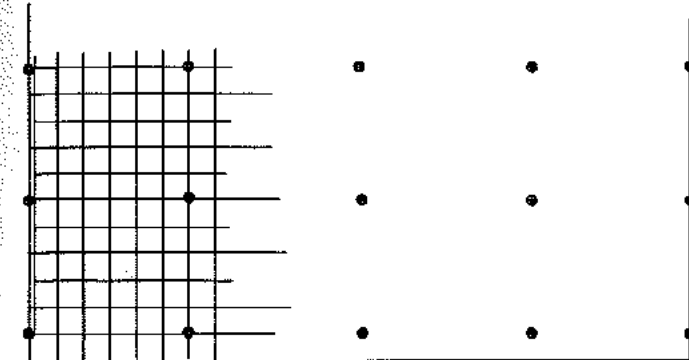
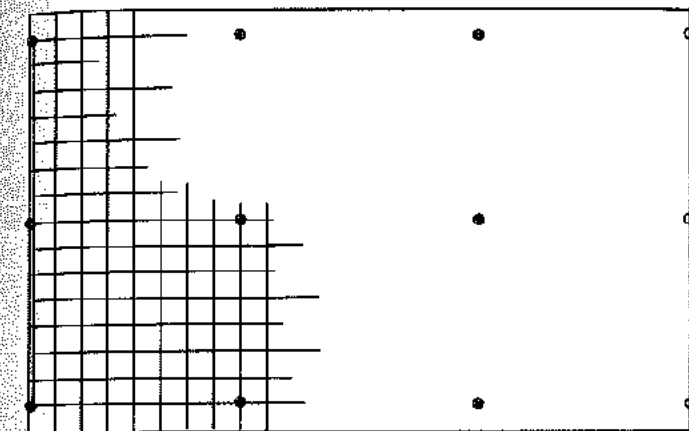


Figure 4

OEEPE - OBERSCHWABEN

Independent models - Block Frankfurt, subblocks
Preliminary planimetric adjustment
Planimetric accuracy as function of block size

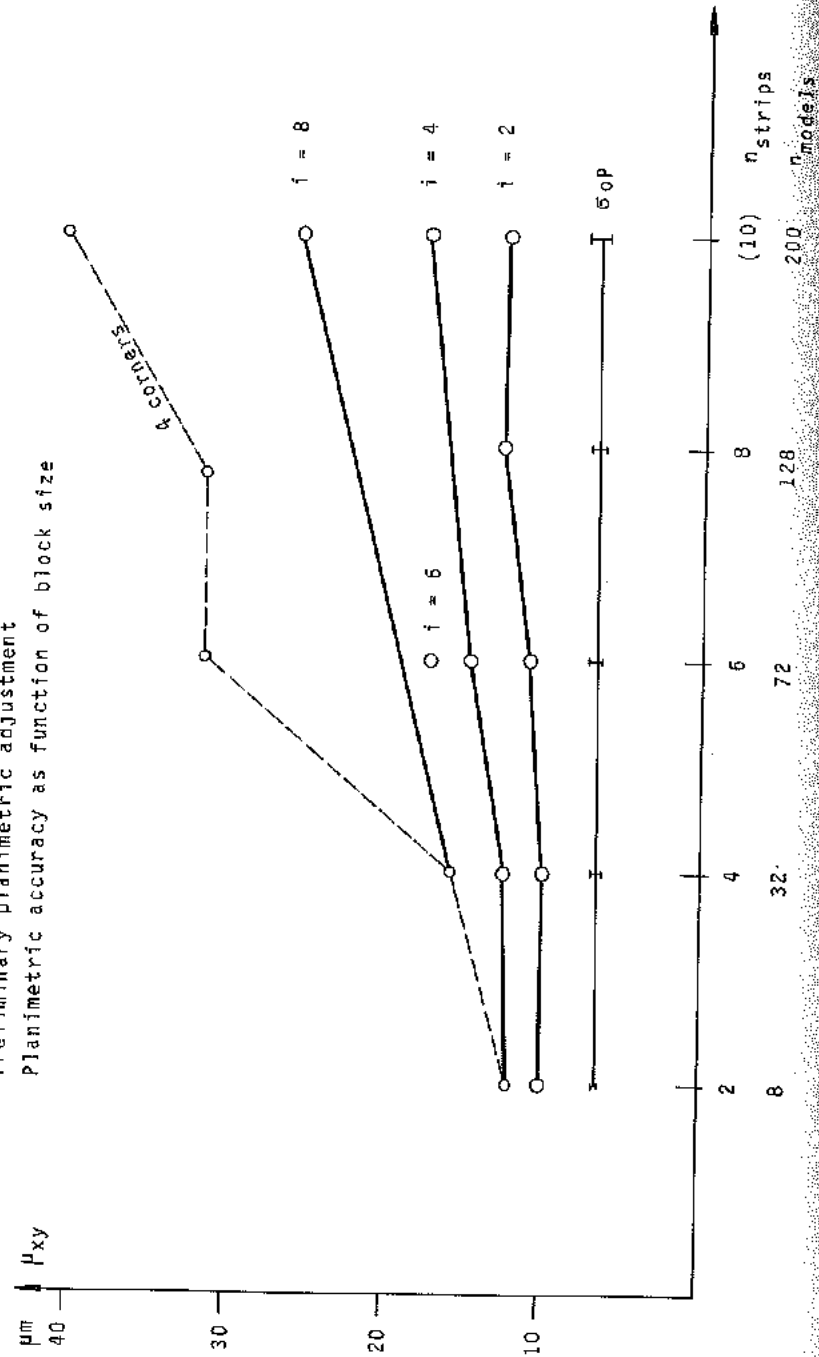


Figure 5

OEEPE - OBERSCHWABEN

Independent models - wide angle
Preliminary planimetric adjustment - Planimetric accuracy (μ_{xy})

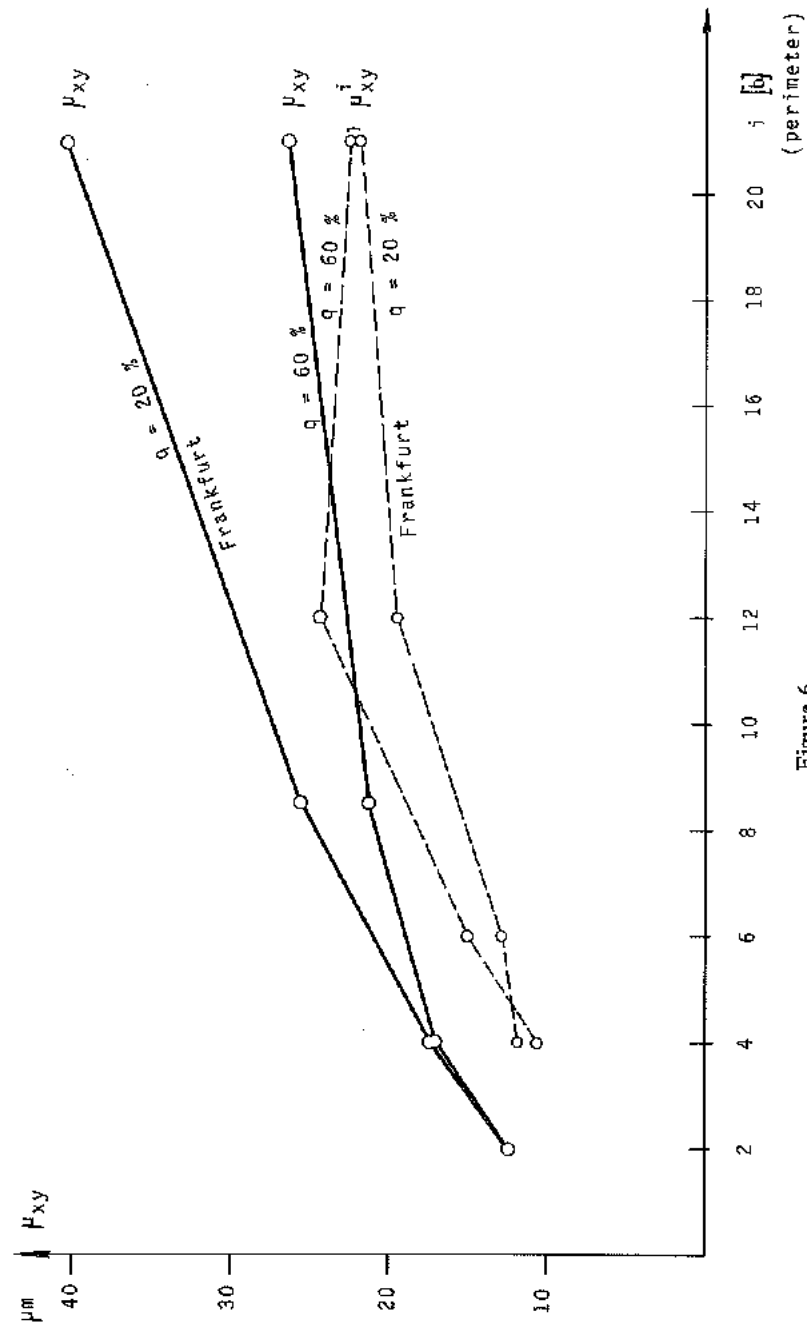


Figure 6

OEEPE - OBERSCHWABEN

Wide-angle, $q = 20\%$ - Independent models
Planimetric accuracy (μ_{xy})

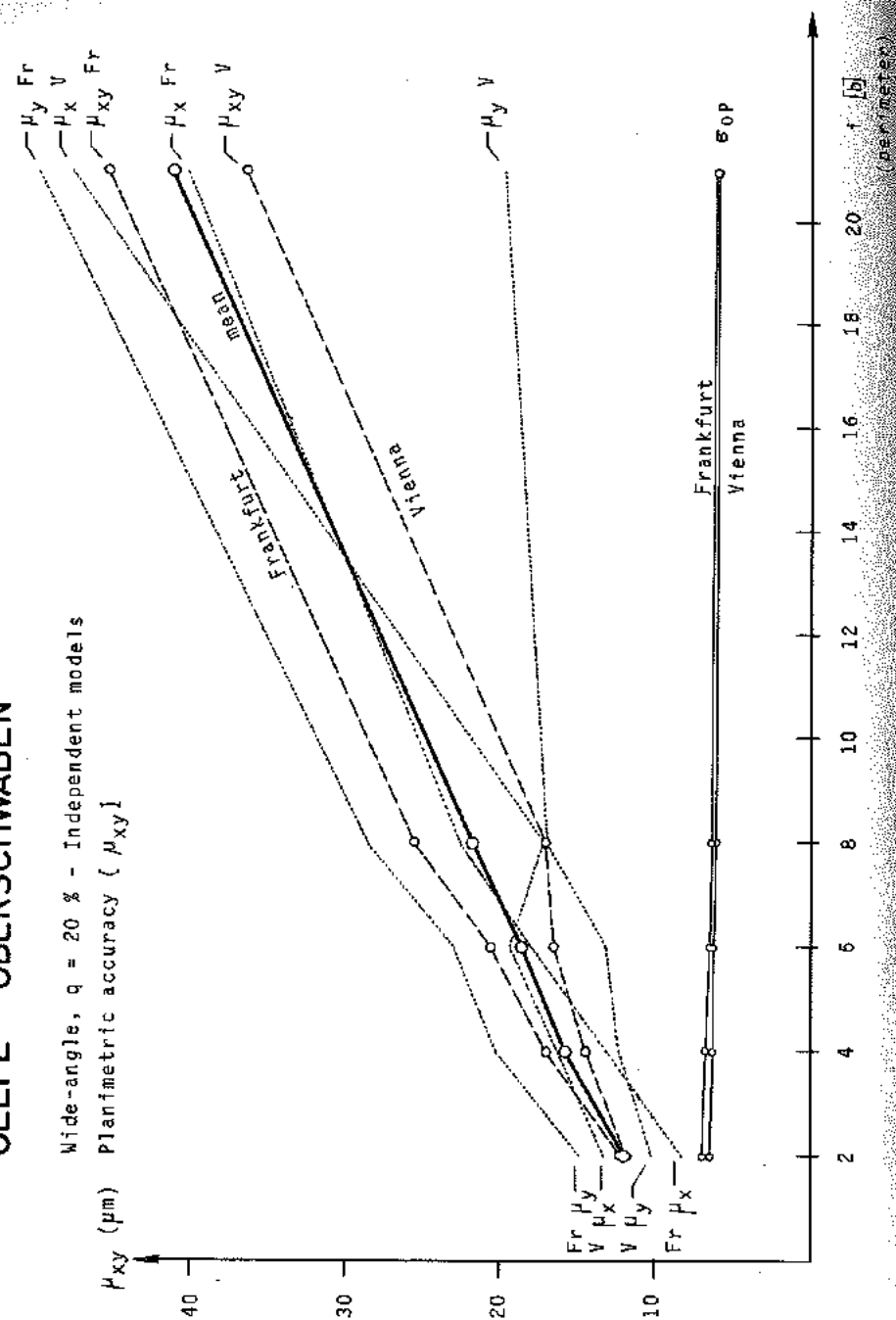


Figure 7

OEEPE - OBERSCHWABEN

Super-wide-angle, $q = 20\%$ - Independent models
Planimetric accuracy (μ_x, μ_y, μ_{xy})

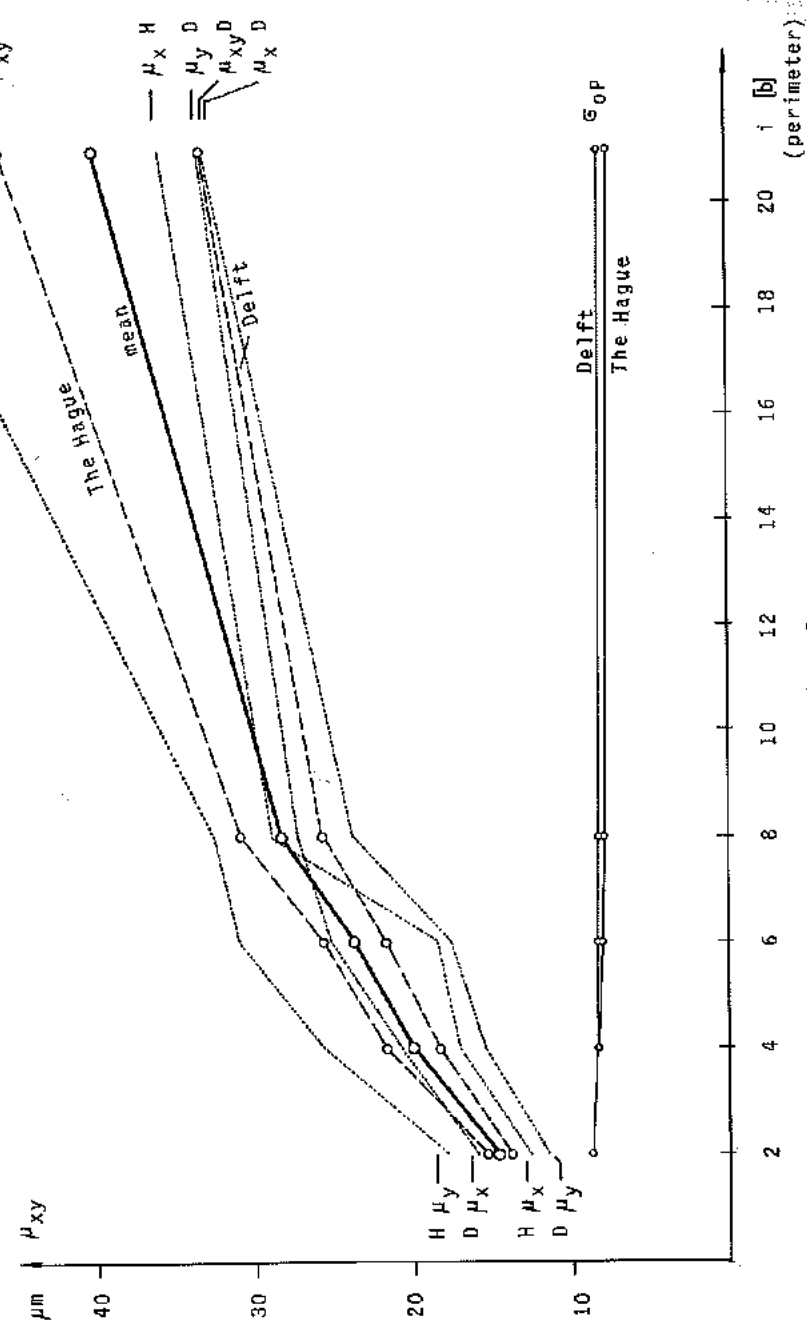
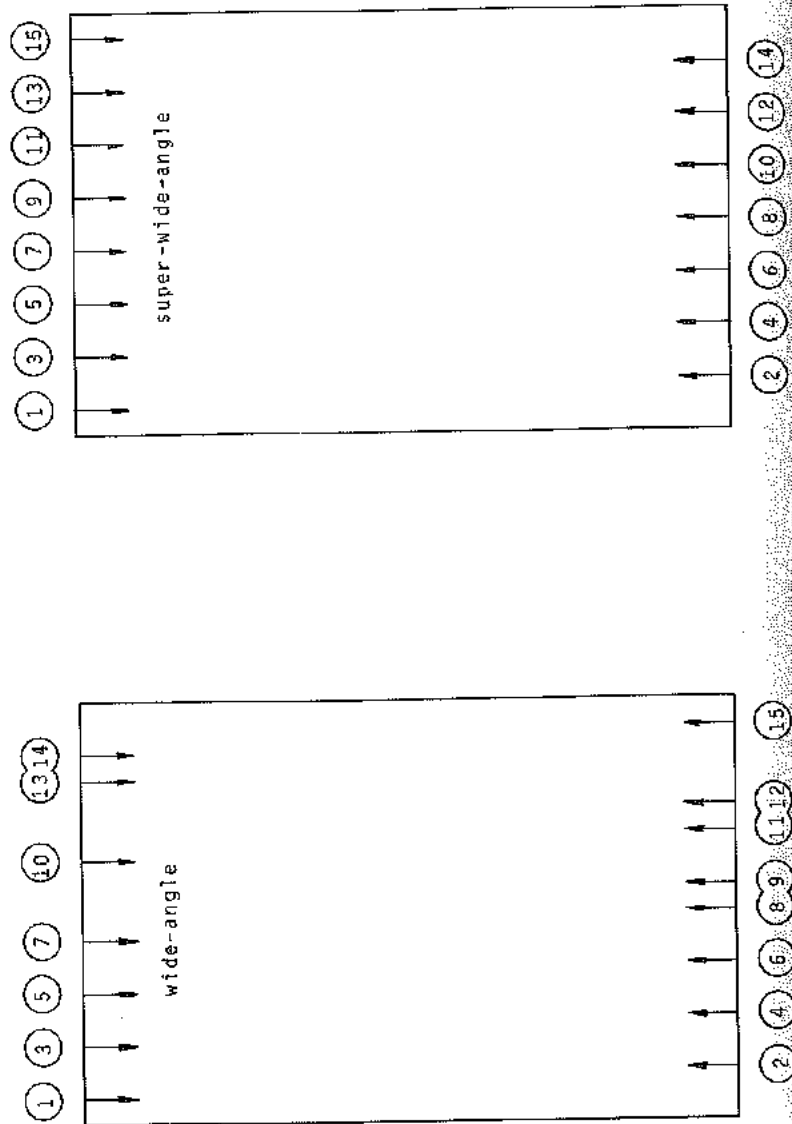


Figure 8

OEEPE - OBERSCHWABEN

Index of strips



OEEPE - OBERSCHWABEN

Independent models - $q = 60\%$
 Planimetric accuracy (μ_{xy})

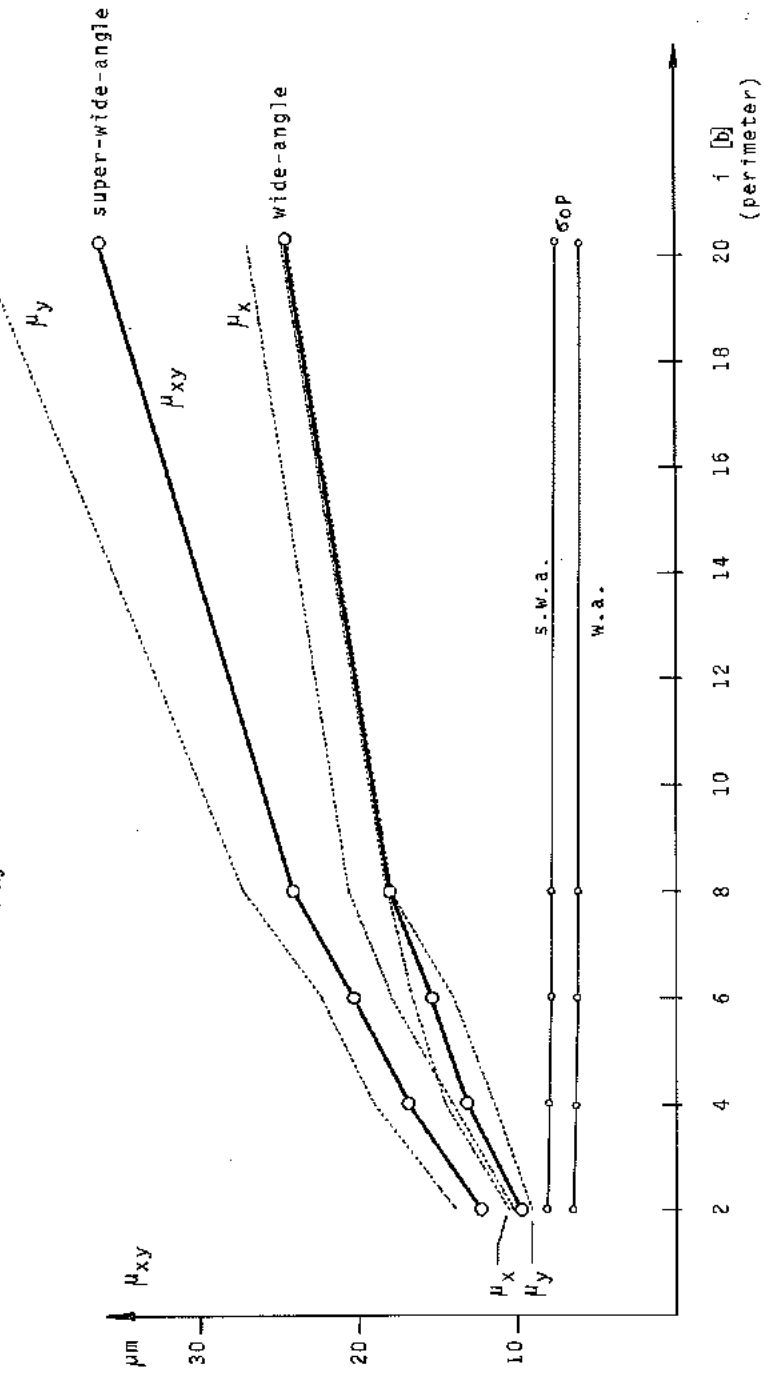
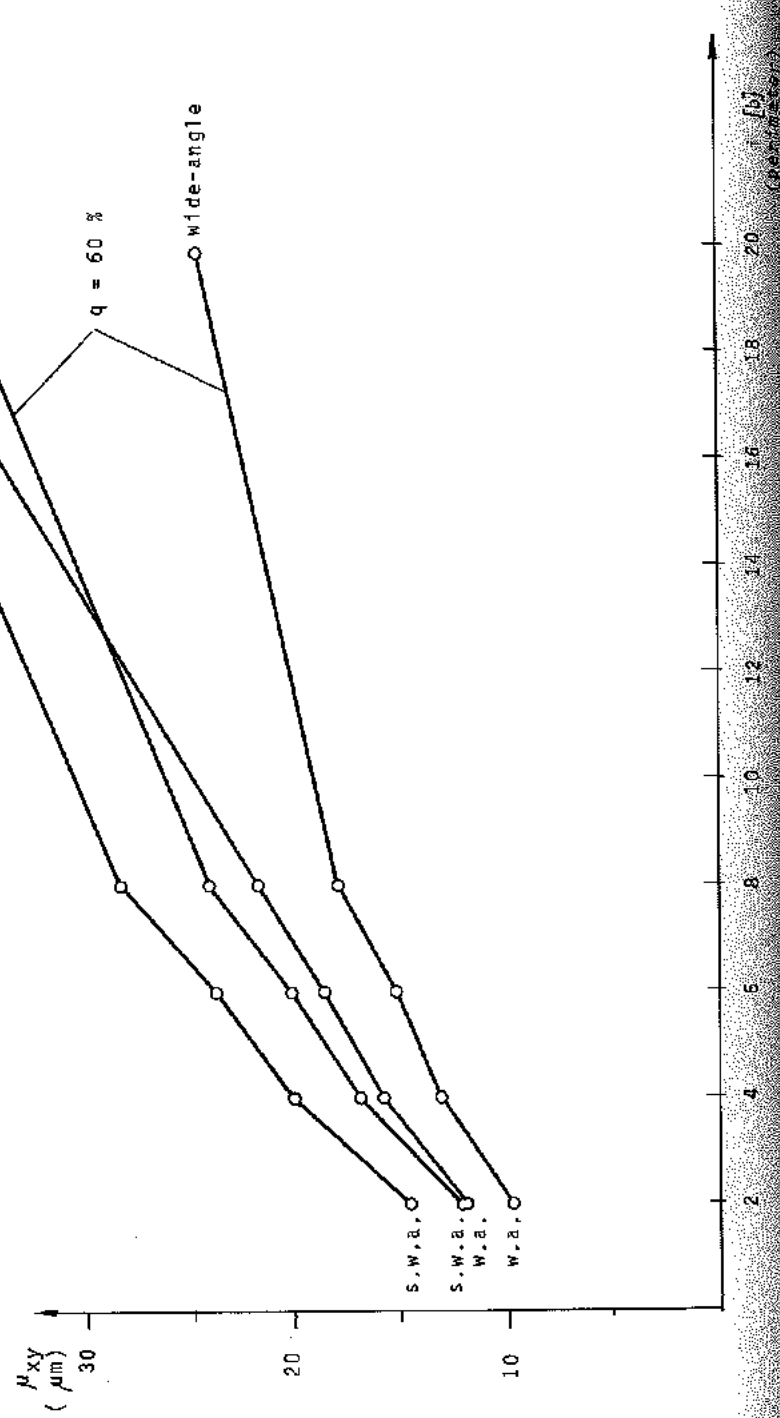


Figure 10

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Independent models
Planimetric accuracy (μ_{xy})



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Wide angle, $q = 20\%$ - independent models
Height accuracy (μ_z)

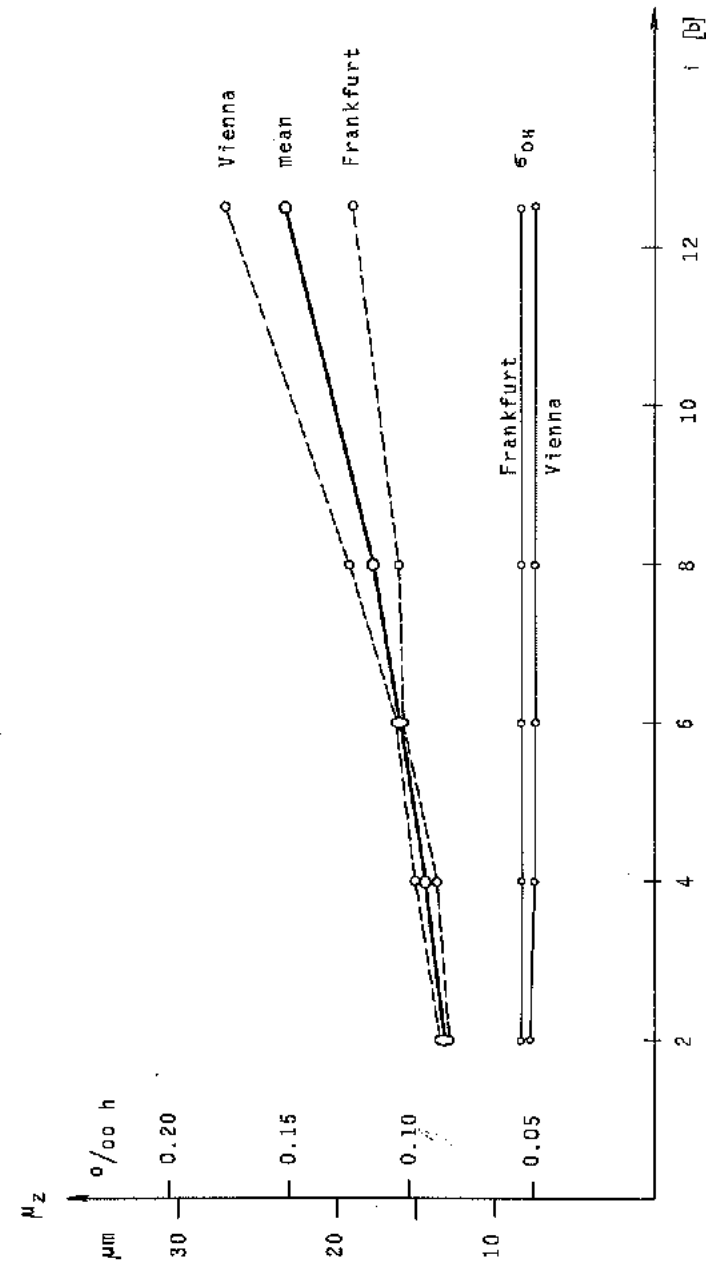


Figure 12

OEEPE - OBERSCHWABEN

Super-wide-angle, $q = 20\%$ - Independent models
Height accuracy (μ_z)

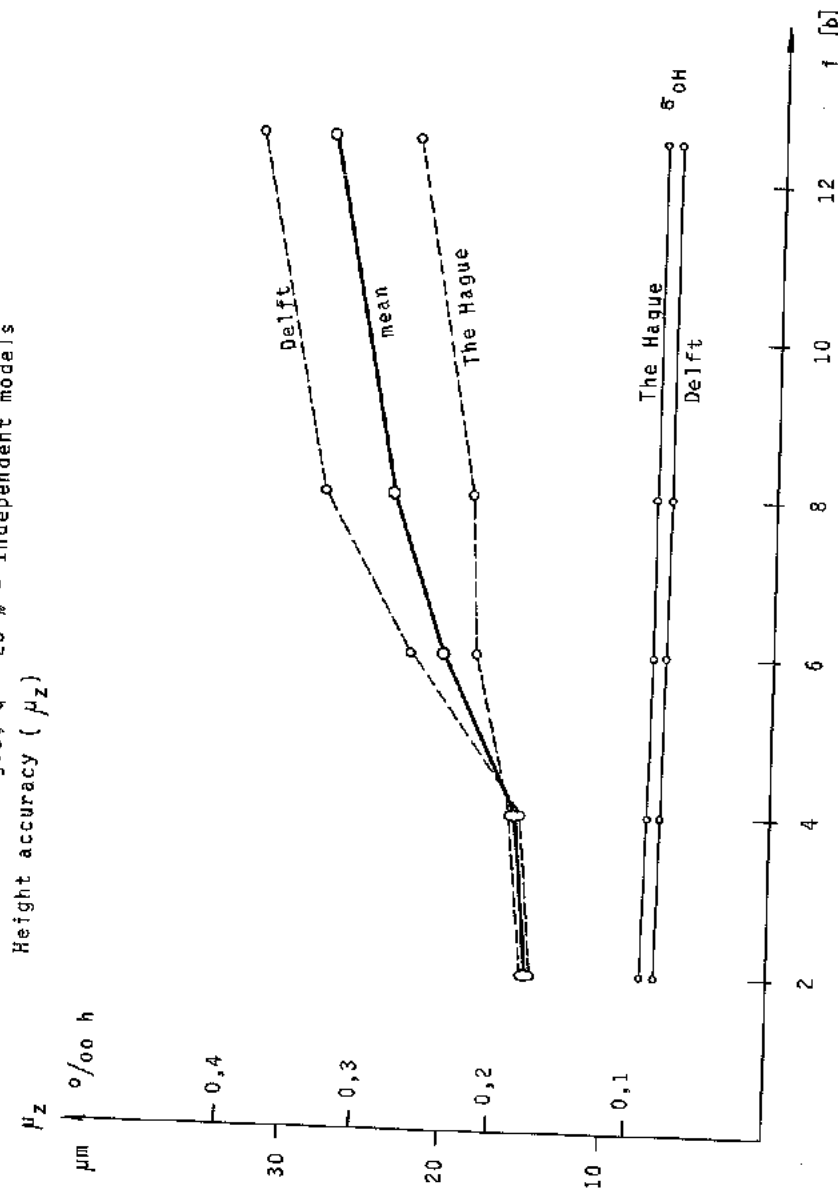


Figure 13

OEEPE - OBERSCHWABEN

Independent models, $q = 60\%$
Height accuracy (μ_z)

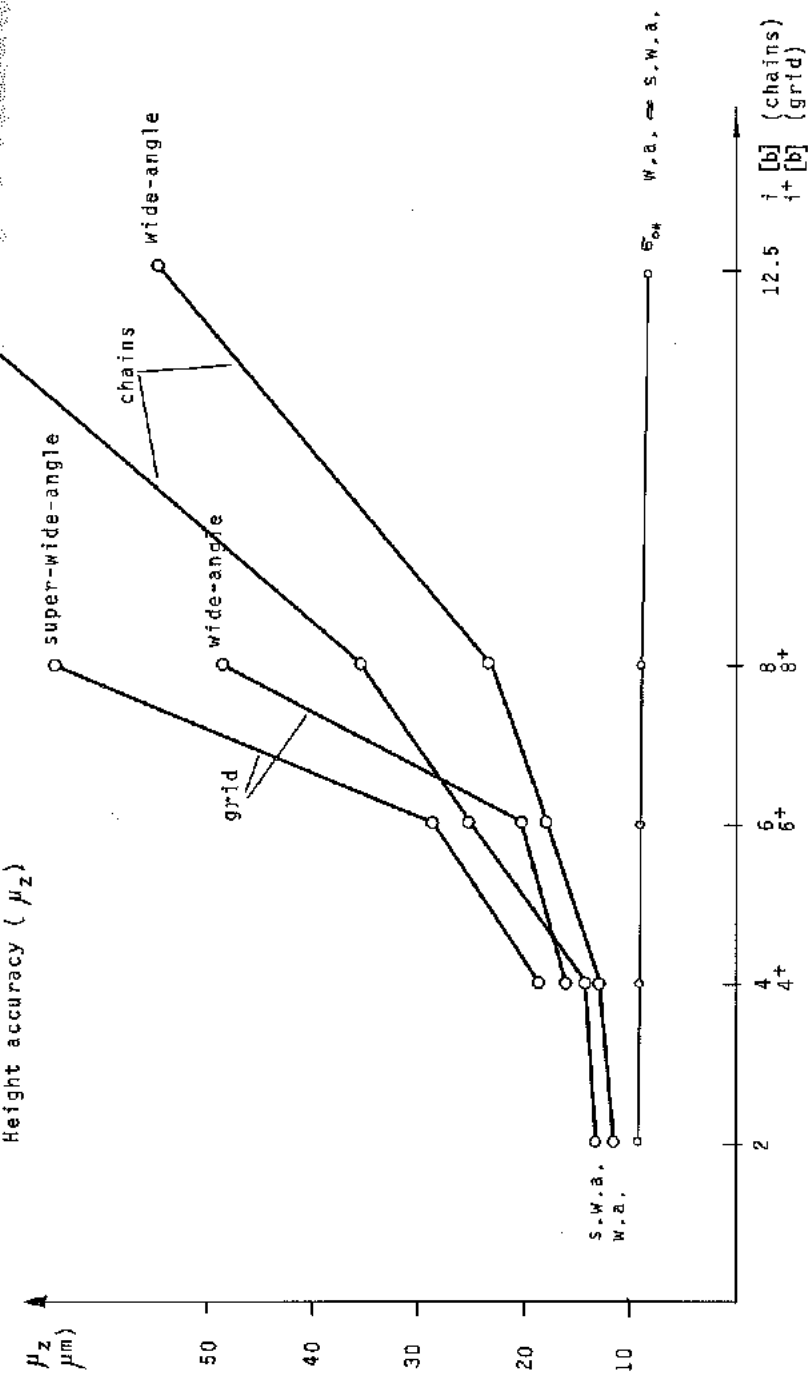


Figure 14

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Planimetric accuracy
Comparison of wide-angle and super-wide-angle

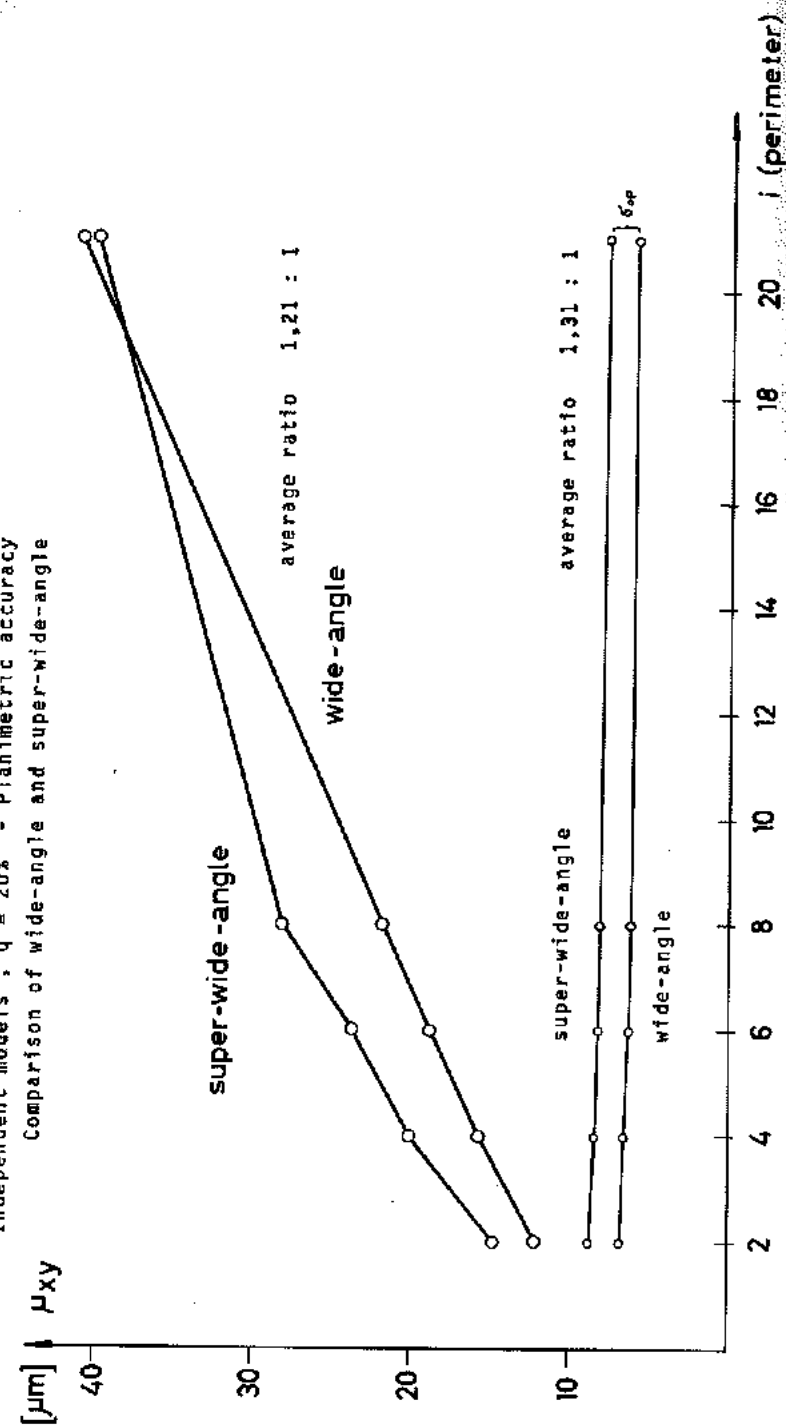


Figure 15

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Planimetric accuracy
Comparison of wide-angle and super-wide-angle

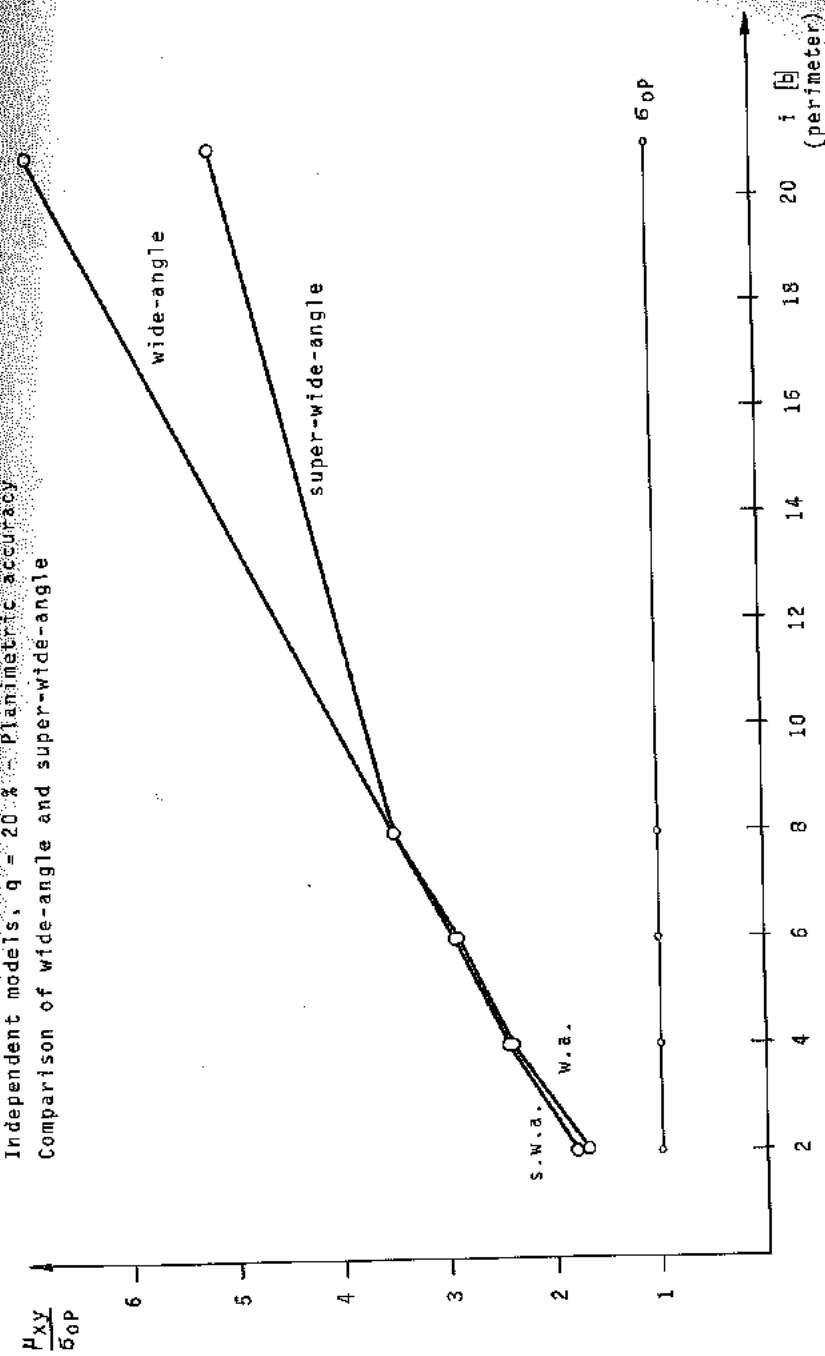


Figure 16

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Height accuracy
Comparison of wide-angle and super-wide-angle

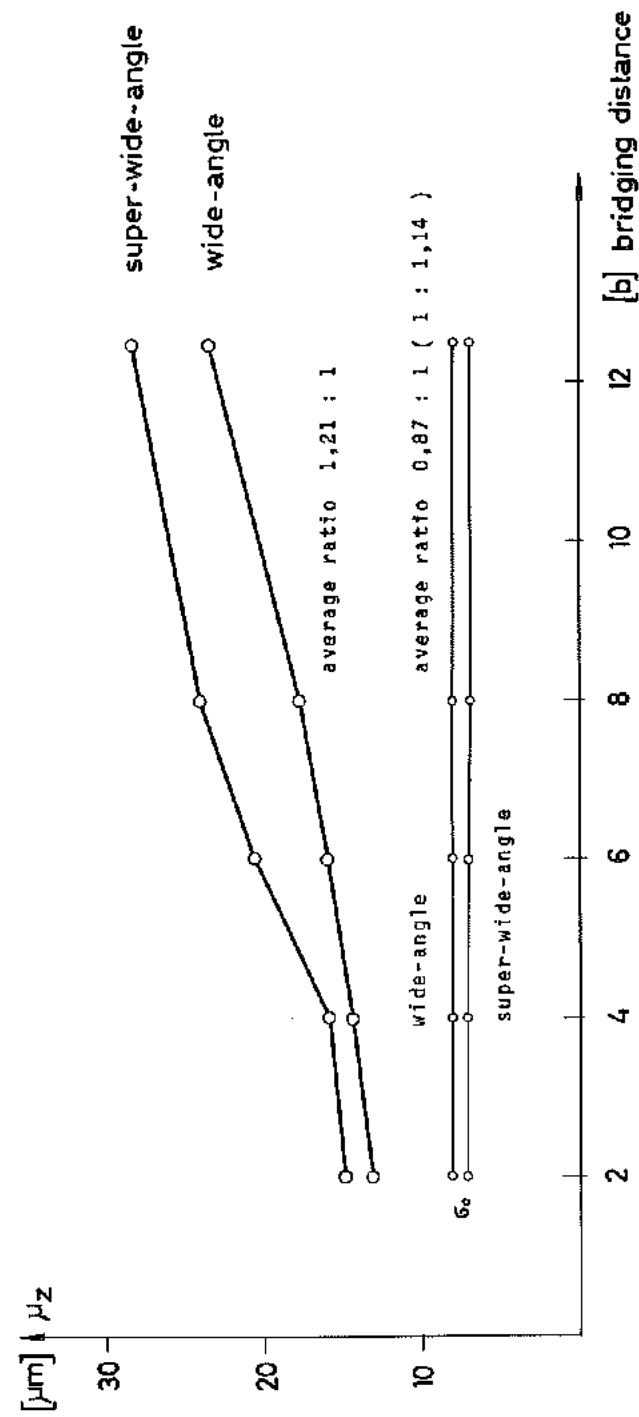


Figure 17

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Height accuracy
Comparison of wide-angle and super-wide-angle

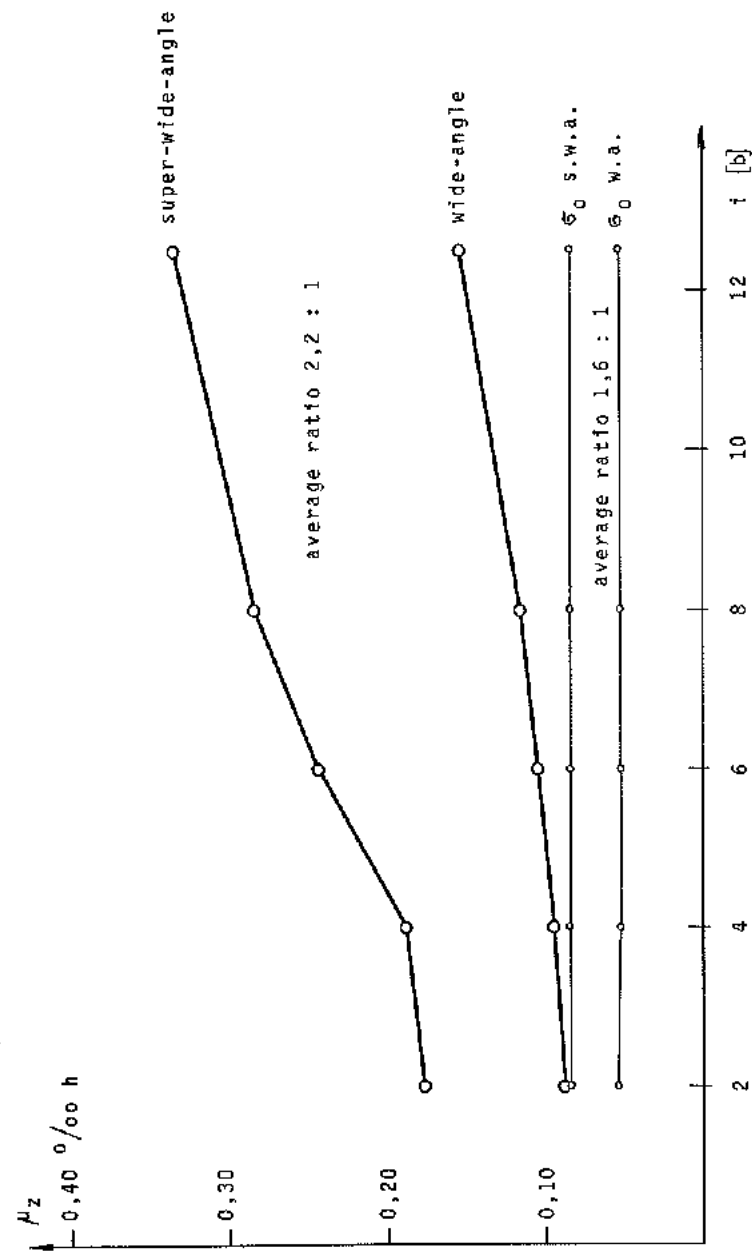
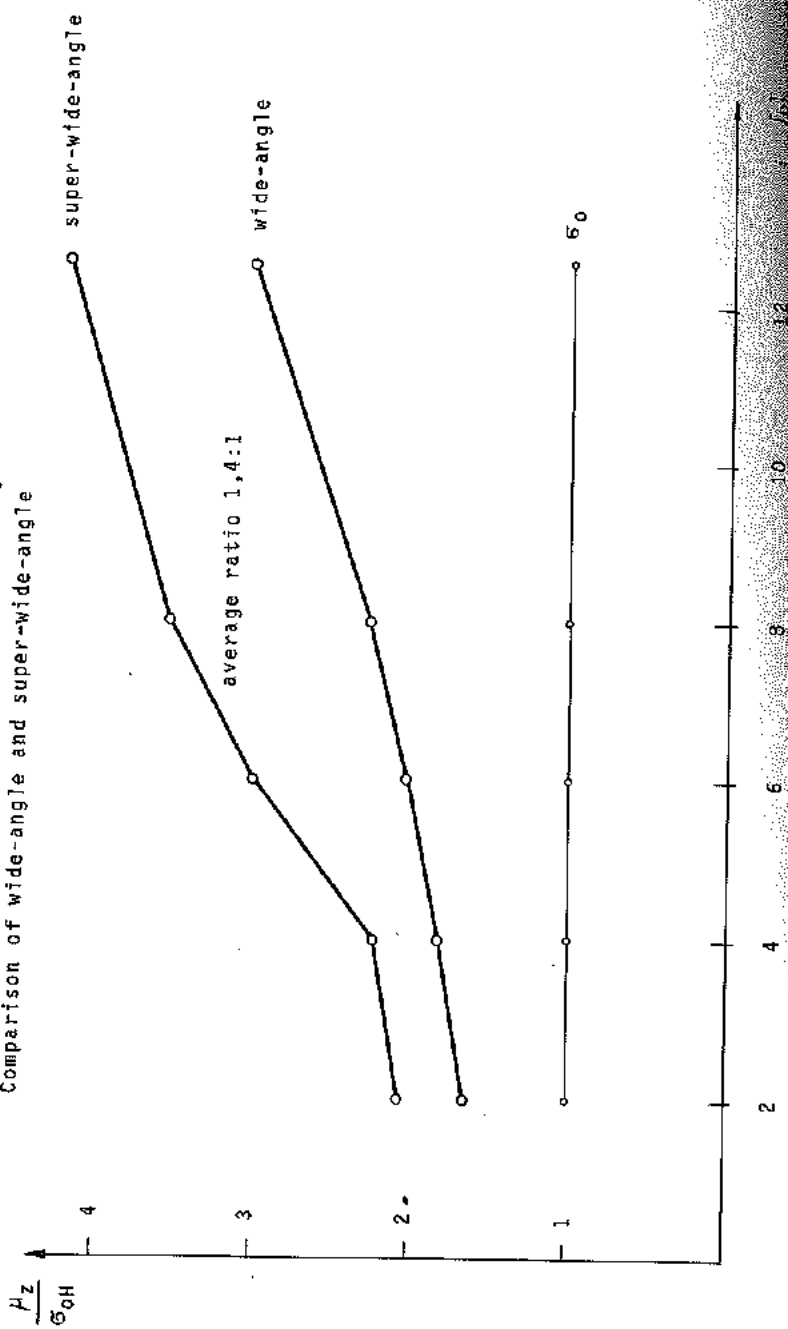


Figure 18

OEEPE - OBERSCHWABEN

Independent models, $q = 20\%$ - Height accuracy
Comparison of wide-angle and super-wide-angle



OEEPE - OBERSCHWABEN

Wide angle, $q = 20\%$ - Independent models
Height accuracy (μ_z / σ_{OH})

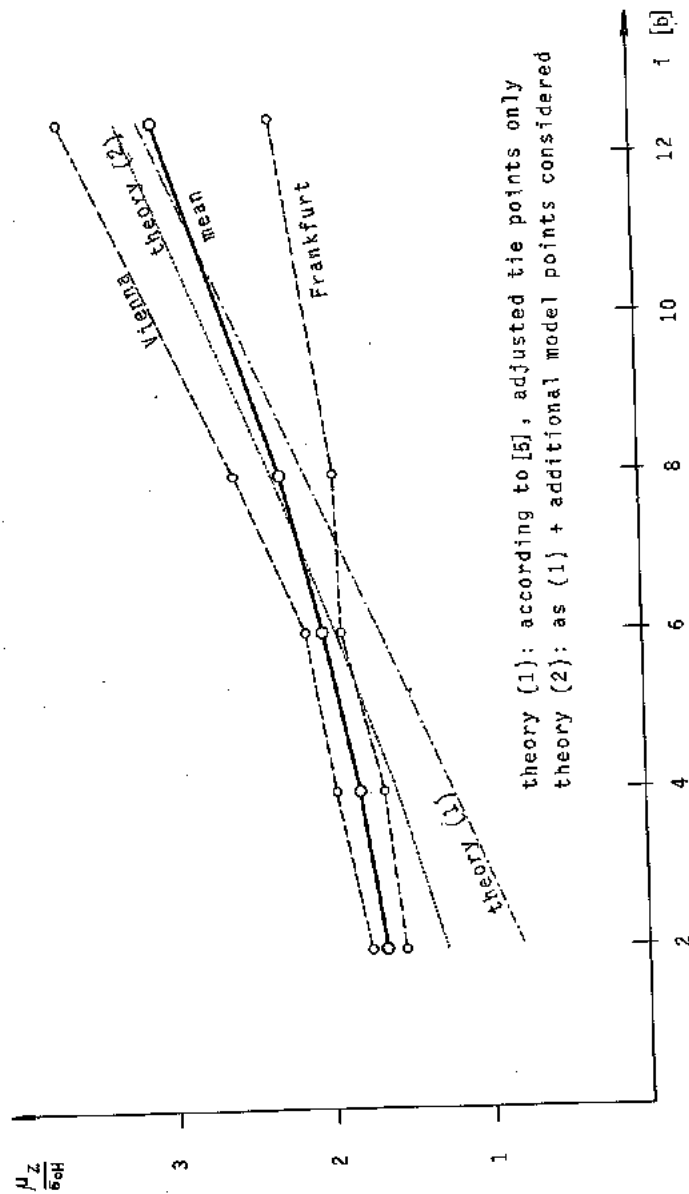
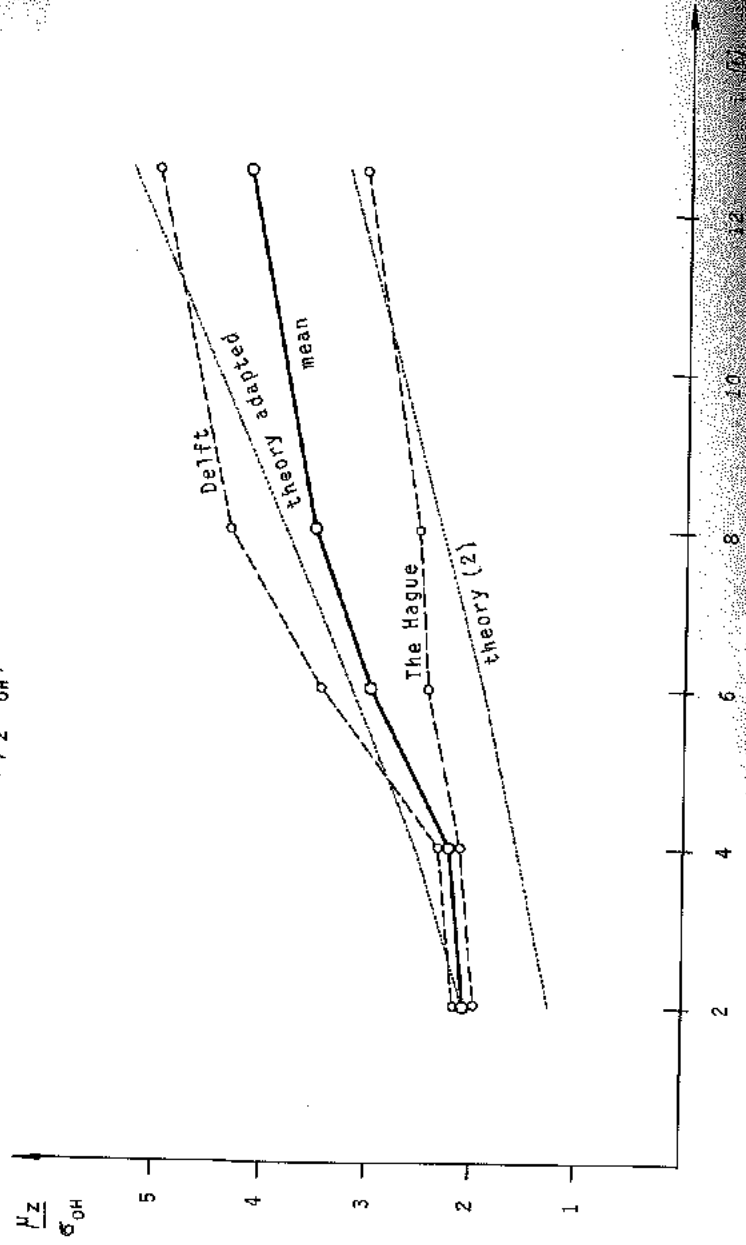


Figure 20

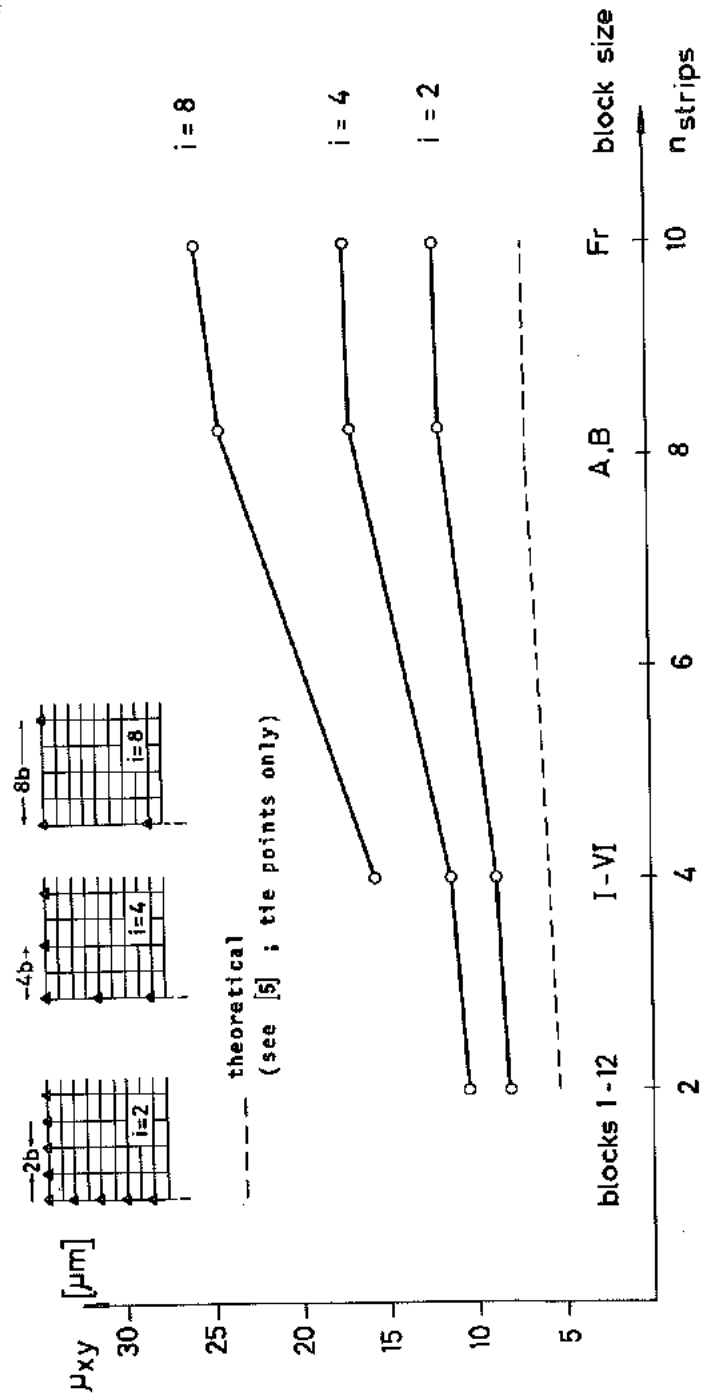
OEEPE - OBERSCHWABEN

Super-wide-angle, $q = 20\%$ - Independent models
Height accuracy (μ_z/σ_{OH})



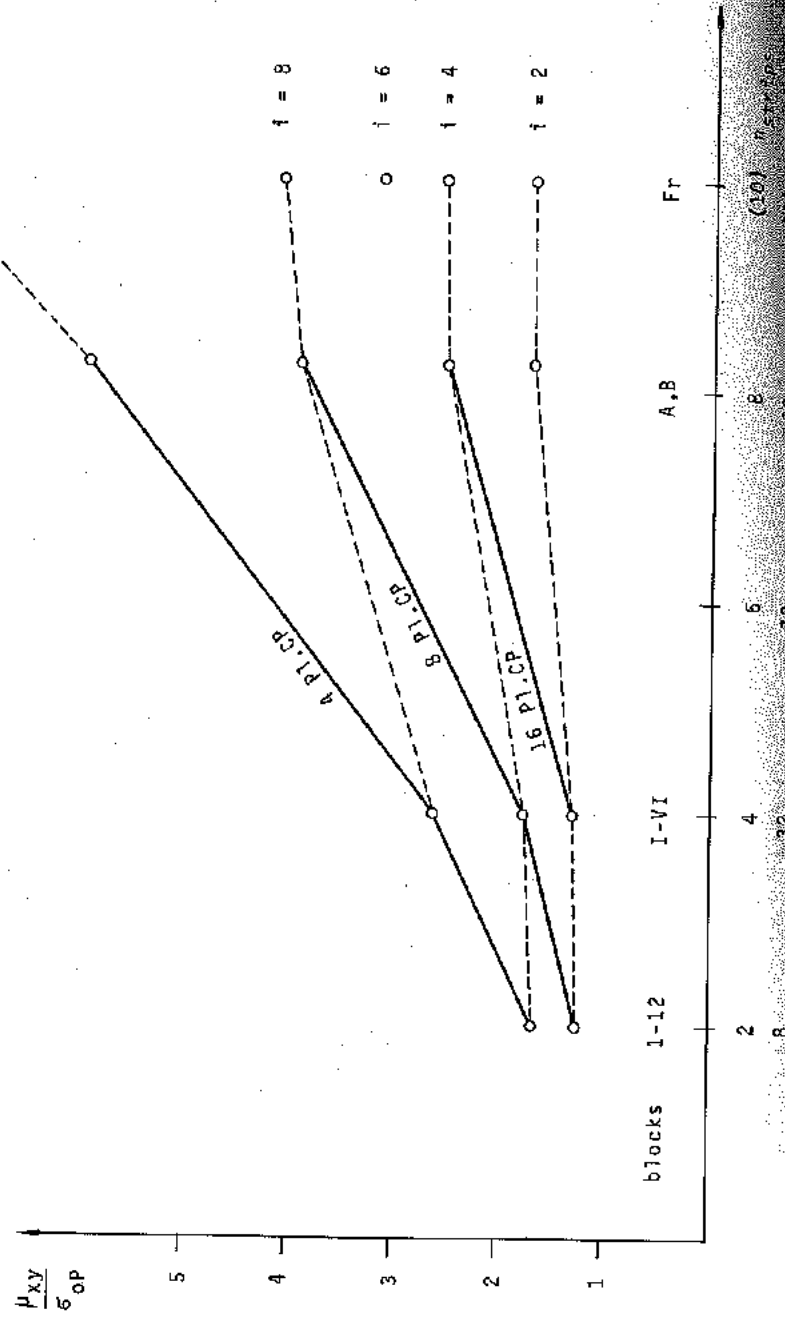
OEEPE - OBERSCHWABEN

Block Frankfurt (w.a.), Independent models, subblocks, perimeter control
Planimetric accuracy as function of block size



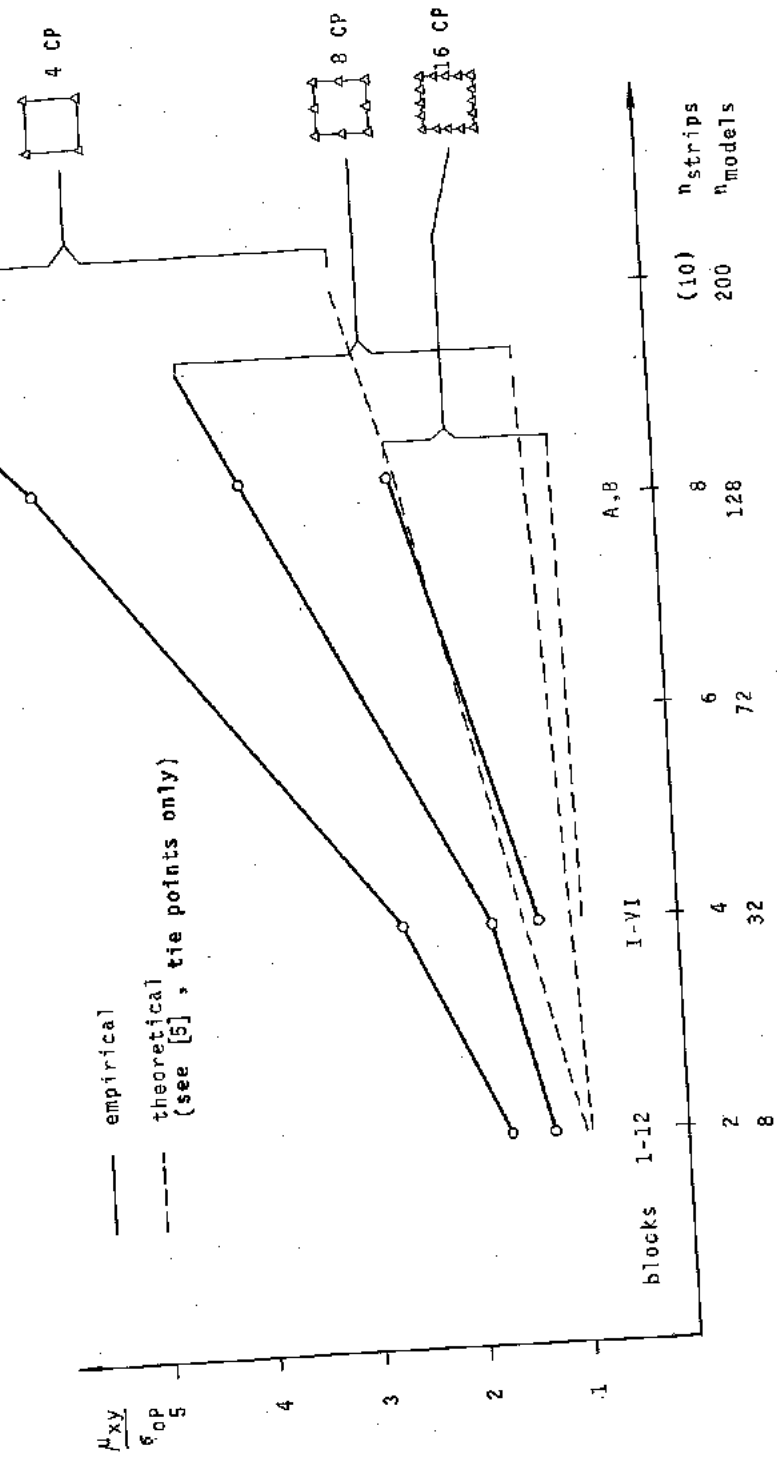
OEEPE - OBERSCHWABEN

Independent models - Block Frankfurt, subblocks
Planimetric accuracy as function of block size



OEEPE - OBERSCHWABEN

Block Frankfurt (wa) - Independent models, subblocks
Planimetric accuracy - Comparison with theory



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Block Frankfurt (w.a.) - Independent models, subblocks
Planimetric accuracy as function of control interval i

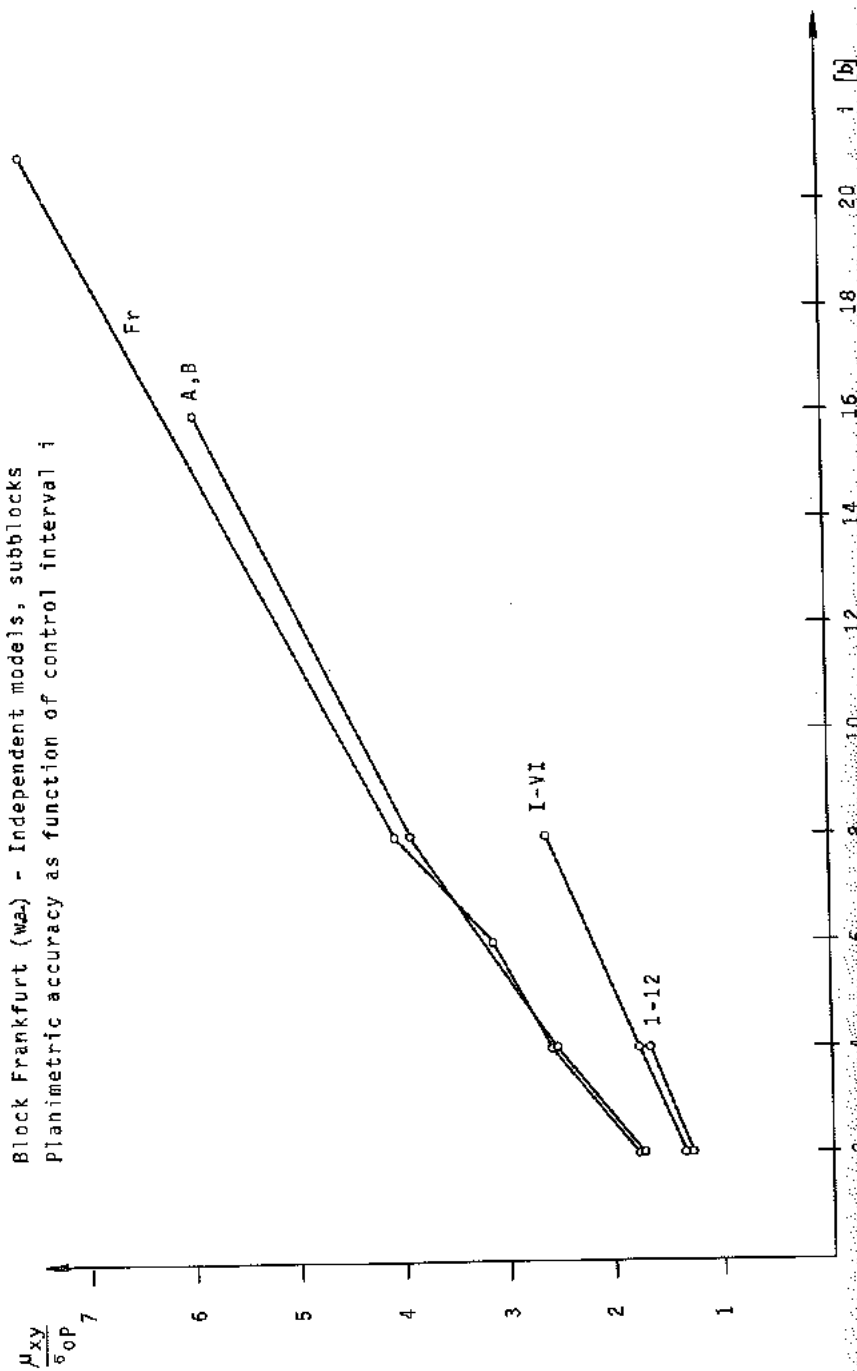


Figure 25

OEEPE - OBERSCHWABEN

Block Frankfurt (w.a.), Independent models, subblocks
Height accuracy as function of block size

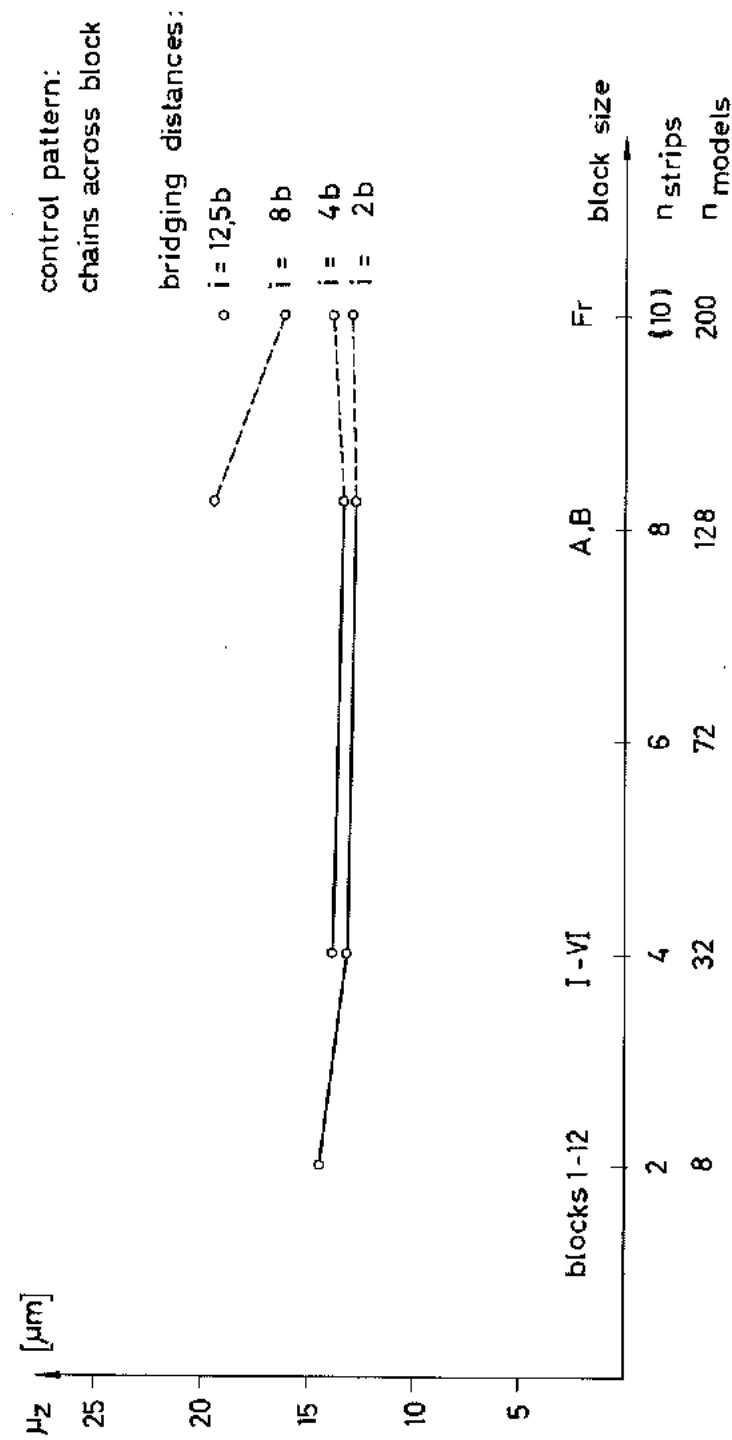


Figure 26

OEEPE - OBERSCHWABEN

Block Frankfurt - Independent models, subblocks
Height accuracy as function of bridging distance i

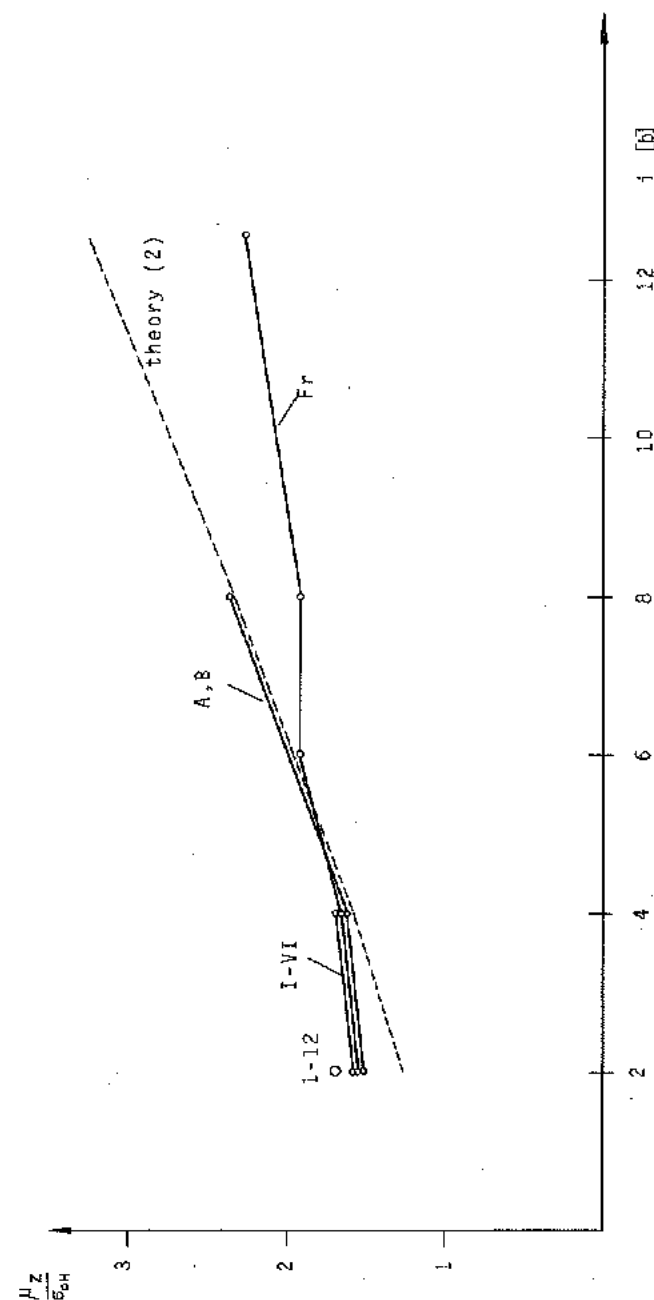


Figure 27

Discussion about the Subject 4 under the Chairmanship of Prof. G. Konecny

Chairman: The main objectives of Prof. Ackermann's paper appear to be the presentation of the practical results and their comparison with photogrammetric theory. Who wishes to open the discussion?

Meissl: I would like to ask Prof. Ackermann whether he considers the discrepancies found between theory and practice to be only due to the theory being wrong. Since we are only dealing here with one block, should we not also compute the reliability of the practical results.

Ackermann: Of course, there are always two effects to be considered: the possibly inadequate theory and the confidence interval due to the limited sample size. Notwithstanding proper statistical testing to come, the empirical results suggest sufficiently inadequacy of the theory. We have, in fact, just started to determine the statistical significance of the results by means of computer simulation.

Jerie: There appears to be a definite anomaly between the strip- and block-adjustment results of the SWA blocks in that the Delft strip-adjustment is more accurate than that of The Hague, but the situation is reversed in the block-adjustment.

Bauer: I have three short questions. What is the relative accuracy of the height control, given that the absolute accuracy is 20 cm? Secondly, how many of the height control-points were elevated? Finally, were the SWA blocks measured stereoscopically?

Ackermann: We have no special information on the relative accuracy of the height control. With regards to the observations, we did not prescribe a measuring procedure. Anyhow, also the SWA blocks were measured on stereocomparators.

Förstner: I can let you have the figures on the percentage of elevated points.

Proctor: If we are dealing with height accuracies in the order of cms, should we not also look into the effects of earth curvature and deformations of the geoid?

Wrobel: First of all, I would like to question Prof. Ackermann's remark that blocks with 60 % side overlap are not as interesting as those with 20 % side overlap. Secondly, would it not be of interest to also publish the maximum values of residuals in addition to the mean values?

Ackermann: There must be some misunderstanding as I did not say that the 60 % side overlap was less interesting in practice but only wished to caution users that the improvement in accuracy with 60 % instead of 20 % side overlap has not been as great as predicted theoretically. As to your second question, we intend to publish both mean and maximum values in the final report.

Jerie: The use of blocks with 60 % side overlap is of practical value for cadastral purposes in view of the double measurements made at each point. While I have the floor I would also like to remark that I do not consider a mesh of height control to be a practical proposition. I consider the establishment of bands of control to be more logical.

Ackermann: In this experiment we are testing a number of possibilities independent of which method of control determination may be preferred. The mesh pattern of control points could have advantages for the field work, be it only because of the greater freedom in placing the control points.

MacDonald: Prof. Jerie mentioned the discrepancy between the Delft and The Hague results in the SWA strips and blocks. A similar discrepancy can be observed between the results of the Vienna and Frankfurt WA blocks in planimetry and height. Finally, I would like to ask whether irregularly shaped blocks give worse results than regular blocks?

Ackermann: I doubt whether the discrepancy you mentioned is significant. As to your second question, the shape of the block is not really relevant provided that the perimeter is well-controlled. It has not been tested, however.

Chairman: I am afraid that we have to close this session now and I want to thank Prof. *Ackermann* for his contribution.

Ackermann: Before closing I would like to point out that the Oberschwaben material is available for individual investigations to OEEPE member countries.

OK 328-735-187(430-313.5):528.14

Comparison of Different Methods of Block Adjustment

(with 13 figures, 8 tables)

Prof. Ebner, Stuttgart

(RESUME see p. 155)

1. Scope and State of the Empirical Investigations

The test Oberschwaben was started to investigate the accuracy of strip and block triangulation, obtainable under practical conditions. The accuracy of blocks depends on the block parameters and on the method used for block adjustment. In the test Oberschwaben the polynomial block adjustment, the block adjustment by independent models and the bundle block adjustment are investigated.

The extensive empirical results of such a test may be used in two different ways. First they give the possibility to check theoretical accuracy models for block triangulation existing so far and secondly they allow for the determination of accuracy relations of the different methods of adjustment, based exactly on the same practical material.

The topic of this lecture is the presentation of the results obtained by polynomial block adjustment and by bundle block adjustment and the comparison with the corresponding results of independent model block adjustment, presented by Prof. *Ackermann* [1].

As a result of the great computational demand up to now only the block Frankfurt (wide angle, 20% side lap) was adjusted by the polynomial method as well as by the bundle method. With this block the effect of the control distribution on the accuracy was investigated completely. Against that the accuracy dependency on the block size isn't studied yet.

For that reasons only an intermediate report without a critical statistical valuation can be given presently. However already the results obtained up to now allow for some important conclusions and indicate very clearly in which direction further research and development activities are necessary.

2. Control Point Distributions

In accordance with the results of preceding theoretical investigations on most favourable control point distributions for planimetry and height the types, presented in figure 1 were used [2]. The only one degree of freedom is the bridging distance i expressed in units of the base length.

To allow for a valid comparison of the different methods of block adjustment not only the same types but even exactly the same control points were used with polynomials, independent models and bundles.

3. Polynomial Block Adjustment

The polynomial block adjustments were performed at the IfAG Institute in Frankfurt, using a Telefunken TR 440 computer. For that I want to thank Prof. *Förstner* and

Mr. Nüßlein very much. The necessary strip formations started from the cleared up model coordinates which had already been used in the independent model block adjustments.

For the polynomial adjustment Dr. Schut's iterative program, version 1966 was used [3]. The basic transformation is conformal in x and y . In planimetry as well as in height second degree polynomials were chosen. According to the independent model block adjustments weight 1 was given to the photogrammetric coordinates of tie points and of control points.

The results obtained with the five different control versions are represented in table 1. In height the control distributions used are practically the same as with the strip adjustments treated by Dr. Stark [4]. A comparison of the corresponding root mean square values μ shows nearly no difference between blocks and strips.

A very important question in combination with the empirical results presented of course is the level of confidence of the different values μ . A few repeated simulations performed with independent models have shown that the standard deviation of $\mu_{x,y}$ is in the order of 5 % of $\mu_{x,y}$ in case of a dense perimeter control, but even 20 % of $\mu_{x,y}$ if 4 control points are used only. However, as far as the accuracy ratios between the different methods of block adjustment are concerned we may expect a much higher level of confidence due to the fact that exactly the same material is used with the different methods.

In figure 2 and 3 the polynomial results are plotted, together with the corresponding results of the block adjustments by independent models. Table 2 contains the accuracy ratios between polynomials and independent models.

In planimetry the degree of inferiority of the polynomial results depends strongly on the number of control points used. With a dense perimeter control the ratio is 2.13. If only 4 control points are available the ratio decreases to 1.36.

In height the polynomial results are inferior too, but there is practically no dependency on the control density. The accuracy ratio varies between 1.66 and 1.85 only.

Further computations have to show whether the accuracy of the polynomial block adjustments can be improved by different weight assumptions or by third degree polynomials.

Valuating the results two aspects have to be distinguished. On the one hand the test confirms that quite good results can be obtained by polynomial block adjustment. On the other hand the improvement of the results by application of block adjustment with independent models, being a different computational treatment only is really remarkable.

4. Bundle Block Adjustment

The bundle block adjustments were performed at the Institute of Photogrammetry in Stuttgart, using a Control Data CDC 6600 computer and the program PAT-B [5]. Previously the original image coordinates were cleared up with the same care as the model coordinates before. The weight 1 was used for all image coordinates. In accordance with the adjustment by independent models the control points were treated as errorfree. For the performance of the bundle block adjustments I want to thank Mr. Schneider heartily.

The results obtained are shown in table 3. The standard deviation σ_0 of the unit weight here represents the mean accuracy of an image coordinate. As we see σ_0 depends strongly on the control distribution used. With independent model block adjustment the corresponding values σ_{op} and σ_{oh} , representing the mean accuracy of model coordinates in planimetry and height, were influenced by the control distribution only to a much smaller extend [1].

A comparison of μ_z in the last column of table 3 with the corresponding values after the bundle strip adjustments, presented by Dr. Stark [4], shows similar results. Only with the bridging distance $l = 12$ the result of block adjustment is significantly superior.

In table 4 theoretical accuracy models for bundle block triangulation are shown. They have been derived very recently and are based on random errors only [6]. The standard deviations σ_{mean} are theoretical expectations of the empirical RMS values μ . Figure 4 and figure 5 give a comparison of the theoretical and empirical accuracy. There is a strong disagreement between theory and test. It indicates the existence of systematic image errors being not compensated by the bundle block adjustment.

Two other facts indicate existing systematic errors too. Firstly, in theory there is practically no influence of the height control configuration on the planimetric accuracy and the height accuracy is nearly independent of the planimetric control distribution. The empirical results however confirm the first theoretical result only. Against that table 5 shows that μ_z is influenced very much by the planimetric control configuration. The same is true for σ_0 .

Secondly, the theory based on random errors only let expect a change to the worse in accuracy when the number of tie points is reduced. But the opposite happened with the bundle block adjustments of the Oberschwaben material. Reducing the number of tie points per each photo from 18 to 9 the accuracy was improved as shown in table 6.

In figure 6 and figure 7 the accuracy obtained by bundle block adjustment is compared with the accuracy after independent model block adjustment. In both figures the corresponding results are plotted, whilst table 7 contains the accuracy ratios of both methods.

Here we have the most surprising results of the test: independent models give a better accuracy than bundles. The accuracy ratio varies between 1.04 and 1.35 in planimetry and between 1.17 and 1.41 in height. The denser the control the more the results of independent models are superior.

This comparison indicates that, in spite of its principal generality the bundle block adjustment obviously is more sensitive against systematic errors than the block adjustment by independent models.

5. Concluding Remarks

Starting from the same practical material of the block Frankfurt and the same level of data clearance the highest accuracy after block adjustment was obtained by the method of independent models. Table 8 furthermore indicates most favourable computer running times for independent models. By that facts the high efficiency of the model block adjustment method is demonstrated under practical conditions.

Taking an average the bundle results are worse at 25 % and the polynomial results at 30 % approximately. Whilst the inferiority of the polynomial block adjustment was expected, the worse results of the bundle block adjustment are most surprising. Responsible for this fact seem to be the systematic errors, being present in the material. This systematic errors obviously impair the bundle block adjustment more than the block adjustment by independent models.

To show the effect of those systematic errors in figure 8 to figure 13 the planimetric difference vectors at the check points are plotted for the control versions 1 and 5 and for the three adjustment methods used. With the same planimetric control all three methods show rather similar vector distributions. Obviously the main error is an affinity in distance and angle. It should be mentioned here that no a priori correction of image or model coordinates was performed. From such corrections based on camera calibrations and fiducial mark measurements a real improvement of accuracy can be expected (see for instance [7]).

The test Oberschwaben demonstrates, that systematic errors can exercise a large influence over the results of block adjustments. Based on theoretical studies this was pointed out already earlier by Dr. Kubik [8].

As far as further research and development activities are concerned from the results obtained it follows that we have to take into account rigorously not only random errors but systematic errors too. The most favourable way doing this today is the introduction of additional parameters into the block adjustment to compensate for the systematic errors as far as possible. A first successful step in this direction was done by Dr. Bauer and Dr. Müller [9].

The main question concerning block adjustment in such an extended form is the proper selection of the type and number of the additional parameters. The solution of this question can be optimised by a concept treating these parameters not as free unknowns but as observations. More about this concept may be found in [10].

Based on this extension general computer programs for bundle block adjustment and for block adjustment by independent models should be developed. Applying these programs in practice we may expect to meet the accuracy results as predicted by the theoretical investigations.

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- [10] Ebner, H.: Zusätzliche Parameter in Ausgleichungen. ZfV (1973), 385–391.

RÉSUMÉ:

On examine la précision empirique de trois procédés de compensation par blocs spatiaux selon la répartition des points de contrôle en se basant sur le même matériel pratique du bloc Francfort (grand'angulaire, 20 % de recouvrement latéral). Comme on pouvait s'y attendre, la compensation par blocs avec polynômes se montre la moins efficace des trois méthodes. Ce qui surprend par contre, c'est que la compensation par blocs avec modèles indépendants donne une plus grande précision que la compensation par faisceaux pourtant théoriquement plus rigoureuse. Ce fait s'explique par les erreurs systématiques qui, apparemment, se propagent moins favorablement dans les blocs à faisceaux que dans les blocs à modèles indépendants. En raison de ces résultats on recommande pour l'avenir une compensation des erreurs systématiques aussi universelle et aussi automatique que possible pendant la compensation par blocs.

Table 1

Version	control points		check points		μ [m]		μ [m]		μ [μ m]	
	x, y	z	x, y	z	x	y	x, y	z	x, y	z
1	40 $i = 2$	117 $i = 2$	472	334	0.590	0.832	0.721	0.668	26	24
2	20 $i = 4$	75 $i = 4$	492	376	0.704	0.958	0.840	0.674	30	24
3	14 $i = 6$	53 $i = 6$	498	398	0.812	0.936	0.876	0.740	31	26
4	10 $i = 8$	42 $i = 8$	502	409	0.728	1.163	0.970	0.769	35	27
5	4 $(i = 20)$	31 $i = 12$	508	420	0.969	2.241	1.727	0.919	62	33

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

μ = RMS of the coordinate differences at check points

Block Frankfurt: wide angle, 20 % side lap

Empirical accuracy after polynomial block adjustment

Table 2

Version	control points		check points		accuracy ratio		accuracy ratio	
	x, y	z	x, y	z	x	y	x, y	z
1	40 $i = 2$	117 $i = 2$	472	334	2.54	1.99	2.13	1.85
2	20 $i = 4$	75 $i = 4$	492	376	1.91	1.69	1.75	1.74
3	14 $i = 6$	53 $i = 6$	498	398	1.61	1.45	1.51	1.66
4	10 $i = 8$	42 $i = 8$	502	409	1.16	1.47	1.35	1.71
5	4 $(i = 20)$	31 $i = 12$	508	420	0.86	1.60	1.36	1.73

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

Block Frankfurt: wide angle, 20 % side lap

Comparison of polynomials and independent models

Table 3

Version	control points		check points		μ [m]		μ [m]		σ_0 [μ m]	μ [μ m]	
	x, y	z	x, y	z	x	y	x, y	z		x, y	z
1	40 $i = 2$	117 $i = 2$	469	330	0.275	0.512	0.411	0.510	5.7	15	18
2	20 $i = 4$	75 $i = 4$	489	372	0.473	0.732	0.616	0.534	5.0	22	19
3	14 $i = 6$	53 $i = 6$	495	394	0.714	0.845	0.782	0.622	4.7	28	22
4	10 $i = 8$	42 $i = 8$	499	405	0.836	0.880	0.858	0.533	4.3	31	19
5	4 $(i = 20)$	31 $i = 12$	505	416	1.285	1.351	1.318	0.623	4.0	47	22

i planimetry: perimeter control points in a distance of i base lengths
 i height: chains of control points in a distance of i base lengths
 μ = RMS of the coordinate differences at check points

Block Frankfurt: wide angle, 20 % side lap
 bundle block adjustment

Table 4

Theoretical accuracy after bundle block adjustment

wide angle 20 % side lap

Planimetry: perimeter control points in a distance of i base lengths

$$\sigma_{x \text{ mean}} \sim \sigma_{y \text{ mean}} \sim (0.7 + 0.011i^2) \sigma_0$$

(block size 10 strips, 21 photos each)

Height: chains of control points in a distance of i base lengths

$$\sigma_z \text{ mean} \sim (1.2 + 0.16i) \sigma_0$$

(independent of block size)

Table 5

control points		σ_0 [μ m]	μ_z [μ m]
x, y	z		
40 $i = 2$	117 $i = 2$	5.7	18
4 $(i = 20)$	117 $i = 2$	4.0	13
40 $i = 2$	31 $i = 12$	5.6	28
4 $(i = 20)$	31 $i = 12$	4.0	22

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

μ = RMS of the coordinate differences at check points

Block Frankfurt: wide angle, 20 % side lap
 bundle block adjustment
 effect of planimetric control on height accuracy

Table 6

control version	x, y	z
1	16 %	5 %
2	9 %	0
3	3 %	1 %
4	0	-2 %
5	2 %	-8 %

Block Frankfurt: wide angle, 20 % side lap
 bundle block adjustment
 accuracy improvement by reducing the number
 of tie points per photo from 18 to 9

Table 7

Version	control points		check points		accuracy ratio	
	x, y	z	x, y	z	x	y
1	40 $i = 2$	117 $i = 2$	469	330	1.19	1.23
2	20 $i = 4$	75 $i = 4$	489	372	1.29	1.29
3	14 $i = 6$	53 $i = 6$	495	394	1.41	1.31
4	10 $i = 8$	42 $i = 8$	499	405	1.33	1.11
5	4 $(i = 20)$	31 $i = 12$	505	416	1.14	0.97
					1.22	1.41
					1.29	1.38
					1.35	1.40
					1.20	1.18
					1.04	1.17

i planimetry: perimeter control points in a distance of i base lengths
 i height: chains of control points in a distance of i base lengths

Block Frankfurt: wide angle, 20 % side lap
 Comparison of bundles and independent models

Table 8

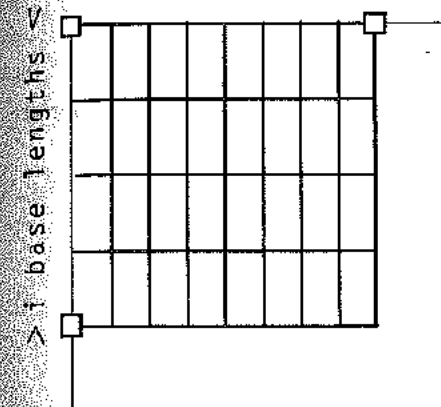
	Telefunken TR 440	Control Data CDC 6600
Polynomials		
10 iteration steps	1.7 CP sec/model	(0.6 CP sec/model)
35 iteration steps	5.9 CP sec/model	(2.0 CP sec/model)
Independent models		
3 iteration steps		0.9 CP sec/model
Bundles		
3 iteration steps		3.4 CP sec/model

CP = central processor time

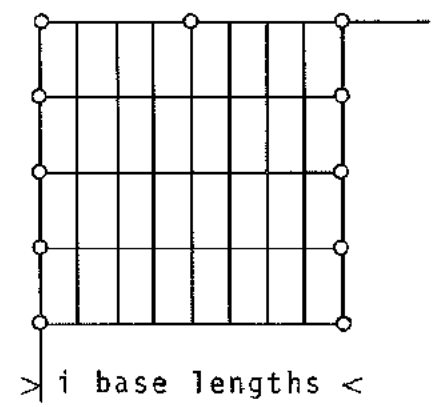
Block Frankfurt: wide angle, 20 % side lap
 Computer running time for block adjustment

Planimetry

> i base lengths <



Height

> i/2 base
> lengths <

Control distributions for 20 % side lap

Figure 1

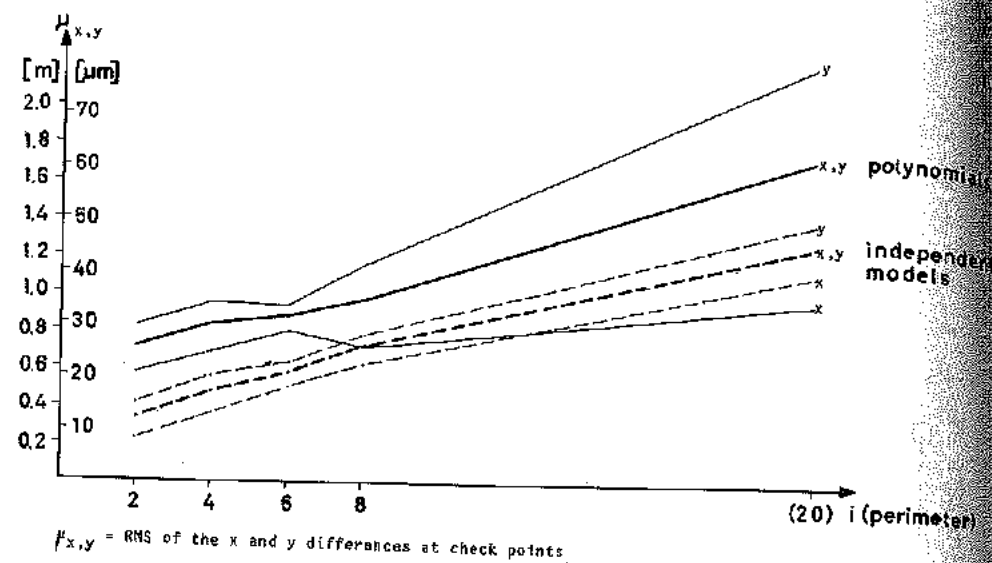


Figure 2

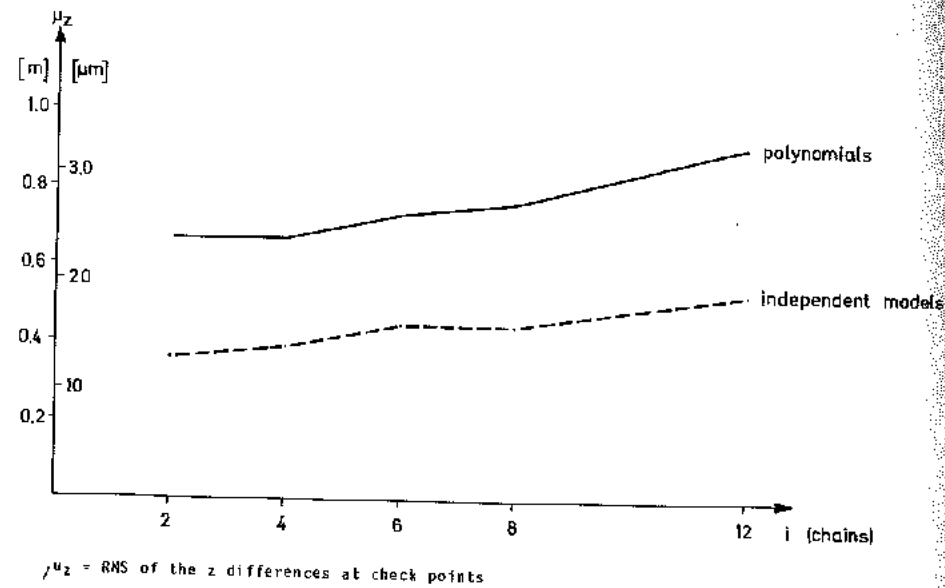


Figure 3

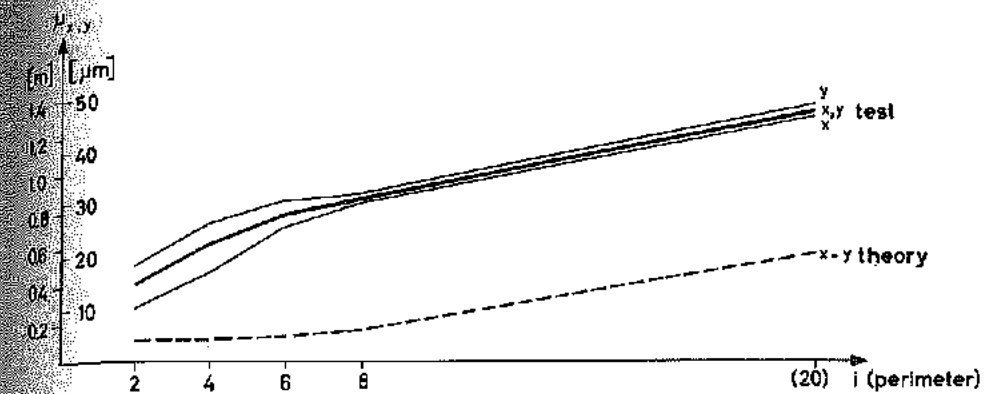


Figure 4

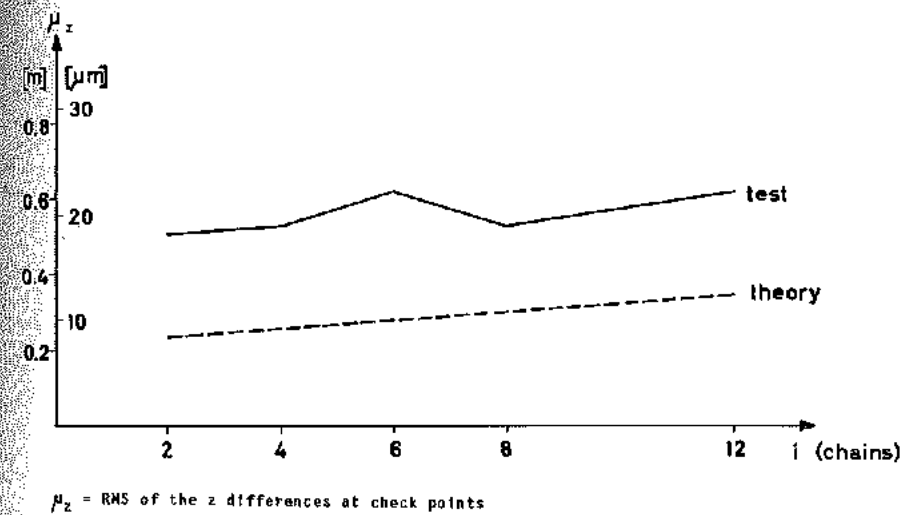
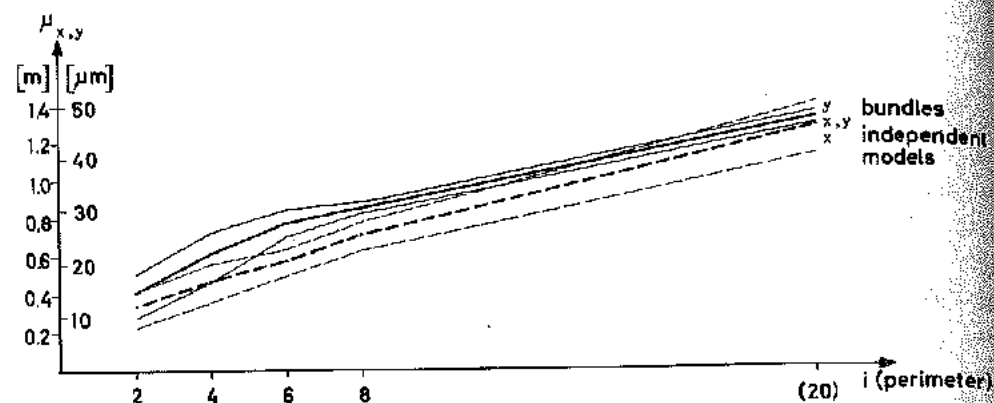


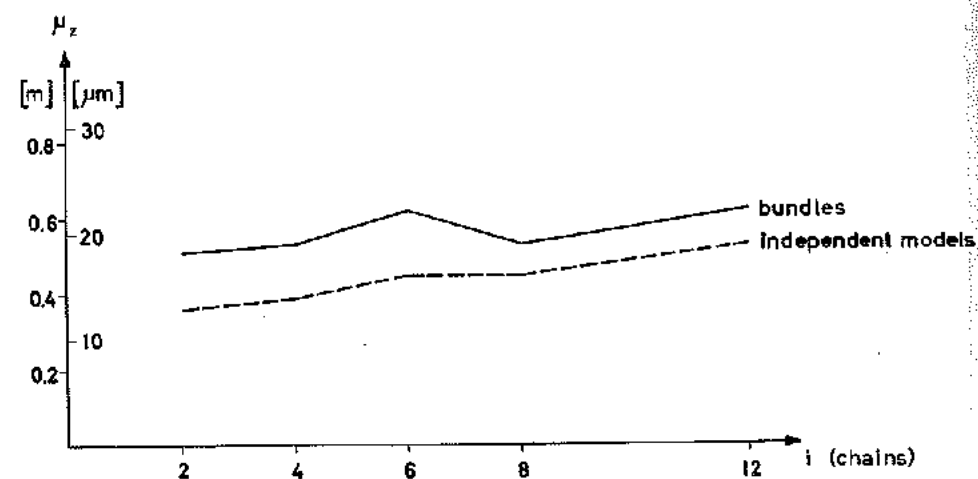
Figure 5



$\mu_{x,y}$ = RMS of the x and y differences at check points

Block Frankfurt: wide angle, 20 % side lap
Empirical planimetric accuracy after block adjustment
Comparison of bundles and independent models

Figure 6



μ_z = RMS of the z differences at check points

Block Frankfurt: wide angle, 20 % side lap
Empirical height accuracy after block adjustment
Comparison of bundles and independent models

Figure 7

BLOCK FRANKFURT Polynomials, 4 Control Points
vector scale 1 : 700, max. 5.29 m

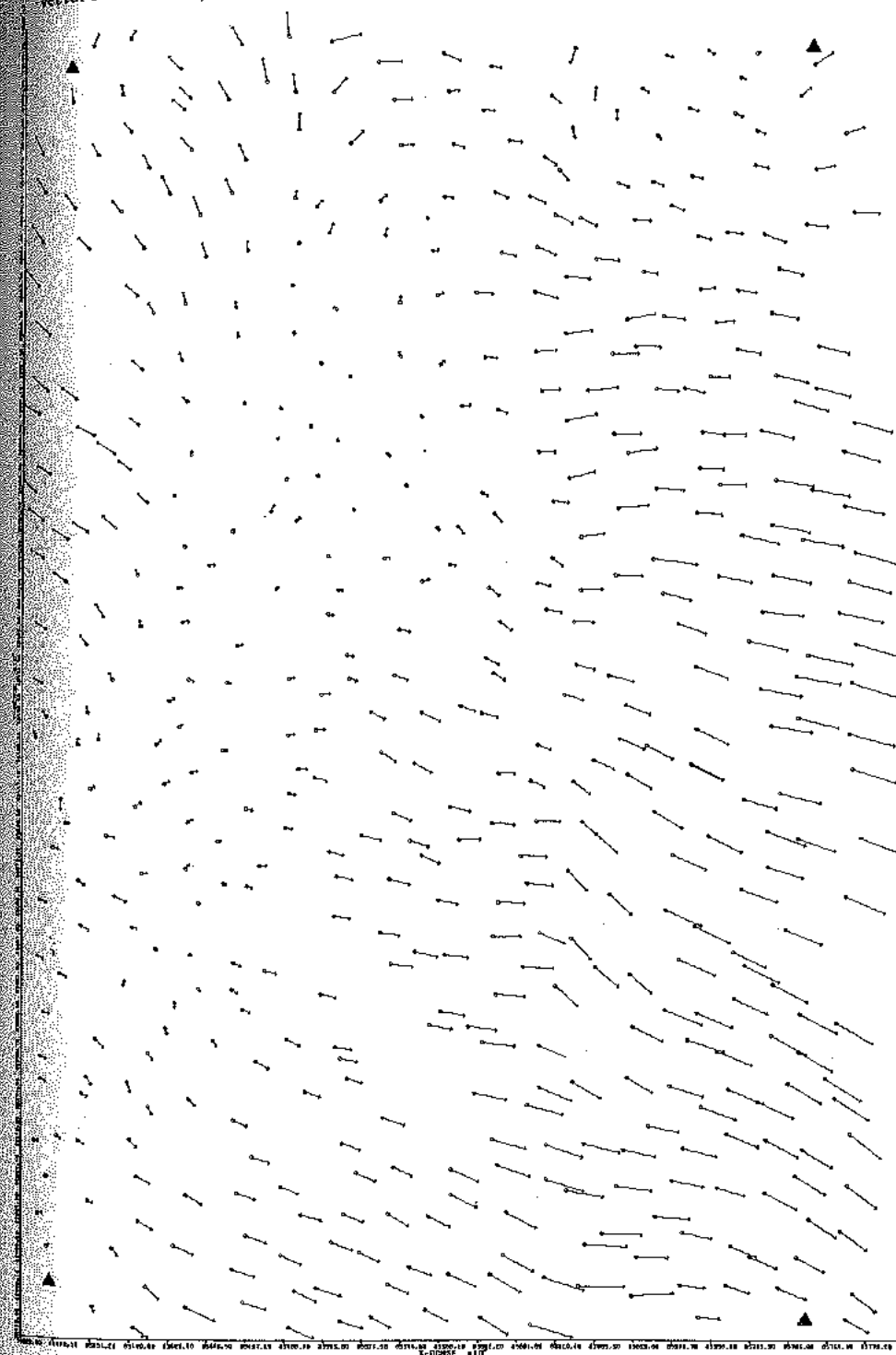


Figure 8

BLOCK FRANKFURT Bundles, 4 Control Points
vector scale 1 : 350, max. 5.04 m

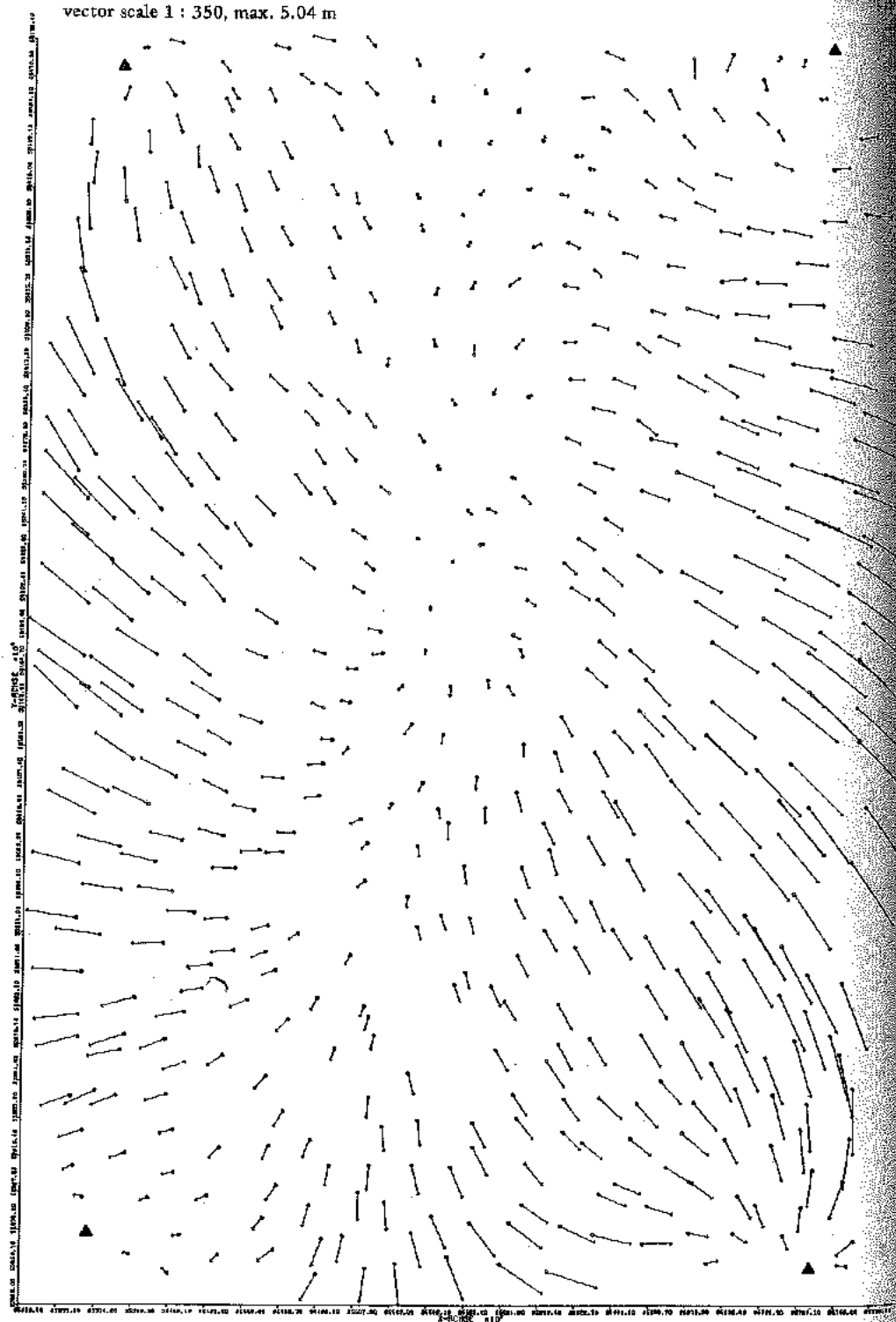


Figure 9

BLOCK FRANKFURT Independent Models, 4 Control Points
vector scale 1 : 700, max. 4.30 m

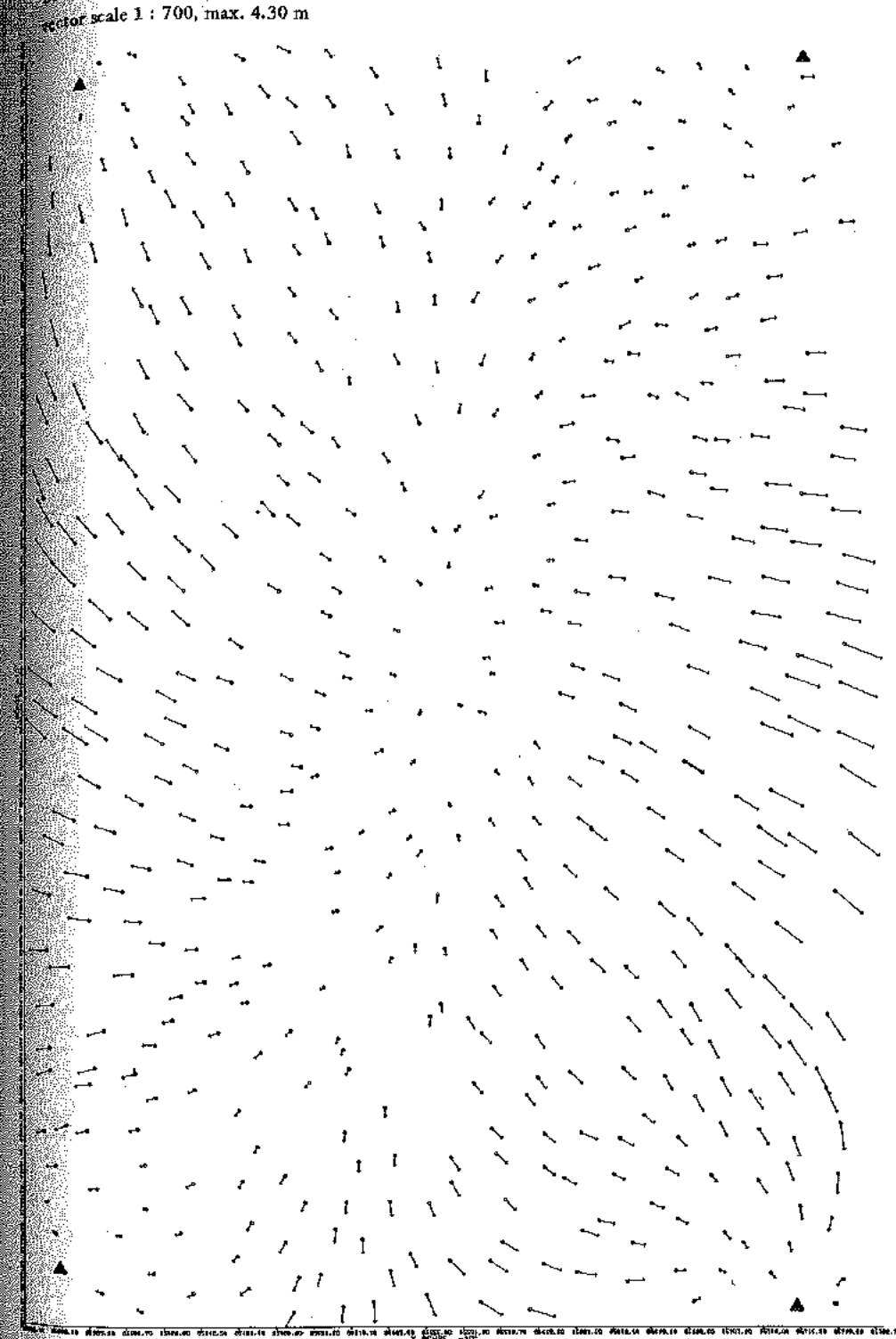


Figure 10

BLOCK FRANKFURT Polynomials, $i = 2$
vector scale 1 : 350, max. 2.63 m

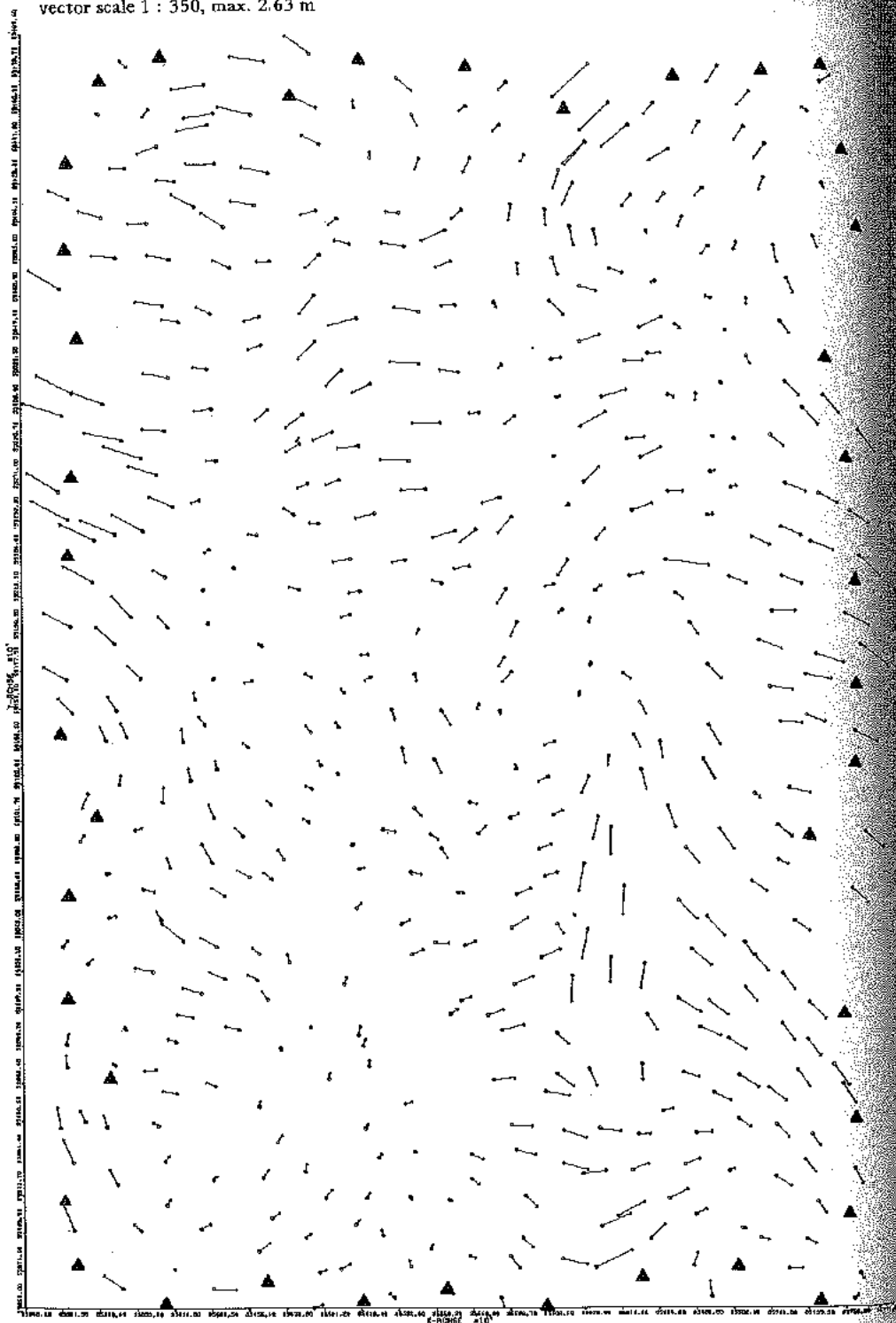
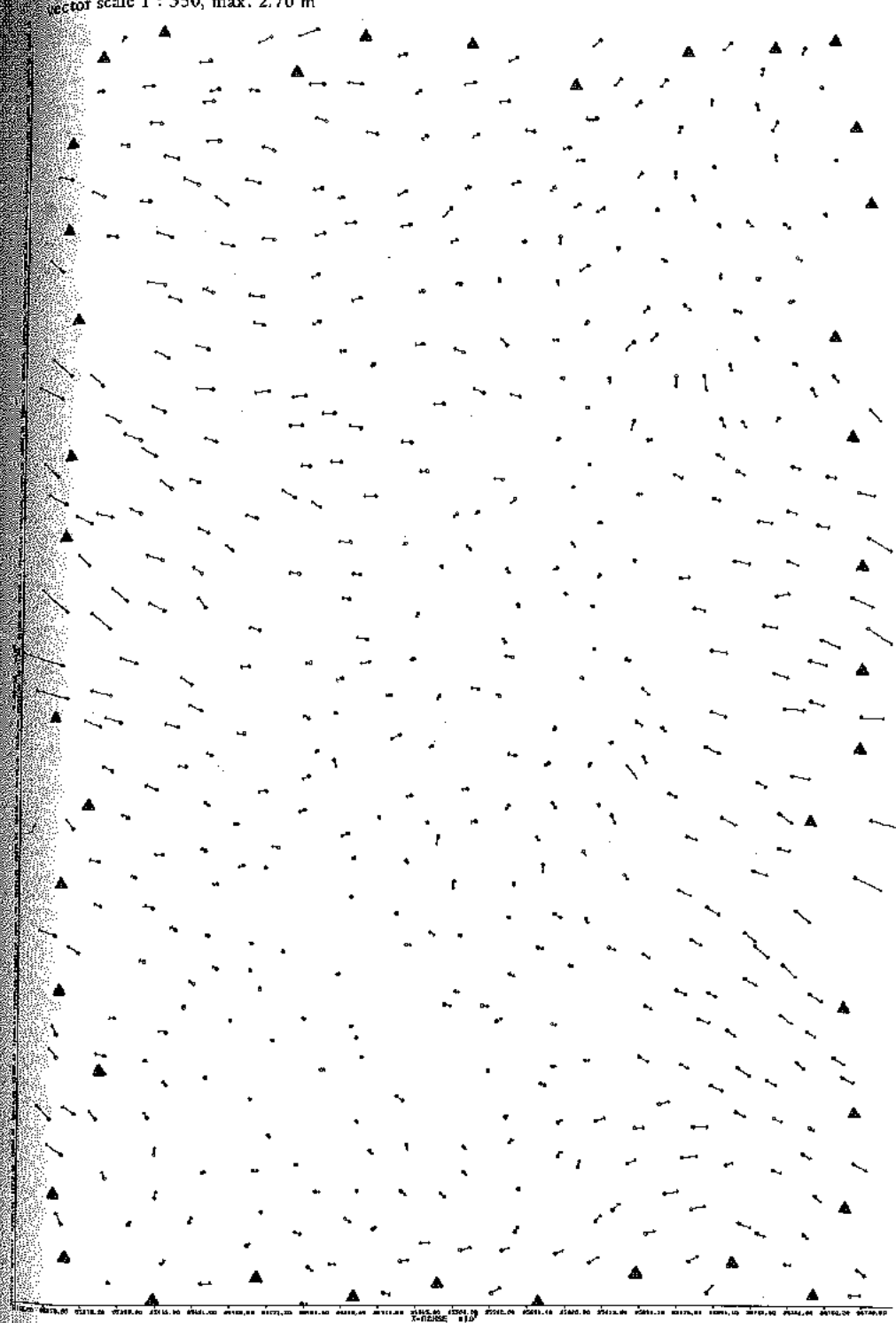
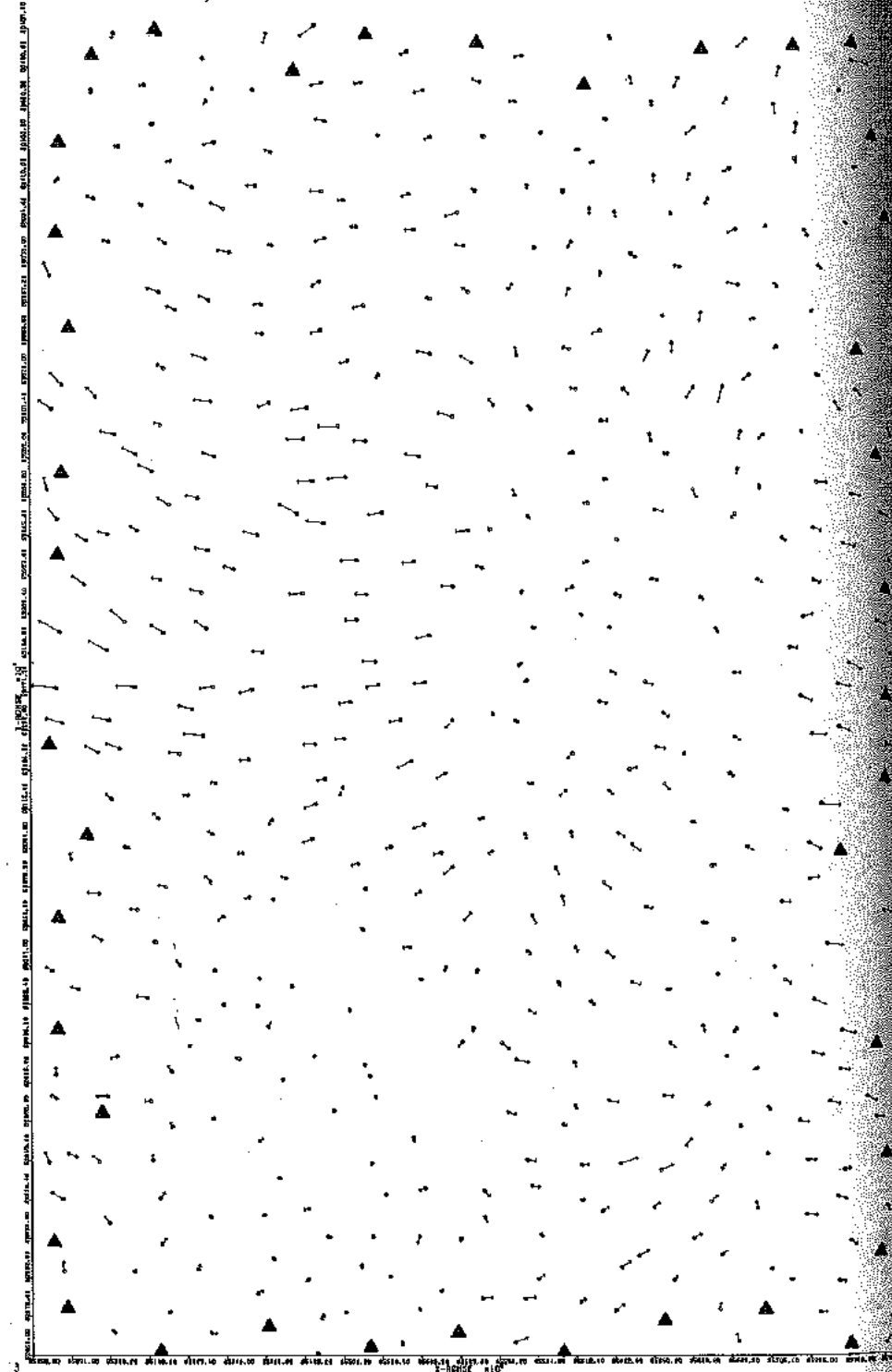


Figure 11

BLOCK FRANKFURT Bundles, $i = 2$
vector scale 1 : 350, max. 2.70 m



vector scale 1 : 350, max. 1.26 m



Discussion about the Subject 5 under the Chairmanship of Prof. G. Konecny

Chairman: Dr. Ebner has shown that it is now possible to make detailed comparisons between various methods of adjustment. Who wishes to open the discussion on the number of questions raised?

Weissl: I think that there is no doubt that the behaviour of σ_0 is influenced by systematic errors and also believe that these systematic effects cannot be computed from the residuals. With respect to the proposal to control the systematic errors by the introduction of additional parameters, I believe this to be only fruitful with the availability of dense control. Finally I would like to remark that I do not think that one can say that the bundle is inferior to the independent model adjustment on the basis of one sample.

Ebner: I can offer no explanation for the small variation of σ_0 in the independent model adjustment. As far as the presence of systematic errors is concerned, we do not know how representative the Oberschwaben block is as regards the ratio between random and systematic errors. Certainly the largest systematic errors are affine errors in scale and angle.

Bauer: The influence of the inclusion of additional parameters increases with a decrease in bridging distance.

Ebner: I do not quite agree that the density of control is the main influencing factor since additional information is obtained from the tie point discrepancies.

Messl: One must also bear in mind that the inclusion of additional parameters reduces the geometric stability of the system.

Ebner: The error propagation itself becomes worse, but the final accuracy can be improved.

Chairman: Has anyone any experience with the inclusion of affine transformations?

Savolainen: We in Finland consider it important to correct for affinity before block adjustment.

Chairman: Is there any comment on the number of parameters that have to be added?

Ebner: The number of additional parameters should be restricted. I may also add that a priori affine corrections were not applied initially since we did not want to disturb the data, but that we now intend to correct for affinity and other systematic errors.

Waldhäusl: What weights were used in the independent model adjustment?

Ebner: Prof. Ackermann outlined this earlier. A unit weight was used for all three coordinates of the model points and for the z coordinate of the perspective centres. The x and y coordinates of the perspective centres had a weight of 0.25.

Chairman: Are the results greatly affected by a variation in the weights used?

Ebner: No.

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Propositions pour le traitement des erreurs non-accidentelles

(2 figures)

M. P. Wiser, Liège

ABSTRACT: p. 182

Ainsi que je le rappelais hier dans une allocution liminaire sur les problèmes de l'aérotriangulation, on peut estimer qu'au niveau de précision actuellement atteint, les progrès ultérieurs dépendront largement d'une meilleure connaissance de la nature des erreurs.

Certes, la question n'est pas neuve et les photogrammètres chevronnés se rappelleront qu'au Congrès de La Haye, en 1948, notre collègue *Roelofs* intitulait une communication importante: «Systematic or accidental errors?». Appliquant une méthode de simulation, il montrait qu'une accumulation aléatoire d'erreurs d'observation purement accidentelles pouvait provoquer, sur les coordonnées obtenues par aérotriangulation, des erreurs résiduelles d'aspect apparemment systématique.

Outre ces constatations empiriques, on pourrait aussi, pour justifier ces résultats, évoquer certains chapitres de la théorie statistique, notamment la loi de l'arc sinus, chère à notre regretté collègue *Hallert*. Il est vrai que la théorie des promenades aléatoires ou «random walks» devrait être généralisée pour couvrir adéquatement le cas de l'aérotriangulation. A ma connaissance, cela n'a pas encore été fait.

Contrairement à cette première opinion, on peut soutenir que les erreurs résiduelles sont principalement de nature systématique, même quand l'aspect désordonné de leur représentation graphique suggère une distribution aléatoire.

Cette controverse a pu longtemps paraître de nature purement académique et je conviens qu'elle n'avait pas grand intérêt pratique à l'époque où la qualité du matériel photographique permettait de négliger l'application de

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Propositions pour le traitement des erreurs non-accidentelles

(2 figures)

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Ainsi que je le rappelais hier dans une allocution liminaire sur les problèmes de l'aérotriangulation, on peut estimer qu'au niveau de précision actuellement atteint, les progrès futurs dépendront largement d'une meilleure connaissance de la nature des erreurs.

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Lindig: Why were about 10 % fewer points used in the bundle than the independent model adjustments?

Ebner: There were only a few points less.

Waldhäusl: I think the difference between the independent model and bundle adjustments is to be expected due to the use of different weights.

Ebner: I consider the difference unexpected since it is contradictory to our previous predictions.

Waldhäusl: The influence of systematic errors differs for planimetry and height measurements. One can expect the results to differ. Furthermore, when comparing the independent model and bundle adjustment in height, the former has an additional degree of freedom and so one could expect a better fit.

Amer: With the bundle adjustment being more sensitive to systematic errors and longer to compute, can one say that the simpler methods such as an independent model with separate plan and height adjustment is more suited to photogrammetric problems?

Ebner: The computing time is certainly longer with the bundle adjustment and I must admit that the independent model adjustment compares favourably with the bundle adjustment in this test.

Chairman: So far only second degree polynomials have been used. Would you expect improvement with third order polynomials?

Ebner: We intend to include third order polynomials and do expect an improvement.

Jerie: For smaller bridging distances should one not also consider sections of the model?

Chairman: In concluding this session, a number of problems raised by Prof. Dr. Ebner have remained unanswered, such as:

remarquable. Toutefois, la chose devait être vérifiée sur une base statistique plus large. Le projet «Oberschwaben» en a fourni l'occasion. Ainsi que M. Camps vous l'exposera tout-à-l'heure, cinq instituts et laboratoires, employant cinq types différents d'instruments analogiques et de comparateurs et travaillant sur huit rubans distincts ont participé à un programme dénommé: «Oberschwaben 2A». Le premier point de ce projet concernait la reproductibilité des erreurs résiduelles, qui a été vérifiée dans toutes les restitutions et dans toutes les variantes d'exploitation des mesures. Vous en verrez tout-à-l'heure quelques exemples, puisés dans un ensemble de résultats beaucoup trop considérable pour être présenté ici dans sa totalité.

On peut se demander pourquoi la reproductibilité, donc la nature principalement systématique des erreurs, n'a pas été plus tôt établie de manière irréfutable. Ce retard est dû à plusieurs causes.

- 1°) L'importance du travail expérimental à fournir. Le projet Oberschwaben 2A, malgré la simplicité de son objet, a réclamé près de mille connexions de photos, connexions qui devaient être exécutées dans les conditions de précision et d'homogénéité les plus rigoureuses.
- 2°) La similitude des diagrammes d'erreurs résiduelles dans les restitutions répétées ne peut se manifester clairement que si l'on dispose d'un réseau très dense de points de contrôle planimétrique et altimétrique soigneusement balisés. Dans le bloc Oberschwaben, il en existe plus de 80 pour chaque ruban de 25 modèles. Je ne pense pas que, dans le passé, une telle densité, sur une surface aussi étendue, ait jamais été disponible pour la recherche.
- 3°) La prépondérance des erreurs systématiques ne peut être mise en évidence que si la composante accidentelle des erreurs est faible, ce qui exige, non seulement un grand soin dans les observations, mais encore une qualité générale très élevée du matériel photographique et des appareils de restitution. Cette qualité est maintenant atteinte, et ce qu'on pouvait seulement deviner dans les observations des années 50 est devenu évident dans les résultats des années 70.

Il résulte des valeurs que vous trouverez dans la communication de M. Camps que le rapport de la partie systématique à la partie accidentelle des erreurs est au moins de 2 à 1. Cette valeur est en bon accord avec le rapport de l'écart-type de contrôle à l'écart-type de l'unité de poids que M. Bauer a constaté dans la compensation par faisceaux du bloc Oberschwaben.

La reproductibilité des erreurs résiduelles et la prépondérance de leur composante systématique entraînent plusieurs conséquences importantes.

- a) L'amélioration des résultats ne peut être obtenue par la répétition pure et simple des observations.
- b) Les procédés de compensation fondés uniquement sur la distribution normale des erreurs sont inadéquats. La composante systématique doit faire l'objet d'un traitement spécial.

Plusieurs méthodes sont concevables pour ce traitement. Elles sont plus ou moins générales, selon qu'elles concernent la totalité des causes d'erreurs systématiques, ou seulement une partie de celles-ci. D'autre part, elles peuvent être fondées sur des principes très différents, à savoir:

- 1°) L'élimination a priori,
- 2°) La détermination directe,
- 3°) La détermination indirecte,
- 4°) L'élimination a posteriori.

Ces quatre catégories de procédés ont été, ou seront évoquées au cours de ce symposium. Par exemple, dans la deuxième, on peut ranger les méthodes fondées sur un étalonnage. Nous y trouvons l'emploi du réseau (cf. Visser et Kure), ainsi que la calibration de chambre au laboratoire ou mieux, par photographie d'un territoire-témoin (cf. Kupfer). Dans la quatrième, se placent les méthodes qui traitent les erreurs systématiques au stade des calculs de compensation. Dans cet ordre d'idées, on peut citer une note de M. Masson d'Authume au Congrès d'Ottawa et la communication que M. Bauer présente à Bruxelles. Enfin, c'est à la première et à la troisième catégorie qu'appartiennent les procédés dont je voudrais vous entretenir.

Le premier vise à l'élimination de la composante systématique des erreurs et à la normalisation de leur distribution. Ce résultat peut être obtenu si la prise de vues est agencée de telle sorte qu'un même point du terrain figure dans plusieurs photos, avec des coordonnées-image systématiquement différentes. Par exemple, en réduisant la base photographique au quart de sa valeur usuelle, on obtient un ruban à 90 % de recouvrement longitudinal décomposable en quatre rubans indépendants à 60 % de recouvrement. Tout point du terrain est alors observable dans quatre modèles différents, à une place différente dans chaque modèle. Toutes les causes d'erreurs systématiques: défauts optiques et mécaniques de la chambre de prise de vues, irrégularités de la réfraction, défauts optiques et mécaniques de l'instrument d'observation, etc. sont donc différentes pour chacune des quatre observations d'un même point. On peut donc penser que la moyennes de ces observations sera dépourvue de systématisme. C'est bien ce qui ressort des résultats que M. Camps vous présentera. Vous constaterez que, sur un ensemble d'observations assez important pour avoir une signification statistique, la précision est plus que doublée.

On sait que, d'une façon générale, le prix de la précision est proportionnel à son carré. Il n'y aurait donc, en principe, rien de déraisonnable à multiplier par quatre le nombre des photos et des observations, dont le coût ne représente qu'une partie, et non la plus importante, des frais totaux. Mais si l'on décidait de quadrupler la couverture photographique minimum, il y aurait lieu de vérifier s'il ne serait pas plus avantageux de combiner la prise de vues à 60 % de recouvrement latéral avec un recouvrement longitudinal de 80 %. Cette vérification est possible au moyen du matériel photographique Oberschwaben. Elle pourrait faire l'objet d'un essai ultérieur.

La deuxième méthode proposée appartient à la troisième catégorie. C'est une version plus générale et plus complète de ce que j'ai développé jadis sous le nom de «méthode des déformations». Son principe consiste à déterminer les erreurs des éléments d'orientation d'un couple stéréoscopique par la mesure des variations d'altitude et de longueur qui subissent des bases auxiliaires transversales observables dans deux modèles adjacents (fig. 1).

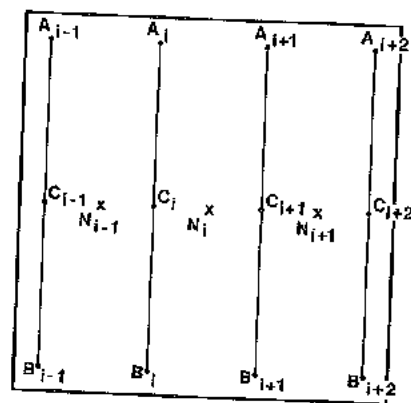


Figure 1 — Cliché P_i

Les calculs sont assez simples mais nécessitent, pour éviter toute ambiguïté, des notations assez lourdes, certaines lettres portant jusqu'à trois indices. Je n'explicitai qu'un exemple.

La photo P_i représentée dans la figure 1, associée à la photo P_{i-1} , forme le modèle M_i . L'erreur d'inclinaison longitudinale $d\varphi_i$ de ce cliché est donnée par:

$$Dz_{i+1,i} + 0,76 Dl_{i+1,i} = -4 d\varphi_i \quad (1)$$

Dans cette expression, le coefficient 0,76 est déterminé par la position des points A et B. Dans cette équation,

$$Dz_{i+1,i} = z_{i+1} - z_i$$

avec: $z_i = Z_{i,i+1} - Z_{i,i}$ et $z_{i+1} = Z_{i+1,i+1} - Z_{i+1,i}$

dans lesquels: $Z_{i,i} = \frac{1}{2} (Z_{A_{i,i-1,i}} + Z_{B_{i,i-1,i}})$

représente l'altitude moyenne de la base $A_i B_i$ mesurée dans le couple M_i . La signification des autres symboles s'établit de même.

Une notation analogue concerne les longueurs. On a:

$$Dl_{i+1,i} = l_{i+1,i} - l_{i,i}$$

avec: $l_{i,i} = L_{i,i,i+1} - L_{i,i,i}$

$$l_{i+1,i} = L_{i+1,i,i+1} - L_{i+1,i,i}$$

dans lesquels, par exemple, $L_{i+1,i,i+1}$ désigne la longueur $A_{i+1} B_{i+1}$ mesurée dans la photo P_i du modèle M_{i+1} .

On calcule les différences Dz et Dl des variations d'altitude et de longueur des bases AB , on forme les sommes de ces variations, désignées par Sz et Sl . On trouve alors, pour l'expression de l'erreur de transmission d'échelle dE_i entre les modèles M_i et M_{i+1} :

$$Sz_{i+1,i} + 0,76 Sl_{i+1,i} = 23 dE_i \quad (2)$$

La détermination de l'erreur d'inclinaison transversale $d\omega_i$ est fondée sur la comparaison des mesures des demi-bases $A_i C_i$, $C_i B_i$, etc. En désignant par t et s des variations de pente et de longueur, on trouve:

$$St_{i+1,i} + 3/2 Ss_{i+1,i} = -24 d\omega_i \quad (3)$$

On peut montrer, et c'est là que réside l'intérêt de la méthode, que les corrections $d\varphi$, dE et $d\omega$ ainsi calculées ne sont pas influencées par les parallaxes longitudinales et transversales engendrées par les déformations du film et par les erreurs systématiques locales, optiques et mécaniques, de la chambre de prise de vues et de l'appareil de restitution. Seule intervient l'erreur accidentelle de mesure de la coordonnée Y des points A, B, C. Or, nous avons vu précédemment que cette erreur purement accidentelle est, en moyenne, sensiblement inférieure aux erreurs systématiques.

Pour être complet, ajoutons que les mesures prévues permettent aussi le calcul des corrections db_z et dk de la composante verticale de la base et du dévers. Toutefois, les valeurs de ces corrections ne jouissent pas du même privilège que les précédentes. Elles doivent cependant améliorer l'orientation relative usuelle sur six points, parce qu'elles font intervenir des points de réglage plus nombreux, ou mieux placés.

Avant de quitter ce sujet, signalons qu'un avantage important du procédé est de traiter d'un seul coup les erreurs systématiques et les erreurs accidentelles, sans distinction de nature ou d'origine. Du point de vue de la métrologie générale, son mérite est de fournir des corrections, au sens strict du mot, et pas seulement des incréments stochastiques tels qu'ils résultent des procédés probabilistes de compensation.

Quant aux inconvénients, les plus évidents sont la multiplication des observations et surtout l'augmentation du nombre de modèles formant un ruban de longueur donnée. Dans le cas du matériel Oberschwaben, par exemple, le raccourcissement nécessaire de la base photographique fait passer le nombre de photos de 25 à 33 par ruban. Il a été trouvé précédemment que cet élément défavorable n'empêchait pas une amélioration des résultats. Comme ce premier essai portait sur un seul ruban de grande longueur, il a paru souhaitable d'appliquer la méthode dans des conditions statistiques plus satisfaisantes; ce que permet le bloc d'Oberschwaben. Ces essais sont compris dans le sous-programme 2B auquel participent trois Centres. Malheureusement, un démarrage très tardif du programme 2 et des contretemps de toute espèce font que les observations ne sont pas terminées.

Un troisième procédé appartient, lui aussi, au groupe des méthodes indirectes. Il n'est pas inédit non plus. Il a été testé sur rubans artificiels, mais c'est seulement dans le cadre du projet Oberschwaben qu'il recevra, éventuellement, une confirmation statistiquement valable.

Cette méthode est fondée sur la notion de «parallaxe latente» que je définis comme suit. Après orientation relative sur six points de réglage, il existe généralement en ces points des parallaxes résiduelles mesurables ou calculables; ce sont des parallaxes apparentes. Bien qu'elles aient un sens statistique, elles ne sont pas la mesure des erreurs des éléments d'orientation du couple. Si celui-ci pouvait, comme c'est en principe le cas en photogrammétrie terrestre, recevoir une orientation relative impeccable grâce à des mesures extérieures, on constaterait dans le modèle ainsi formé des parallaxes d'origine diverses: déformations du film, défauts du matériel de prise de vues et de restitution, etc. Au moyen de ces parallaxes, on pourrait calculer les erreurs des éléments d'orientation relative commises dans le cas précédent. Ce sont ces parallaxes, qui restent cachées dans le procédé usuel d'orientation relative, que j'appelle «parallaxes latentes». Il s'agit de les déterminer, bien qu'elles ne soient pas directement observables.

Cette détermination est possible en utilisant des combinaisons linéaires de parallaxes mesurées en des points auxiliaires judicieusement choisis. Plusieurs dispositions, plus ou moins compliquées, sont possibles. On peut, par exemple, faire correspondre quatre points auxiliaires à chacun six points usuels de réglage. Une formulation particulièrement simple sera obtenue si les points auxiliaires se trouvent sur des horizontales et des verticales passant par le point principal correspondant, à une distance de celui-ci égale au quart de la base photographique (fig. 2).

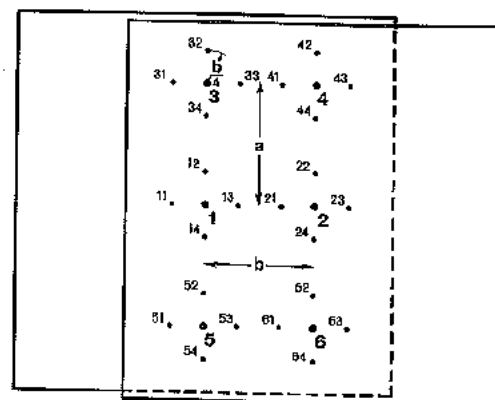


Figure 2

Pour alléger les notations, nous désignons les parallaxes par le seul indice du point où elles sont mesurées ou calculées. Dans ces conditions, et sous une réserve que nous aurons à examiner, l'erreur latente de convergence du couple est donnée par:

$$d\gamma = k \cdot (31 - 33 + 32 - 34 + 51 - 53 + 54 - 52 + 43 - 41 + 42 - 44 + 63 - 61 + 64 - 62) \quad (4)$$

avec

$$k = \frac{f}{4a \cdot b}$$

On peut établir des formules analogues pour l'erreur d'inclinaison transversale $d\Delta\omega$ et pour l'erreur de déversement dk .

Comme je viens de l'indiquer, la validité de la formule (4) est soumise à une condition. Il faut que la parallaxe latente soit la même au point principal et aux quatre points auxiliaires qui l'entourent. Physiquement parlant, et pour les conditions des photos d'Oberschwaben, cela signifie que les erreurs systématiques sont constantes à l'intérieur d'un cercle de 20 mm de rayon, à l'échelle des clichés. Cette hypothèse de travail paraît admissible en ce qui concerne les déformations du film. Par exemple, les observations de Kissler et Kure dans leur étude des réseaux montrent que le vecteur «déformation du film» évolue progressivement et peut être considéré comme stable à l'intérieur d'un domaine restreint. En ce qui concerne la distorsion optico-mécanique secondaire, les écartements de chambres et les réglages d'instruments d'observation conduisent généralement aux mêmes conclusions. Enfin, on peut supposer que les turbulences atmosphériques qui altèrent localement la réfraction normale sont localement stables.

La constance des erreurs systématiques dans une zone limitée de la photo peut encore se déduire d'un fait expérimental bien connu. Quand on augmente le nombre des points de réglage de l'orientation relative, en les prenant par groupes dans les zones centrées sur les points principaux 1, 2, ... 6 et quand on introduit ces parallaxes surabondantes dans le calcul usuel de l'orientation relative, le gain de précision est nul ou faible, en tout cas très inférieur à ce qu'on obtiendrait si les parallaxes latentes n'étaient pas constantes, ou presque, dans les zones où les points surabondants sont choisis.

Quoi qu'il en soit, la validité et l'efficacité de la formule (4) dépendent de deux facteurs contradictoires: d'une part, la constance de la parallaxe latente, qui demande que les points auxiliaires soient compris dans un petit domaine; d'autre part, la minimisation du facteur k qui suppose que les points auxiliaires soient choisis aussi loin que possible du point principal. Une distance égale au quart de la base photographique paraît un compromis acceptable.

Une autre question se pose: avec quelle précision la formule (4) et les formules analogues permet-elle la détermination de l'erreur totale, latente et apparente?

Dans le procédé usuel d'orientation relative, la valeur finale de la convergence résulte de variations successives qui peuvent s'exprimer par:

$$\Delta\gamma = \frac{f}{4a \cdot b} (3 - 5 + 6 - 4)$$

La valeur de ces variations dépend donc de la mesure de quatre parallaxes aux points principaux 3, 4, 5 et 6. Au contraire, la détermination de l'erreur latente de convergence, avec le même coefficient k , demande la mesure de seize parallaxes. L'écart-type de cette détermination est donc deux fois plus grand. Pour que le calcul de la convergence latente soit avantageux, il faut que l'erreur à corriger soit, en moyenne, au moins deux fois plus grande que sa partie accidentelle. C'est bien ce qui semble se dégager des résultats obtenus dans les programmes Oberschwaben 1 et 2, où l'on traite cependant des questions de nature bien différente.

Pour conclure, je dirai que j'ai voulu attirer votre attention sur un éventail de méthodes présentant une grande souplesse d'application et même susceptibles d'être combinées dans une procédure plus ou moins complexe. Je me garderai de les présenter comme des panacées. En ce qui concerne les deux dernières, elles doivent encore recevoir la sanction d'une expérimentation étendue et rigoureuse. Plusieurs centres de recherche y sont occupés. Si les résultats obtenus le justifient, les procédés que je viens d'évoquer feront l'objet de publications détaillées.

ABSTRACT:

Experimental evidence has been gained of the preponderancy of non-accidental errors in aerotriangulation (e. g. in *F. Camps'* paper: "Résultats obtenus . . ."). Consequently, the efficiency of purely probabilistic processes of adjustment is likely to be impaired.

Three different methods for the previous treatment of non-accidental errors are proposed, i. e.:

- a) independent strips,
- b) measurement of model deformations,
- c) determination of "latent parallaxes".

Experimental results of method a) are to be found in *F. Camps'* paper quoted above.

Discussion: p. 207.

Résultats obtenus dans le cadre du projet Oberschwaben 2A

(avec 15 figures)

par *F. Camps, Liège*

SUMMARY: p. 194

Introduction

Depuis de nombreuses années, le mystère qui plane sur la nature systématique, accidentelle ou pseudo-accidentelle des erreurs en aérotriangulation ainsi que sur leur comportement reste entier. Comme dans tout domaine de mesure, il existe certainement des erreurs de toutes natures mais, jusqu'à présent, on n'est pas encore parvenu à délimiter la part de chacune d'entre elles. Le problème est d'une extrême complexité imputable à la fois aux interférences des erreurs les unes sur les autres et à leur mode de propagation.

Le programme 2A n'a évidemment pas la prétention de vider complètement la question: son but est plus modeste et plus restreint, il ne vise à soulever, si c'est possible qu'un tout petit coin du voile.

Plus précisément, l'idée directrice du programme 2A est de se faire une opinion sur l'influence que peut avoir sur la précision des résultats la localisation dans le cliché des points à mesurer. Il s'agit donc d'un programme de recherche essentiellement expérimental, ce qui implique des mesures très soignées, longues, fastidieuses qui ne sont d'ailleurs pas encore complètement terminées. Les résultats que nous présentons aujourd'hui sont donc forcément incomplets et les conclusions qui en découlent doivent être considérées comme de simples indications générales.

Méthode d'investigation

Un ruban à 90 % de recouvrement longitudinal est décomposé en quatre rubans à 60 %. On obtient ainsi quatre rubans indépendants présentant tous les mêmes points de contrôle, mais ceux-ci sont chaque fois situés dans des zones différentes des clichés. Ces rubans sont restitués successivement soit par voie analogique, soit par voie analytique. Pour contrôler que les variations constatées sont bien le fait de la localisation des points dans les clichés, un des rubans est restitué quatre fois de suite dans des conditions aussi identiques que possible.

Au total, on dispose donc des résultats fournis par:

- 1° les restitutions des quatre rubans indépendants,
- 2° les quatre restitutions répétées d'un même ruban.

On compare enfin les résultats moyens de chaque groupe de quatre restitutions.

Matériel photographique

Il est constitué de deux rubans du bloc d'Oberschwaben, ce sont les rubans n° 12 et 14 décomposés d'une part en 12/1, 12/2, 12/3 et 12/4 et d'autre part en 14/1, 14/2, 14/3 et 14/4.

Les rubans 12/3 et 14/3 ont été répétés quatre fois.

Les rubans étant composés d'environ 25 couples, il en résulte qu'un essai comporte la restitution de 175 couples. Enfin, dans chaque ruban, il y a à peu près 80 points de contrôle.

Centres participants

Les mesures ont été effectuées dans les centres de Francfort, Liège, Milan, Mons et Munich. Tous les participants ont traité le ruban 14 sauf à Mons où l'on a restitué le ruban 12.

- I. A Francfort, la restitution est analytique sur stéréocomparateur PSK. Les mêmes mesures ont donné lieu à trois versions selon le mode de calcul de l'orientation relative.
 1. Orientation relative fondée sur six points naturels choisis par l'opérateur et identifiés par des croquis ainsi que sur tous les points marqués au sol.
 2. Orientation relative comme au 1°, mais en plus on a englobé des points «artificiels» préalablement piqués sur les clichés.
 3. Orientation relative sur six points naturels.
- II. A Liège, la restitution est analogique sur autographe Wild A7. Le centre de Liège ayant la charge de centraliser les résultats des différents participants et d'exécuter les calculs de compensation et de comparaison, les mesures ne sont pas encore terminées et ne figurent pas dans le présent rapport.
- III. Le centre de Milan a procédé aussi par voie analytique, les mesures ont été exécutées sur le stéréocomparateur TA3/P. Deux versions sont présentées:
 1. l'une où la transmission de l'échelle se fait à l'aide d'un seul point au nadir du cliché,
 2. l'autre où elle a lieu à l'aide de trois points.
- IV. Le centre de Mons a traité le ruban 12 analogiquement sur un Wild A7. Les orientations relatives sont basées sur six points naturels et la connexion des couples se fait de la manière classique.
- V. Enfin, le centre de Munich a employé la méthode analogique pour le ruban 14. L'appareil de restitution est le Zeiss C8.

Calculs

Chaque participant a fourni au centre pilote de Liège les coordonnées ruban exprimées en mètres des points de contrôle après ajustement linéaire.

Pour la compensation on a employé la formule:

$$\Delta = Ax^2 + Bxy + Cx + Dy + E.$$

Chaque coordonnée est compensée séparément. Les coefficients A, B, C, D, E sont calculés selon les moindres carrés en se fondant sur six points: deux au début, deux au milieu et deux à la fin du ruban. Il eût été souhaitable de conserver les mêmes points d'appui pour la compensation des quatre rubans indépendants; malheureusement, la chose n'a été possible que pour le ruban 12. Pour le ruban 14, il aurait fallu choisir des points situés beaucoup trop loin des extrémités du ruban. Force fut donc de choisir deux groupes de points d'appui, l'un pour les rubans 14/1 et 14/2, l'autre pour les rubans 14/3 et 14/4. De plus, l'un de ces points n'ayant pas été mesuré dans tous les rubans par certains centres, on a été contraint de le remplacer dans ce cas par un autre moins bien situé. Les écarts résiduels sont alors portés en graphiques et l'on en calcule la moyenne arithmétique et la dispersion.

On considère ensuite les deux groupes de rubans: les quatre rubans indépendants et les quatre rubans répétés. Dans chaque groupe, on calcule pour chaque point la moyenne arithmétique des écarts résiduels. On obtient ainsi des écarts moyens que l'on porte aussi en graphiques et dont on calcule également la moyenne et la dispersion.

Toutes ces opérations ont été effectuées à l'aide d'un ordinateur IBM 370. Les programmes sont en Fortran IV.

Les résultats numériques sont rassemblés dans les tableaux qui suivent. Pour chaque expérience et pour chaque coordonnée, nous donnons la moyenne des écarts et leur dispersion exprimées en centimètres. A titre indicatif, nous avons fait figurer également la moyenne des quatre restitutions aussi bien pour les rubans indépendants que pour les rubans répétés, ainsi que le rapport:

$$r = \text{moyenne des dispersions} / \text{dispersion des moyennes}.$$

Quant aux graphiques des écarts résiduels, nous nous sommes limité aux plus représentatifs d'entre eux. Le lecteur les trouvera en annexe.

Ils concernent tous, les écarts sur la coordonnée Z, la plus importante. Les écarts correspondant aux points de contrôle situés dans la zone centrale du ruban sont reliés par un trait plein; le trait interrompu et le trait d'axe représentent les écarts sur les points latéraux.

Nous avons choisi de ne donner le détail des sept restitutions que pour un seul centre: celui de Mons. C'est là en effet que la différence entre les restitutions des quatre rubans indépendants et les quatre restitutions du même ruban apparaît le plus clairement.

Ensuite, pour les autres centres participants nous ne présentons que les résultats globaux, c'est-à-dire les graphiques des écarts moyens pour les rubans répétés et pour les rubans indépendants.

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	44	84	-47	75	57	142
14/2	13	98	-1	108	-47	145
14/4	34	70	8	80	4	83
14/3/1	-16	55	13	87	9	122
Moyennes	19	77	-7	88	6	123
Ecart moyen	16	37	-8	27	4	45
r		2,08		3,26		2,73

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	-16	55	13	87	9	122
14/3/2	-42	85	9	57	-20	181
14/3/3	-28	59	-15	76	14	95
14/3/4	-18	121	26	70	-20	108
Moyennes	-26	80	8	72	-4	127
Ecart moyen	-26	74	8	66	-3	113
r		1,08		1,09		1,12

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	53	75	-39	68	-2	174
14/2	42	121	27	80	-39	89
14/4	20	102	42	80	51	101
14/3/1	19	44	1	44	-26	153
Moyennes	33	86	8	68	-4	129
Ecart moyen	31	49	5	28	-5	75
r		1,75		2,34		1,72

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	19	44	1	44	-26	153
14/3/2	-20	47	25	47	-6	166
14/3/3	1	33	14	43	-44	206
14/3/4	-4	100	34	59	-7	148
Moyennes	-1	56	19	48	-21	168
Ecart moyen	-1	42	17	37	-20	164
r		1,33		1,30		1,02

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	- 8	122	-59	90	77	150
14/2	-28	68	3	105	-29	164
14/4	- 8	47	1	67	5	68
14/3/1	-38	129	14	89	- 1	116
Moyennes	-20	91	-10	88	13	124
Ecart moyen	-19	37	-11	37	10	51
r		2,46		2,38		2,43

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	-38	129	14	89	- 1	116
14/3/2	-34	196	17	72	-37	184
14/3/3	-27	112	-25	79	- 8	104
14/3/4	-14	159	10	69	-54	130
Moyennes	-28	149	4	77	-25	133
Ecart moyen	-29	144	6	73	-25	120
r		1,03		1,05		1,11

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	21	70	-49	134	49	158
14/2	-29	61	- 7	85	39	148
14/4	30	86	-13	46	-18	101
14/3/1	24	63	16	105	64	110
Moyennes	12	70	-13	92	33	129
Ecart moyen	12	33	-11	39	34	67
r		2,12		2,36		1,93

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	24	63	16	105	64	110
14/3/2	49	112	-62	139	-17	59
14/3/3	76	125	-29	114	-82	119
14/3/4	43	89	- 9	149	61	153
Moyennes	48	97	-21	127	6	110
Ecart moyen	48	88	-22	120	6	69
r		1,10		1,06		1,59

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	- 8	122	-59	90	77	150
14/2	-28	68	3	105	-29	164
14/4	- 8	47	1	67	5	68
14/3/1	-38	129	14	89	- 1	116
Moyennes	-20	91	-10	88	13	124
Ecart moyen	-19	37	-11	37	10	51
r		2,46		2,38		2,43

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	-38	129	14	89	- 1	116
14/3/2	-34	196	17	72	-37	184
14/3/3	-27	112	-25	79	- 8	104
14/3/4	-14	159	10	69	-54	130
Moyennes	-28	149	4	77	-25	133
Ecart moyen	-29	144	6	73	-25	120
r		1,03		1,05		1,11

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	21	70	-49	134	49	158
14/2	-29	61	- 7	85	39	148
14/4	30	86	-13	46	-18	101
14/3/1	24	63	16	105	64	110
Moyennes	12	70	-13	92	33	129
Ecart moyen	12	33	-11	39	34	67
r		2,12		2,36		1,93

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	24	63	16	105	64	110
14/3/2	49	112	-62	139	-17	59
14/3/3	76	125	-29	114	-82	119
14/3/4	43	89	- 9	149	61	153
Moyennes	48	97	-21	127	6	110
Ecart moyen	48	88	-22	120	6	69
r		1,10		1,06		1,59

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	-30	77	-42	135	45	117
14/2	-55	93	-2	83	34	81
14/4	32	65	-9	42	1	88
14/3/1	13	33	7	39	42	129
Moyennes	-10	67	-11	75	30	104
Ecart moyen	-8	29	-9	50	30	64
r		2,31		1,50		1,63

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	13	33	7	39	42	129
14/3/2	46	75	-40	85	-16	92
14/3/3	59	86	37	89	-99	200
14/3/4	11	68	28	78	34	159
Moyennes	32	65	9	73	-10	145
Ecart moyen	33	53	8	43	-10	126
r		1,23		1,70		1,15

Rubans indépendants						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
12/1	-45	99	20	112	94	142
12/2	-62	103	-41	66	67	145
12/4	2	52	5	67	-170	295
12/3/1	-63	114	-62	271	66	146
Moyennes	-42	92	-20	129	14	182
Ecart moyen	-41	66	-15	63	15	51
r		1,39		2,05		3,57

Rubans répétés						
Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
12/3/1	-63	114	-62	271	66	146
12/3/2	-14	64	-15	104	149	322
12/3/3	24	91	-36	67	71	255
12/3/4	21	46	37	105	80	199
Moyennes	-10	79	-19	137	91	230
Ecart moyen	-8	36	-15	109	85	225
r		2,20		1,26		1,02

Rubans indépendants

Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/1	184	250	-111	172	-195	384
14/2	-64	222	73	226	-204	324
14/4	-8	168	65	105	386	482
14/3/1	-83	374	131	353	-21	206
Moyennes	8	253	39	214	-8	349
Ecart moyen	7	150	31	62	-5	119
r		1,69		3,45		2,93

Rubans répétés

Ruban	X_0	σ_x	Y_0	σ_y	Z_0	σ_z
14/3/1	-83	374	131	353	-21	206
14/3/2	117	237	-12	213	-35	249
14/3/3	-1	97	-117	182	83	345
14/3/4	13	338	66	164	91	384
Moyennes	12	262	17	228	32	296
Ecart moyen	12	228	25	161	28	237
r		1,15		1,42		1,25

Analyse des résultats

Le temps nous a manqué pour nous livrer à une analyse statistique approfondie et minutieuse des résultats. D'autre part, il serait abusif de vouloir commenter un à un les 69 graphiques résultant de l'expérience. Néanmoins, quelques tendances générales se dégagent.

1. L'allure des courbes d'erreurs résiduelles est extrêmement variée: si l'on trouve quelques sinusoides régulières, on trouve aussi pas mal de courbes présentant des ondulations à caractère tout à fait irrégulier. Cette diversité affecte aussi bien les restitutions analytiques que les restitutions analogiques.

2. En ce qui concerne la restitution d'un ruban isolé, la supériorité des appareils analytiques s'affirme de façon indiscutable. Pour la coordonnée Z, on trouve deux écarts quadratiques moyens égaux ou supérieurs à 2 m pour les restitutions analytiques contre dix pour les restitutions analogiques.

3. La comparaison des résultats obtenus selon différentes versions de calcul semble montrer que:

1° On obtient des résultats assez dissemblables selon le nombre de points utilisés pour le transfert de l'échelle. Il apparaît que, dans l'ensemble, le nombre de points de transfert apporte une très légère amélioration.

2° En ce qui concerne les variantes de calcul de l'orientation relative, il est très difficile de tirer des conclusions valables aussi bien au sujet de la précision globale que de la forme des courbes. On trouve autant de concordances que de discordances dans les graphiques d'erreurs. Nous laissons aux responsables des centres intéressés le soin d'une analyse plus fouillée de ces résultats.

4. Venons-en maintenant à la comparaison des rubans indépendants et des rubans répétés.

Ici, les résultats paraissent nettement significatifs. On constatera à la lecture des tableaux que, systématiquement, la dispersion des écarts moyens est plus faible pour les rubans indépendants que pour les rubans répétés. Une seule exception: la coordonnée X de la première restitution analogique (Mons). Les graphiques d'erreurs résiduelles confirment ce caractère systématique. Du côté des rubans indépendants, la distribution des points est telle qu'en général, il n'est guère possible de tracer une courbe moyenne: les points semblent disséminés au hasard autour de l'axe. Tout au plus peut-on trouver de temps en temps quelques vestiges de sinusoides.

Au contraire, les graphiques relatifs aux rubans répétés montrent des formes nettement plus accusées, parfois même des sinusoides bien régulières.

D'autre part, la dispersion des moyennes est toujours plus faible que la moyenne des dispersions, qu'il s'agisse des rubans indépendants ou des rubans répétés, mais le rapport:

$$r = \text{moyenne des dispersions} / \text{dispersion des moyennes}$$

est, à deux exceptions près, toujours favorable aux rubans indépendants. Il vaut en moyenne 2,29 pour les rubans indépendants contre 1,25 pour les rubans répétés. De plus, le contraste est particulièrement marqué dans le cas des restitutions analogiques.

Conclusion

On peut donc considérer que la restitution de plusieurs rubans indépendants est préférable à des restitutions répétées d'un même ruban.

A notre avis, ce phénomène trouve son explication dans l'influence de la localisation des points dans le cliché. D'une part, cette localisation est un paramètre variable et il entraîne des erreurs accidentelles qui ont tendance à se compenser lorsque l'on prend la moyenne des résultats. Dans le cas de restitutions répétées au contraire, la localisation des points étant fixe, il en résulte toujours les mêmes erreurs qui, forcément, se retrouvent dans la moyenne. On en arrive ainsi à reconsidérer le problème de la formation du modèle ou, si l'on préfère, celui de l'orientation relative.

Il est à noter aussi que l'avantage des rubans indépendants semble, toutes proportions gardées, plus marqué dans le cas des restitutions analogiques, ce qui conduit à penser que les clichés ne sont pas seuls à incriminer, mais que les instruments, eux aussi, peuvent être le siège d'erreurs locales. Mais c'est là une opinion qui demanderait à être vérifiée par des expériences plus nombreuses.

SUMMARY:

The purpose of the program Oberschwaben 2A was an experimental research about the influence that the position of the measured points in the photographs can have on the final result of an aerotriangulation.

A 90 % overlap strip is divided into four 60 % overlap strips. So, in each of these four independent strips, the control points are the same but their places in the photographs change from one strip to another.

Then, the aerotriangulation of the four strips is made either analytically or analogically, and one of them is repeated four times. This experiment has been done in Frankfurt, Liège, Milano, Mons and München.

We have thus two groups of aerotriangulations:

- one for the four independent strips,
- one for the four repetitions of the same strip.

In each group, and for each control point, the mean of the four results is computed and the mean errors curves are drawn for each coordinate.

The comparison of these curves shows that the influence of the position of the points on the plates is very important. The mean square error is more than two times greater for the repeated strip than for the independent strips.

In the case of the repeated strip, the mean errors curves have about the same systematic character as an isolated strip. On the contrary, this systematic character practically disappears in the case of independent strips.

This may be explained by the fact that the variations of the position of the measured points on the plates involve accidental errors which become systematic when the position of the points remains unchanged.

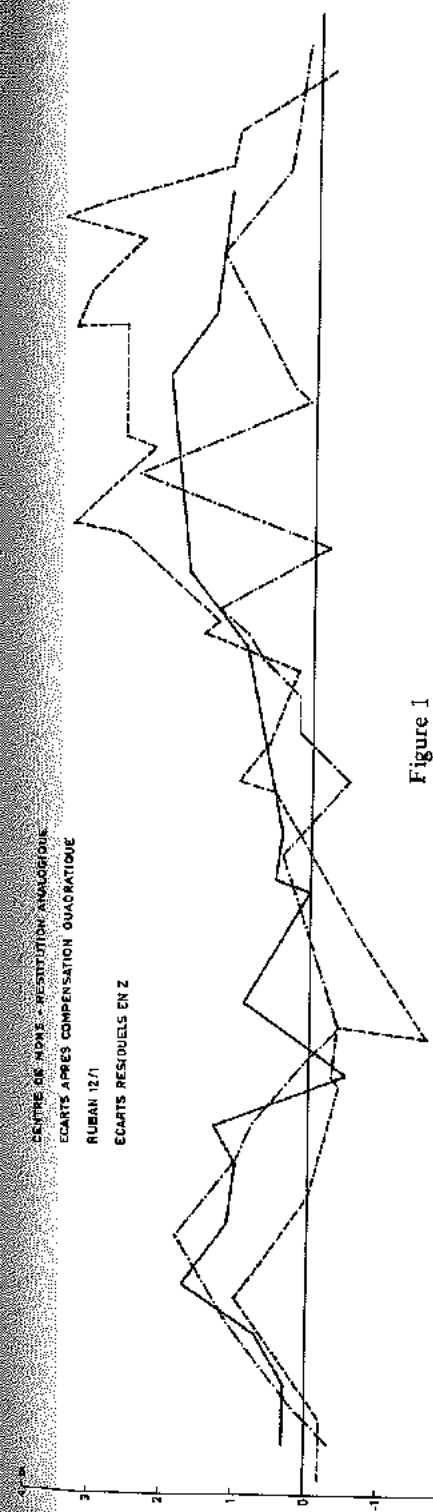


Figure 1

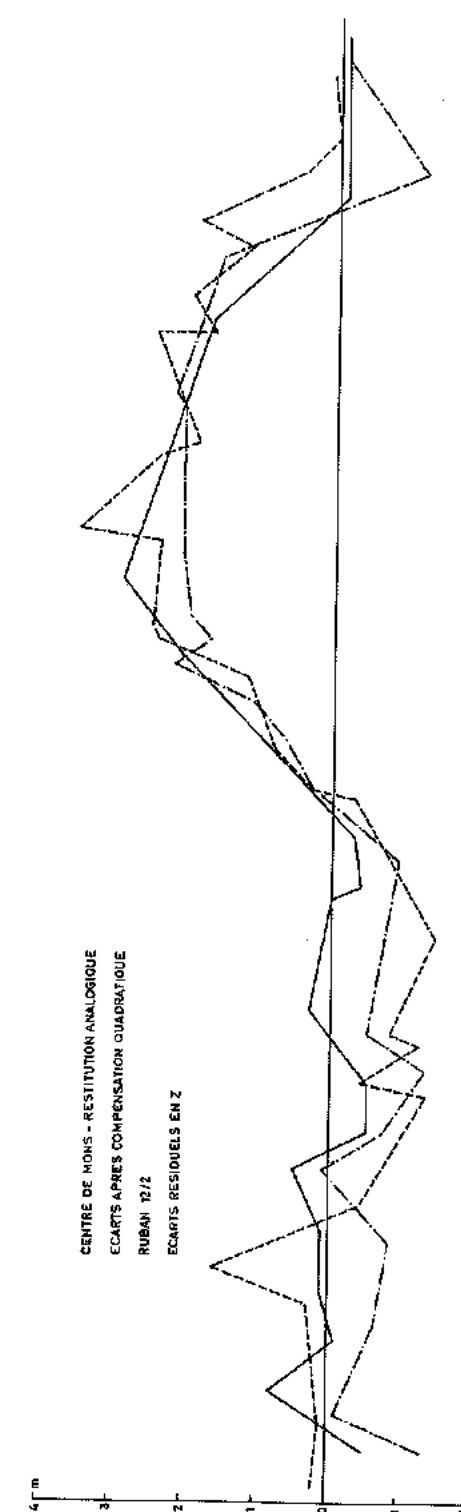


Figure 2

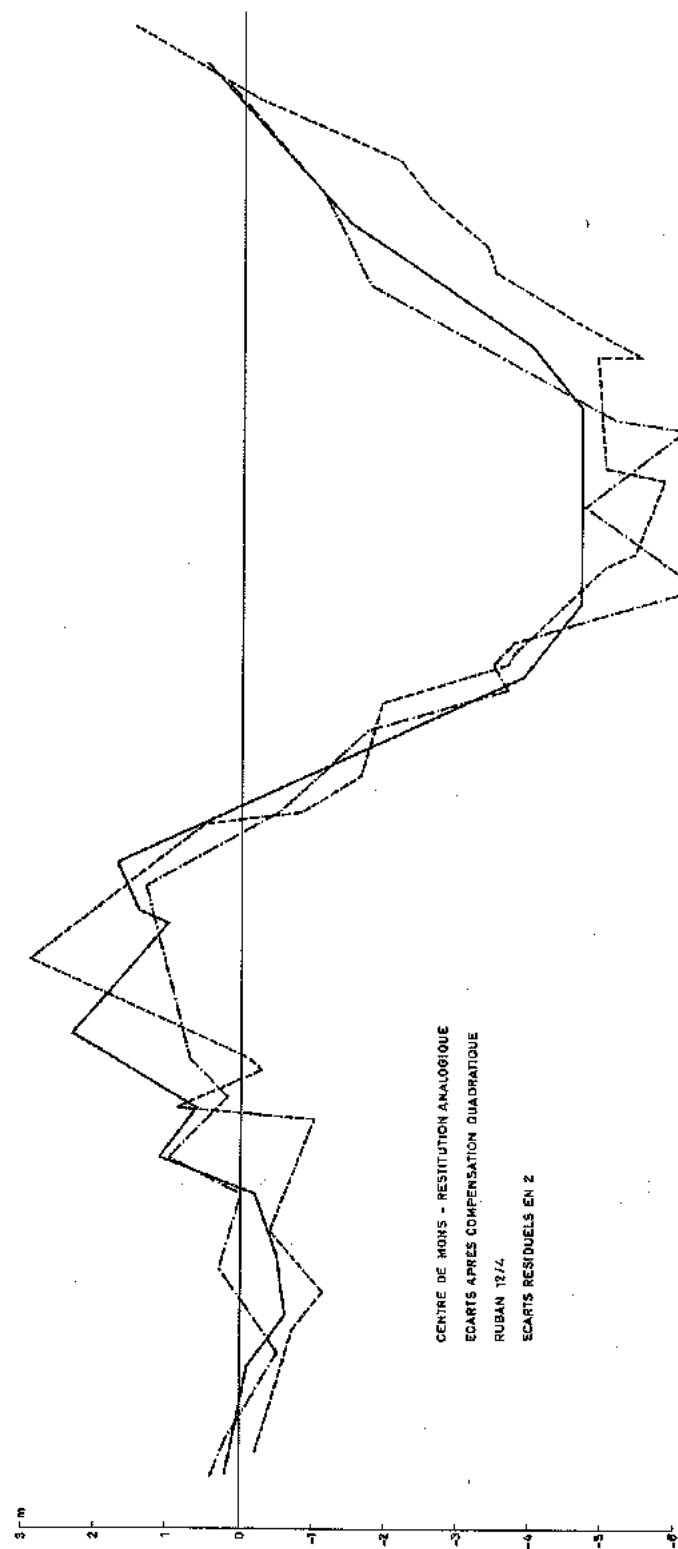


Figure 3

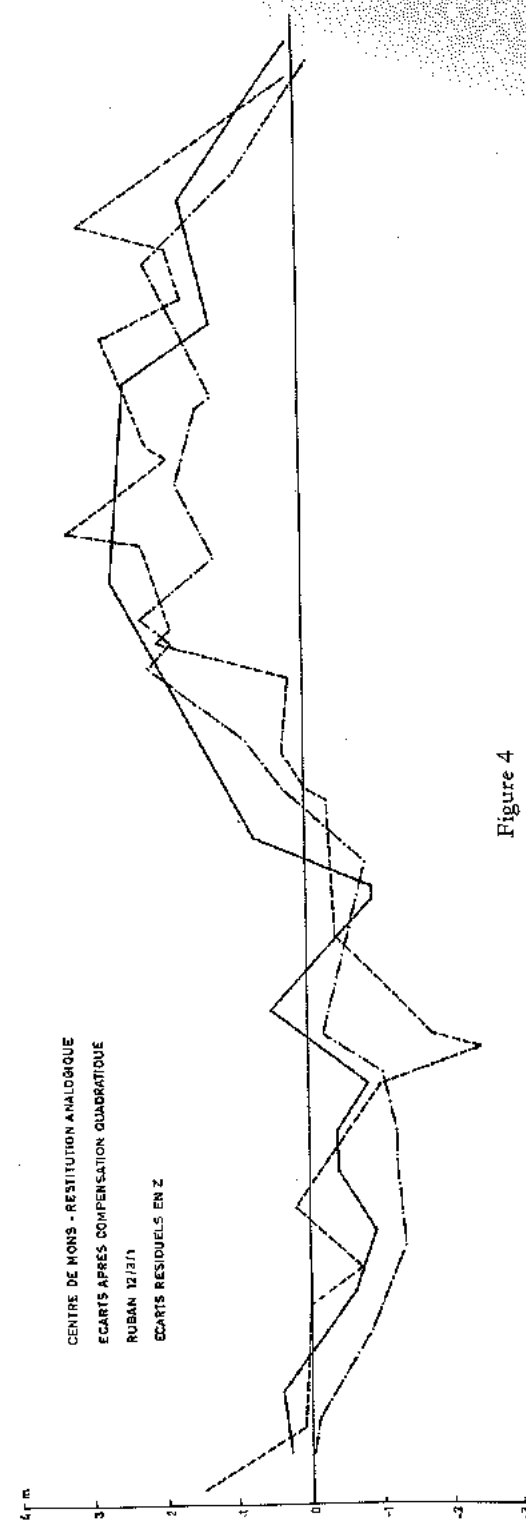


Figure 4

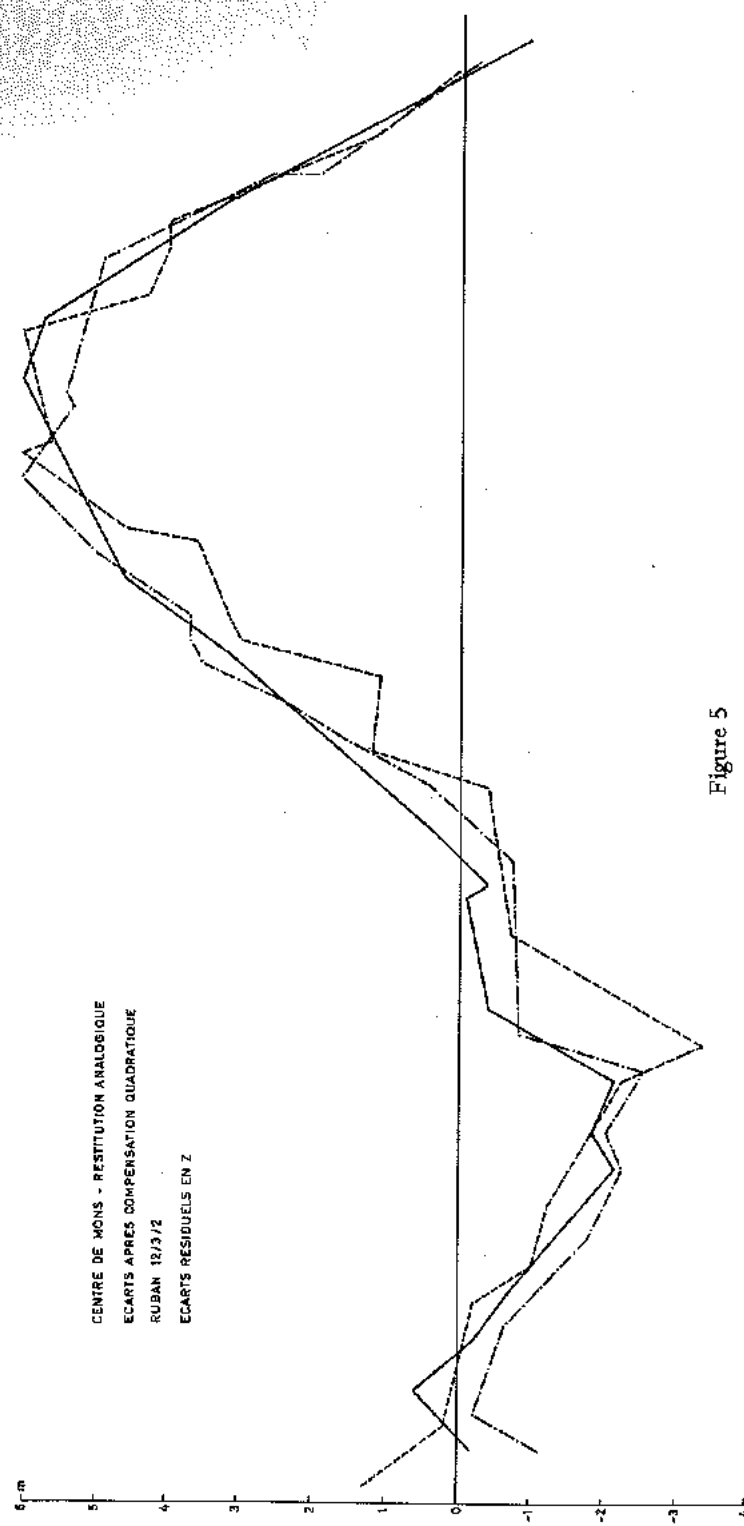


Figure 5

CENTRE DE MONS - RESTITUTION ANALOGIQUE
 ECARTS APRES COMPENSATION QUADRATIQUE
 RUBAN 12/3/3
 ECARTS RESIDUELS EN Z

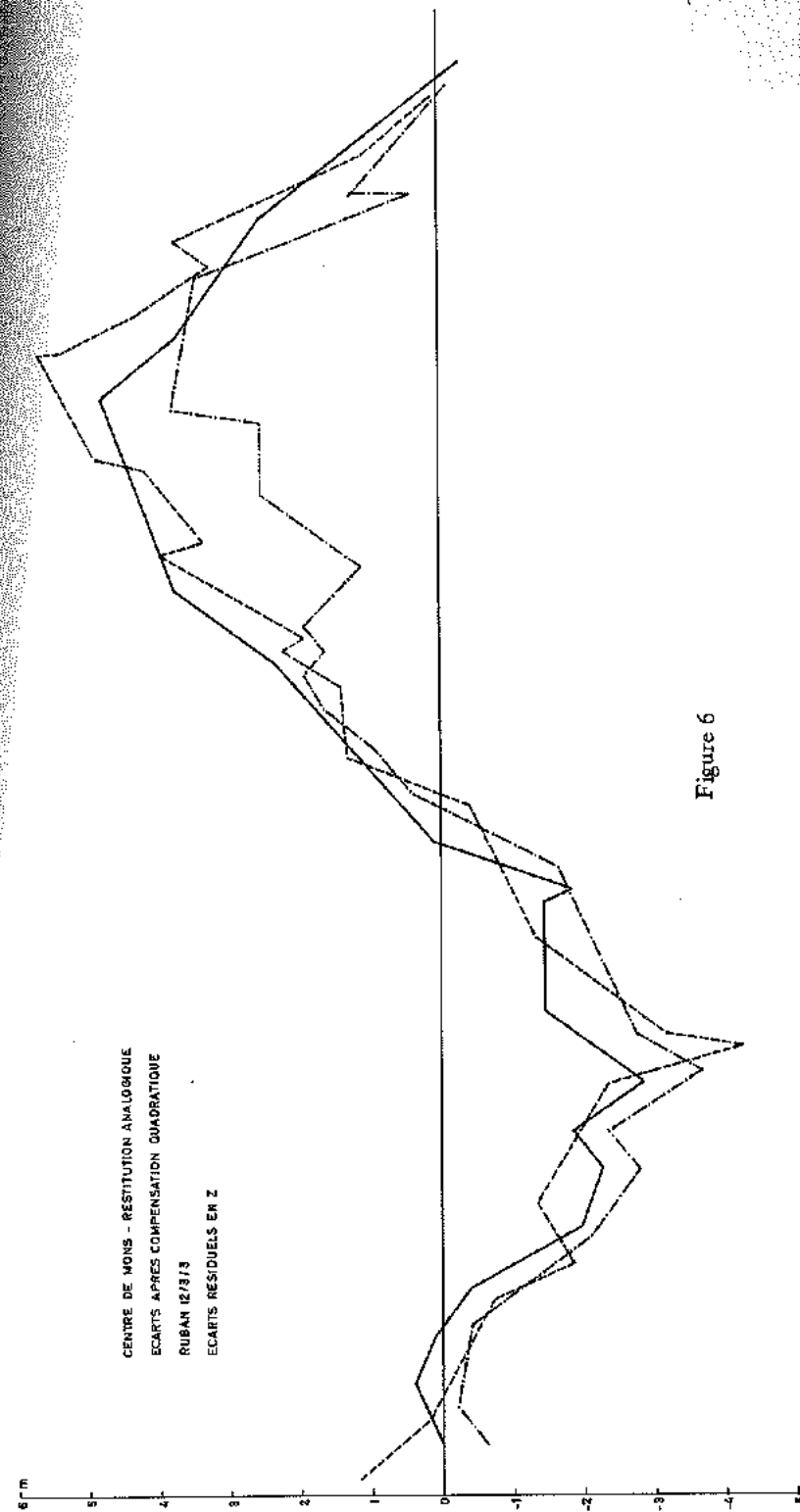


Figure 6

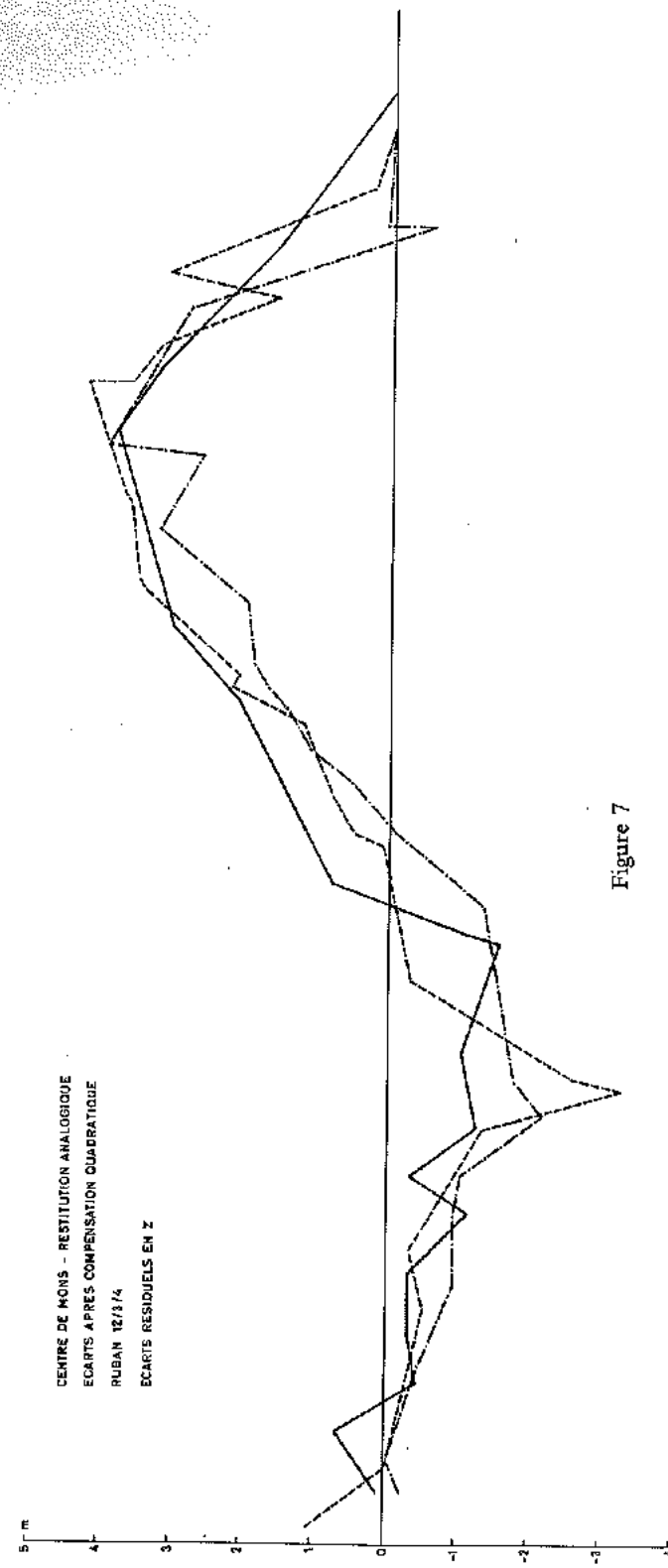


Figure 7

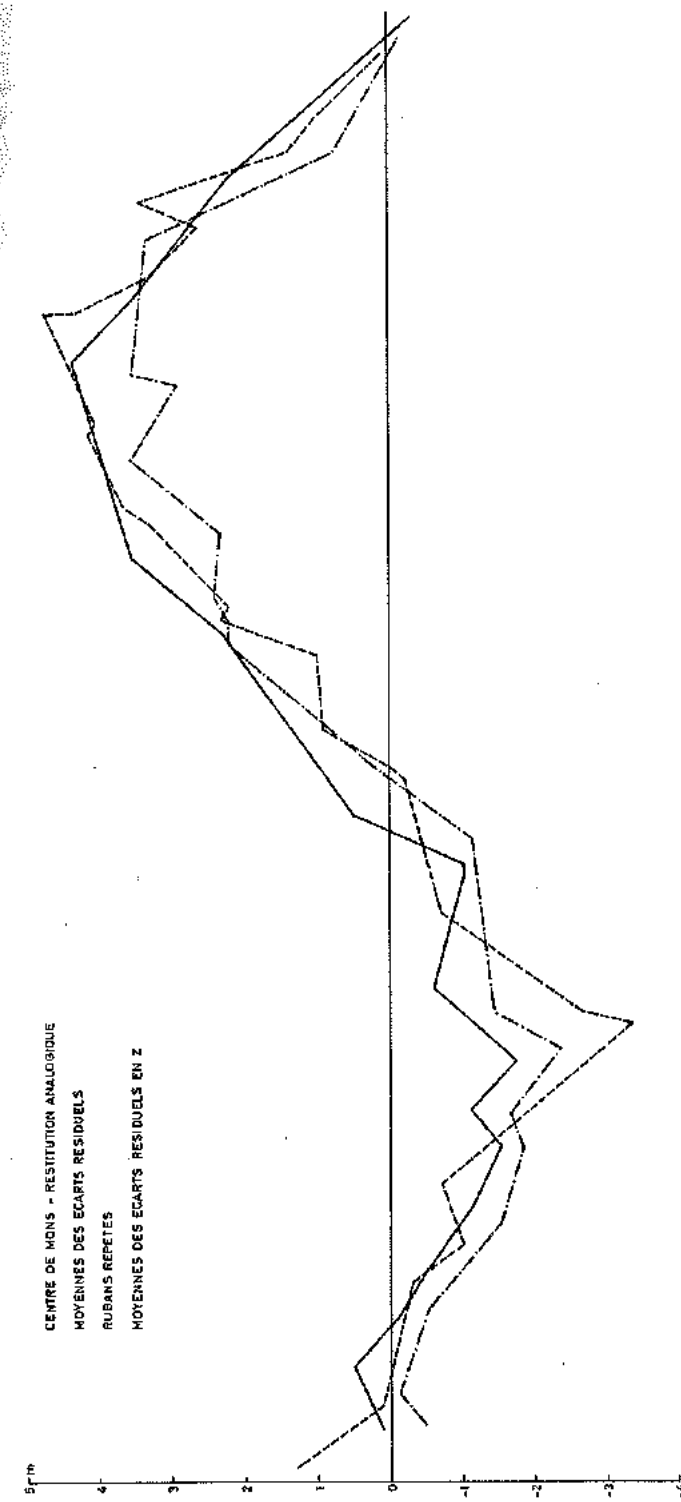


Figure 8

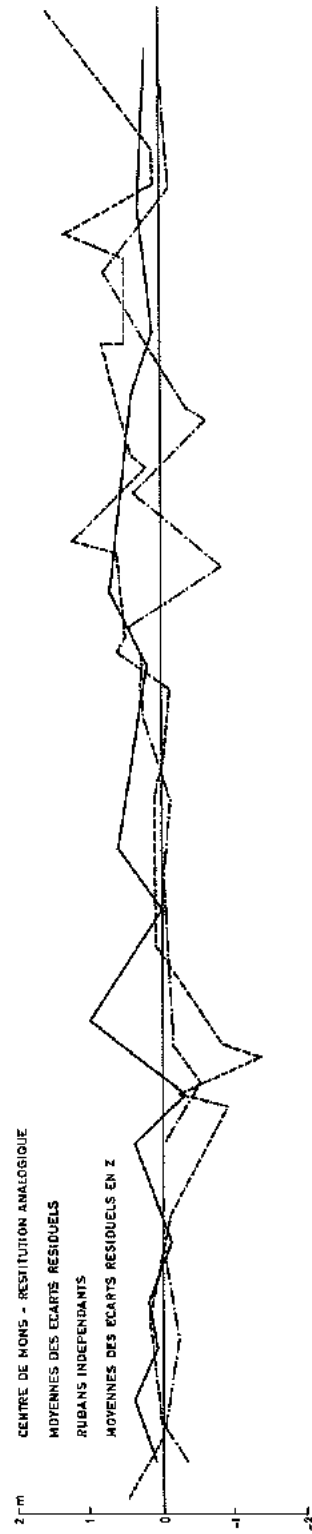


Figure 9

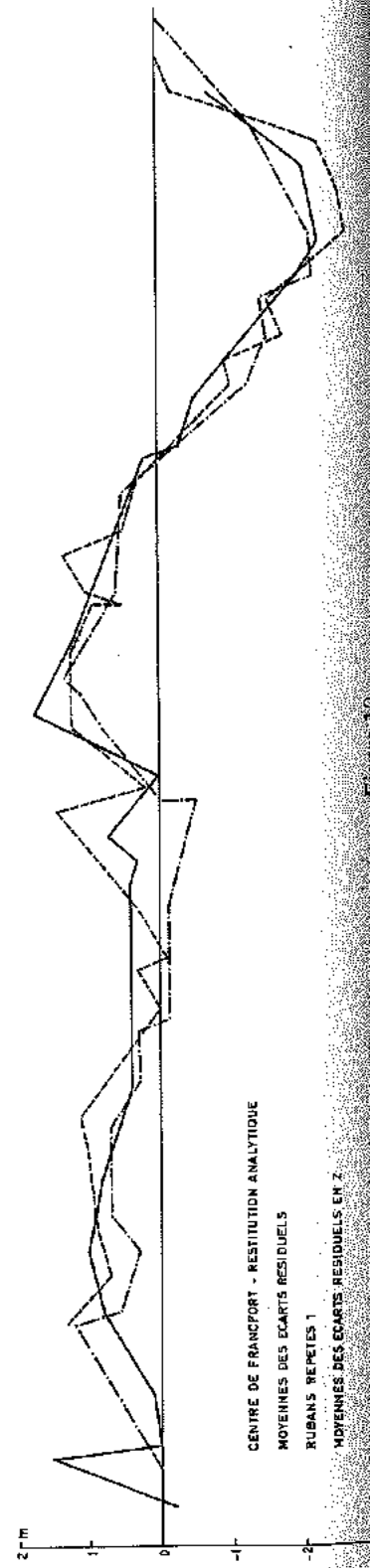


Figure 10

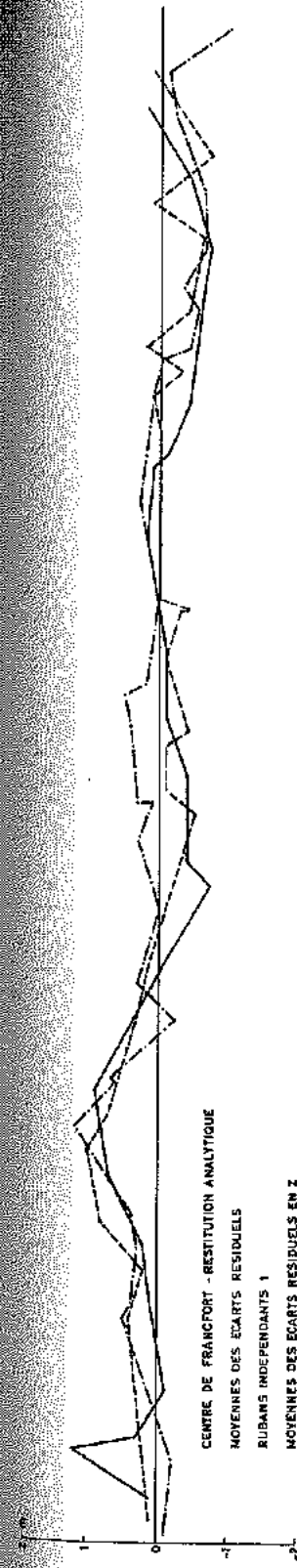


Figure 11

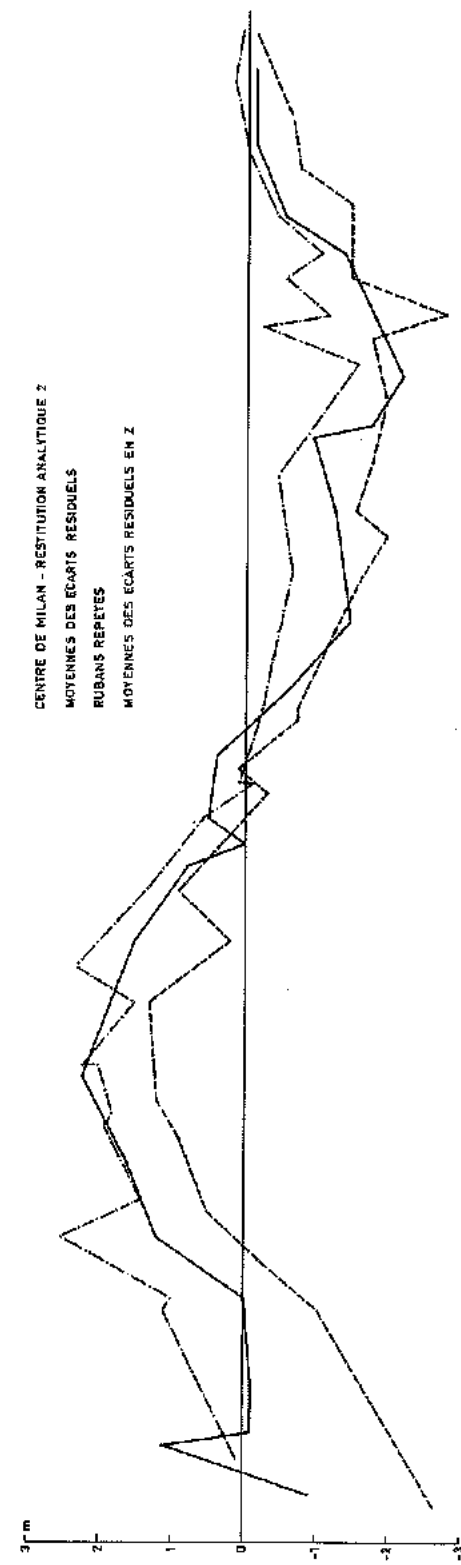


Figure 12

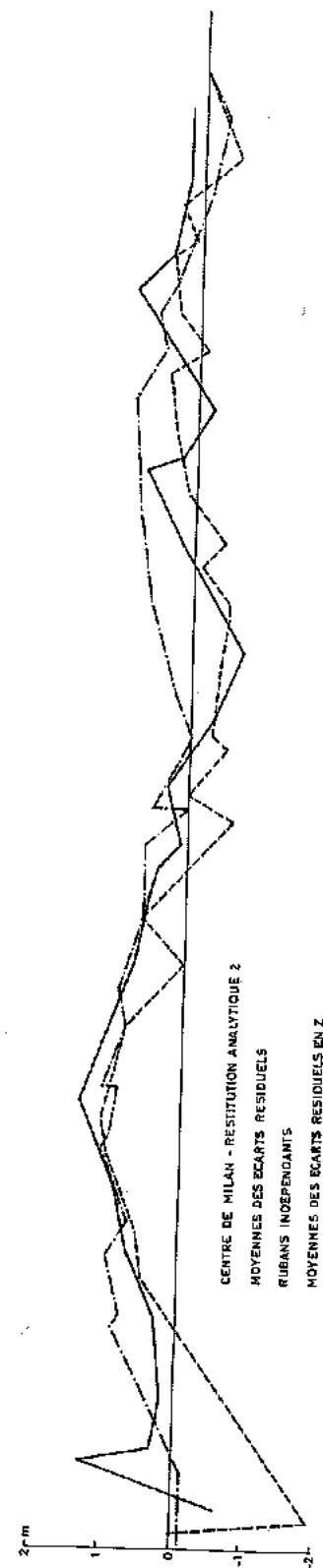


Figure 13

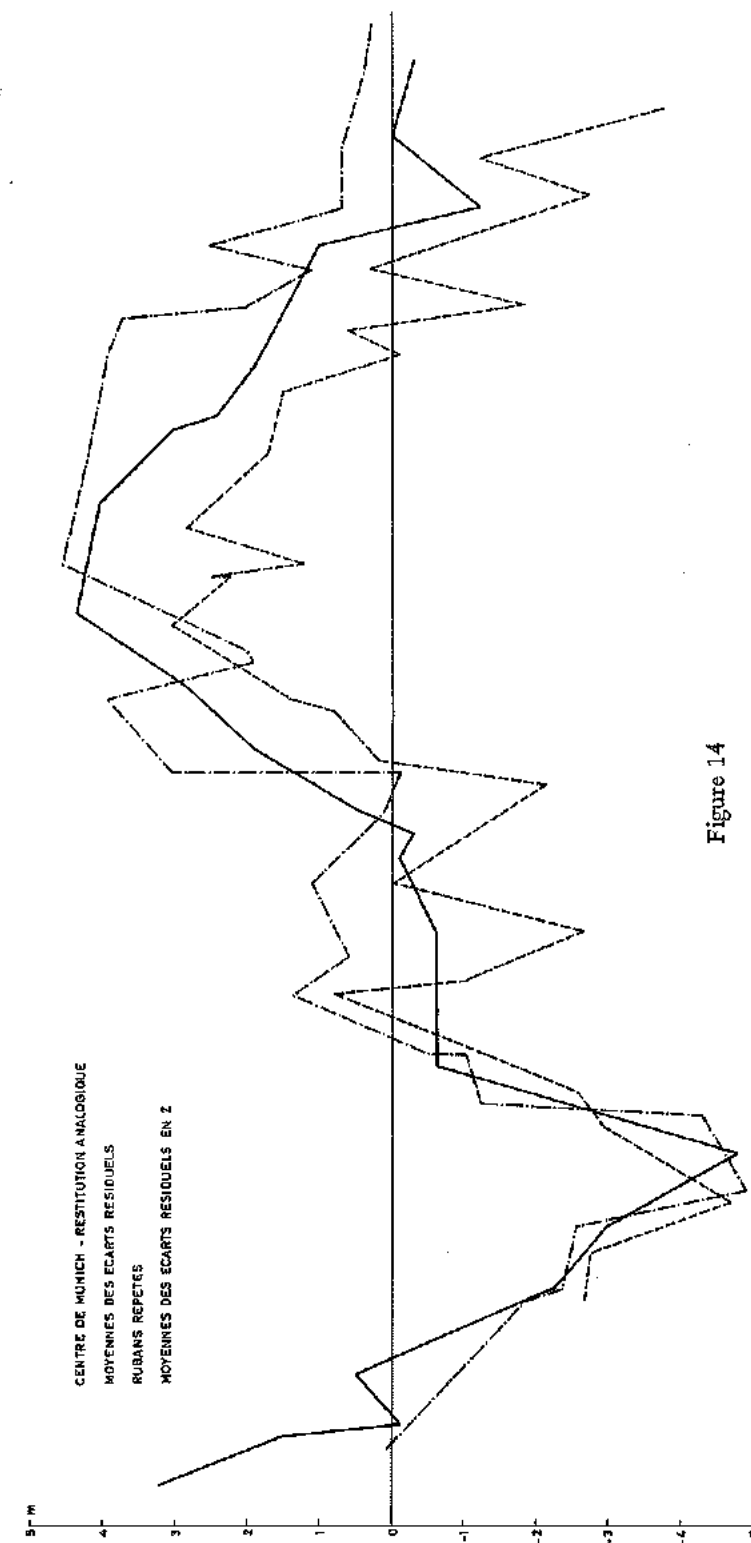


Figure 14

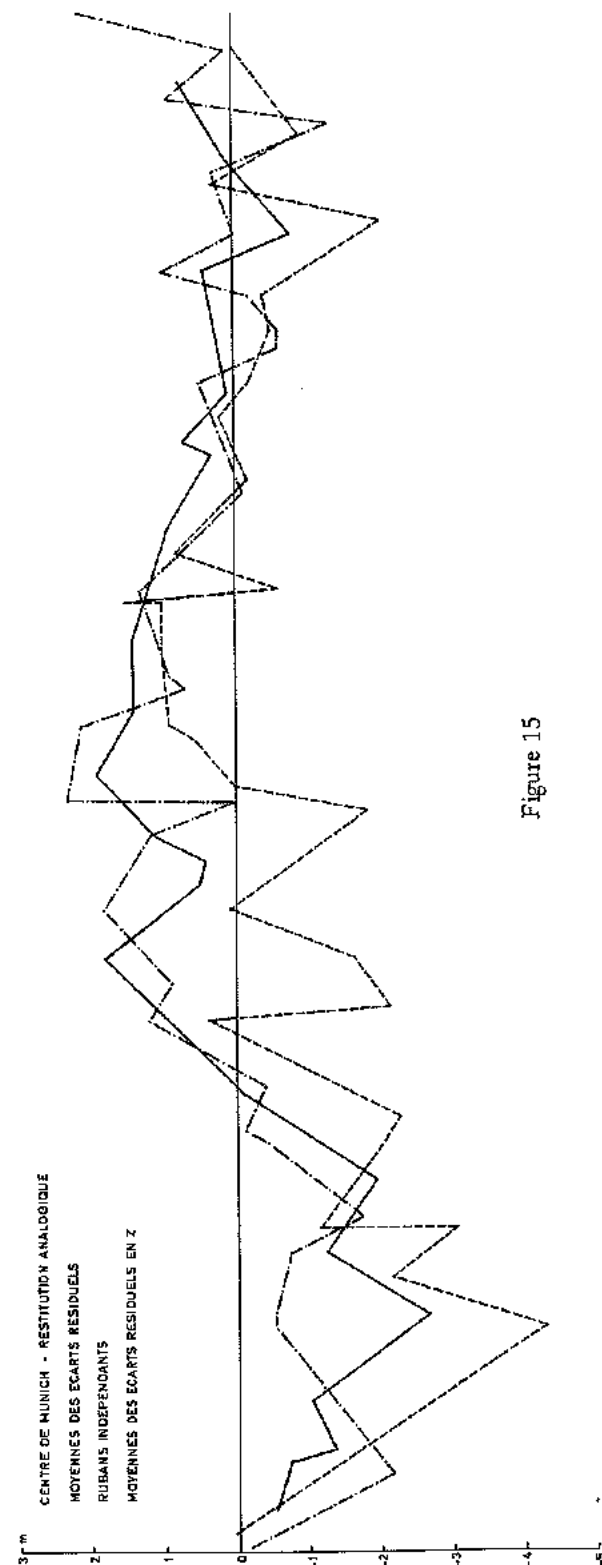


Figure 15

Discussion about the Subjects 6 and 7 under the Chairmanship of Prof. W. K. Bachmann

Chairman: In opening the discussion on the papers of Prof. *Wiser* and Dr. *Camps* I wish to make two observations. First of all, systematic errors have been largely neglected in the overall treatment of aerial triangulation. We know of their existence and now suddenly we hear that the systematic errors are twice as large as the accidental. How are we going to cope with them?

Gotthardt: Is there an explanation for the differences in the four sets of analytical measurements of Frankfurt since with large systematic errors one would expect σ to be fairly constant?

Camps: I can offer no explanation as yet since I have not yet had time to analyse the results.

Wiser: A partial explanation can be given by the various types of points used. In addition one must expect a certain difference between tests due to the summation of accidental errors.

Jerie: One of the problems is what constitutes a systematic and what an accidental error? Most of the errors can be determined, including purely observational errors, and thus very few irregular errors are left over.

Wiser: The ratio of systematic to random components was in the order of 2 : 1. Thus, 4 repetitions would only reduce the total error by a factor of 1.25 instead of a factor 2 if only random errors occurred.

Jerie: What practical conclusions can be drawn from this?

Wiser: Since the errors no longer form a random distribution, something must be done to reduce the systematic component, prior to any adjustment. This is the aim of the so-called Programme II.

Jerie: I am afraid that very little can be done unless a systematic component can be determined which is constant over a large number of photographs.

Chairman: It is certain that these results confirm the existence of systematic errors and I believe that this type of investigation should be continued. However, we must also be realistic when comparing different results as even the theory of errors appears to be subject to different interpretations, the late Prof. *Tienstra* claiming that systematic errors did not exist and Prof. *Jerie* now claiming the same for accidental errors.

DK 528.735-187:528.11

Etude statistique expérimentale des erreurs d'enchaînement des photogrammes

(avec 6 figures, 17 tableaux)

Par M. Cunietti, Milan - A. Vanossi, Milan*)

ABSTRACT: p. 226

1. Justification et but de la recherche

La recherche proposée ici est si simple qu'elle peut même sembler naïve. D'où la nécessité d'une analyse de ses motivations et de ses justifications.

Répéter plusieurs fois un même enchaînement, accumuler des données sur la dispersion pratique des erreurs d'enchaînement, tout cela constitue un travail peu séduisant. Il se présente tel qu'il est en réalité: un travail d'analyse expérimentale sans ambition, visant uniquement à constituer la base de départ d'intuitions futures moins indépendantes de la réalité que ne le sont souvent celles d'aujourd'hui.

De ce fait, la première justification réside dans la modestie de ces intentions, qui peuvent se résumer ainsi: documenter le plus largement possible le comportement des erreurs d'enchaînement, leur variabilité à l'intérieur de la bande, leur importance expérimentalement vérifiée, leur corrélation éventuelle avec les conditions d'ambiance de prise de vues ou de restitution.

Le but documentaire ne peut toutefois être abandonné aux caprices du hasard, car la recherche risquerait alors de naufrager dans la marée des données dépourvues de toute coordination.

Il est donc nécessaire d'encadrer logiquement la recherche afin de lui permettre de parvenir plus rapidement à son but.

A la base de la recherche nous trouvons la conviction que nos connaissances sur le comportement des nombreux phénomènes convergeant dans l'opération d'orientation des photogrammes sont encore aujourd'hui élémentaires et grossières.

La structure fine, le comportement particulier de chaque opération nous échappent; nous ne voyons que la structure globale et apparente, révélée par la multiplication des effets au cours de l'enchaînement. Le résultat global nous amène souvent à considérer la responsabilité du résultat comme susceptible d'être répartie en moyenne sur les différents maillons de la chaîne et nous ignorons les responsabilités localisées, souvent tout aussi insidieuses.

La question qui se repropose au sujet des erreurs accidentelles et systématiques devient bien plus complexe et hasardeuse quand on examine le produit final au terme d'une

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longue chaîne d'opérations; elle devient par contre plus simple et plus claire dès qu'on analyse les différents maillons de la chaîne, sans les détacher toutefois de la chaîne qui la unit et constitue de ce fait leur milieu naturel.

Le traitement *a posteriori* des résultats de l'enchaînement part généralement de l'hypothèse que le comportement accidentel des erreurs dans le cadre de la bande ou du bloc s'avère pratiquement homogène. Selon cette hypothèse, ni les bandes ni les blocs ne présentent des points et des zones de connexion plus faibles, où l'erreur quadratique moyenne des éléments d'enchaînement entre les modèles a une valeur plus élevée. Cette hypothèse n'a jamais été contrôlée directement, mais elle s'est toujours basée uniquement sur la conviction que les conditions de prise de vues demeurent à peu près entièrement constantes tout le long de la bande, et que par conséquent l'enchaînement lui-même est obtenu avec des erreurs à peu près constantes en tous les points de la bande ou du bloc.

De même, dans le cadre d'un bloc de bandes, toutes les bandes sont supposées avoir une égale précision, et toutes sont par conséquent traitées de la même manière. Cette supposition répond-elle à la réalité? Il est très raisonnable de penser que les conditions de prise de vues et celles du territoire, évidemment différentes, aboutissent également à des situations différentes de précision de chaque bande.

Dans le cas des modèles enchaînés, les déformations provoquées par les erreurs commises dans les éléments d'orientation sont estimées d'importance à peu près homogène, car les erreurs quadratiques moyennes des éléments d'orientation des modèles sont toutes estimées *a priori* du même ordre de grandeur. Cette hypothèse exige, elle aussi, une confirmation.

2. Méthodologie d'exécution de la recherche expérimentale

Aucune difficulté conceptuelle n'entrave la méthodologie d'exécution de la recherche, sinon la difficulté — commune à toutes les recherches statistiques expérimentales — d'obtenir des résultats dignes de foi et significatifs. Cette validité et cette significativité ne peuvent que dériver de l'étendue de l'échantillon examiné, de la variété des conditions de prise de vues contenues dans cet échantillon et de celle des procédés de calcul qui président à l'élaboration de l'enchaînement, du nombre des répétitions des essais sur chaque bande, de la minutie des procédés d'enquête statistique adoptés et des tests utilisés. Cela entraîne comme conséquence que le travail de recherche devient très long et doit être réparti sur plus d'un centre de recherche et que les résultats peuvent sembler en définitive disproportionnés à la longueur et à l'étendue du travail ainsi qu'aux énergies employées.

Il est hors de doute, dans ces circonstances, que des données peu nombreuses — si intéressantes soient-elles — ne peuvent aboutir à aucune conclusion.

Ne nous étonnons donc pas si les argumentations rapportées ici ne peuvent constituer que le début d'une recherche, début susceptible tout au plus de nous donner quelques indications sur la façon de procéder au cours de l'étude.

Sous l'angle des opérations à effectuer, il faut choisir, pour entreprendre la recherche, le matériel à utiliser, les instruments et les méthodes de mesure et d'enchaînement, le nombre de répétitions de chaque enchaînement, le traitement analytique des résultats aux fins de l'analyse statistique.

a) La recherche ne présente aucune difficulté en ce qui concerne le matériel photographique à utiliser. Toute bande obtenue à des fins pratiques ou pour des études expérimentales peut être employée. Il n'est point nécessaire de disposer d'un nombre considérable de points de contrôle sur le terrain pour l'étude détaillée des erreurs. Il suffira de disséminer, le long de la bande, quelques points à coordonnées connues, en nombre suffisant pour contrôler les déformations systématiques les plus manifestes des bandes. La longueur de ces dernières pourra être variable; il est estimé toutefois qu'un maximum de vingt-vingt-cinq photogrammes par bande pourra suffire. Dans le choix du matériel photographique il faudra avoir soin de varier au maximum les conditions de la prise de vues ainsi que la conformation et la situation du terrain photographié.

b) Chaque Centre participant à la recherche devra choisir un procédé d'exécution des opérations et sera tenu de le respecter scrupuleusement, non seulement pendant toutes les répétitions successives de la même bande, mais aussi pour tout un groupe de bandes examinées. On doit évidemment choisir des procédés d'enchaînement susceptibles de mettre en évidence les erreurs de restitution, comme celles d'orientation et d'enchaînement, de la façon la plus directe possible. A ces fins il y a lieu de conseiller les méthodes d'enchaînement analytique et, parmi ces dernières, les plus simples et les moins sophistiquées, c'est à dire celles où le lien géométrique entre les photogrammes est obtenu sans compensations susceptibles de masquer les erreurs.

Naturellement, l'instrument employé pour les mesures — qu'il soit analogique ou analytique — doit être soumis à un contrôle périodique pour connaître son instabilité éventuelle dans le temps.

c) L'un des éléments à établir dans la recherche, c'est le nombre de répétitions d'une même bande dans les mêmes conditions d'opération, afin que les éléments obtenus de l'analyse des résultats aient une signification statistique.

Les données statistiques qui nous intéressent sont: les moyennes, les écarts quadratiques moyens (é. q. m.), les coefficients de corrélation.

Pour les moyennes il faut disposer d'un nombre de répétitions permettant d'établir la significativité d'une moyenne (ou de la différence entre deux moyennes), comparée à l'hypothèse nulle, au moins lorsque la moyenne (ou la différence des moyennes) est du même ordre de grandeur que l'é. q. m. de la population d'où l'on a tiré les éléments qui ont abouti au calcul de la moyenne ou des moyennes correspondantes; c'est-à-dire M (ou $M_2 - M_1$), approximativement égal à l'é. q. m. σ . Si nous considérons que la moyenne est significative quand:

$$\frac{M \sqrt{n}}{\sigma} = t \geq t_0 \quad (1)$$

où t_0 est la valeur du test t de Student, valeur qui a une probabilité de 0,05 d'être dépassée pour quelque cause accidentelle, en introduisant l'hypothèse choisie sur la valeur de la moyenne ($M = \sigma$), nous obtenons

$$\sqrt{n} = t \pm t_0$$

Il résulte des tableaux du test de Student, en fonction des degrés de liberté, que la plus petite du nombre n de répétitions à effectuer est 8.

En ce qui concerne la significativité de la différence entre deux é. q. m. on peut raisonner de façon analogue, en partant de l'hypothèse que le nombre de répétitions doit être de nature à nous permettre de considérer deux é. q. m. comme provenant de populations à dispersion différente, lorsque le rapport entre le plus grand et le plus petit est approximativement égal à 1,7 ($\sigma^2_1 / \sigma^2_2 = 3$). En pareil cas, le test z de Fischer prend la valeur

$$z = 1,15 \log_{10} \frac{\sigma^2_1}{\sigma^2_2} = 1,15 \log_{10} 3 = 0,55 \quad (3)$$

Une fois en possession de cette valeur, on analyse le tableau de la fonction de Fischer calculée en fonction des degrés de liberté des deux é. q. m.: dans l'hypothèse estimant significative une différence lorsque la valeur de z correspondante a une probabilité de 0,05 % d'être dépassée pour des causes accidentelles, on obtient qu'un total de 12 essais est nécessaire pour mettre en évidence des différences significatives éventuelles du type indiqué.

Pour ce qui concerne les coefficients de corrélation linéaire le tableau n. 1 nous montre, dans le cas théorique de $r = 0$, que les limites du champ que le coefficient r ne peut pas dépasser pour des causes purement accidentelles (avec une limite de confiance de 0,95) varient lentement à mesure que le nombre des répétitions augmente. Pour obtenir des intervalles assez limités autour de la valeur théorique, il faut augmenter énormément le nombre des essais.

Le tableau n. 2 reproduit, pour certaines valeurs théoriques du coefficient de corrélation linéaire, les limites des zones non critiques lorsque le nombre des répétitions sur lesquelles sont effectués les calculs arrive à 15. Nous tirons de ce tableau que la limite de la zone non critique atteint son maximum pour la valeur théorique $r = 0$.

Il résulte clairement de cette étude l'impossibilité pratique de procéder à des essais suffisants pour limiter la zone non critique pour n'importe quelle valeur de r .

Il est toutefois possible, pour obvier à cet inconvénient, de considérer non point les valeurs de r pour les différents modèles, mais sa valeur moyenne pour des groupes de cinq ou six modèles consécutifs de la bande. De telle façon on perd une partie des informations analytiques, mais on augmente la significativité des résultats.

En admettant de subdiviser les bandes en groupes de cinq ou six modèles, avec 15 répétitions de la bande, nous pouvons atteindre des limites de significativité assez remarquables, même dans le cas le plus défavorable ($r = 0$).

En conclusion de cette analyse on peut fixer à 15 le nombre minimum de répétitions permettant de porter à des résultats suffisamment significatifs pour les moyennes, les é. q. m. et les coefficients de corrélation obtenus expérimentalement.

Les résultats numériques des essais répétés d'enchaînement d'une même bande se résument dans les données suivantes:

- parallaxes résiduelles sur les points d'orientation relatifs des photogrammes;
- cassures sur les points de jonction des modèles le long de la bande;
- valeur des éléments d'orientation angulaire des photogrammes;
- valeur des composantes de la base des différents modèles;
- coordonnées-bande des points restitués.

Il est évident que ces données numériques ne sont pas directement utilisables et qu'il faut les réélaborer par des procédés opportuns de calcul que nous énumérons ci-après.

Pour la compréhension des symboles utilisés il est bon de tenir compte des bases de départ suivantes:

$i = 1, \dots, m:$	numéro indice du modèle dans la bande de m modèles.
$j = 1, \dots, n:$	numéro indice de la répétition de l'enchaînement.
$k = 1, \dots, r:$	numéro indice du point utilisé pour l'orientation.
$w = a, N, b:$	indice du point de connexion entre modèles consécutifs de la bande: a = en haut, N = au centre, b = en bas.
$z = x, y, z:$	indice de la coordonnée considérée.
$O = \varphi, \omega, \chi, bx, by, bz:$	éléments d'orientation brutes des photogrammes.
$O' = \Delta\varphi, \Delta\omega, \Delta\chi, R, \Delta\Phi, \Delta\Theta:$	éléments d'orientation angulaire et d'enchaînement des modèles.
$p = 1, \dots, s:$	numéro indice du groupe de modèles successifs de la bande sur lesquels on a calculé la valeur moyenne des coefficients de corrélation.
$h = 1, \dots, N:$	numéro indice du point restitué le long de la bande.
$c = 1, \dots, M:$	coefficient du terme du polynôme interpolateur des corrections.

Avec les parallaxes résiduelles P sur les points d'orientation, nous calculons:

- 1) le $\sigma_{0,ij}$ du système d'équations d'orientation relatif aux divers modèles et aux diverses répétitions.
- 2) La moyenne $M(P)_{ik}$ des parallaxes résiduelles P dans les n répétitions de l'enchaînement, relatif au modèle i -ième et au point d'orientation k -ième.
- 3) L'é. q. m. $\sigma(P)_{ik}$ des parallaxes résiduelles P dans les n répétitions de l'enchaînement, relatif au modèle i -ième et au point d'orientation k -ième.

Les cassures R sur les points de jonction des modèles le long de la bande permettent de calculer:

- 4) La moyenne $M(R)_{iwt}$ des cassures R entre le modèle i -ième et le modèle $(i+1)$ -ième aux points communs à deux modèles successifs, dans les n répétitions de l'enchaînement relatif à chaque modèle, à chaque point de liaison w et à chaque coordonnée t .
- 5) L'é.q.m. $\sigma(R)_{iwt}$ des cassures entre modèles consécutifs dans les n répétitions, relatif à chaque modèle, à chaque point et à chaque coordonnée.

L'analyse statistique des éléments d'orientation angulaires des photographes et des composantes de la base est effectuée, pour une première phase, en utilisant les valeurs brutes directement tirées de l'enchaînement; on calcule pour chacun des éléments d'orientation O :

- 6) la moyenne $M(O)_i$ de l'élément d'orientation O dans les n répétitions de l'enchaînement pour chaque modèle.
- 7) L'é.q.m. $\sigma(O)_i$ de l'élément d'orientation O dans les n répétitions relatif à chaque modèle.

Au cours d'une deuxième phase, les éléments d'orientation des photographes sont dûment traités de façon à rendre chaque élément de chaque modèle indépendant des erreurs existant dans les modèles qui le précèdent.

Pour les éléments angulaires des photographes des modèles nous calculons:

$$\begin{aligned}\Delta\varphi_i &= \varphi_{i+1} - \varphi_i \\ \Delta\omega_i &= \omega_{i+1} - \omega_i \\ \Delta\chi_i &= \chi_{i+1} - \chi_i\end{aligned}\quad (4)$$

pour les composantes de la base:

$$\begin{aligned}R_i &= \frac{bx_{i+1}}{bx_i} \\ \Delta\Phi_i &= \arctang \frac{bz_{i+1}}{bx_{i+1}} - \arctang \frac{bz_i}{bx_i} \\ \Delta\Theta_i &= \arctang \frac{by_{i+1}}{bx_{i+1}} - \arctang \frac{by_i}{bx_i}\end{aligned}\quad (5)$$

Ces différences contiennent les erreurs d'enchaînement d'un couple de modèles consécutifs de la bande, mais elles sont indépendantes des erreurs d'enchaînement des modèles précédents.

Pour le premier modèle, nous devons raisonner à part. Dans le cas du premier modèle, les (5) ont été ainsi calculées:

$$\begin{aligned}R &= \frac{bx_1}{bx_0} \\ \Delta\Phi &= \arctang \frac{bz_1}{bx_1} - \arctang \frac{0}{bx_0} = \arctang \frac{bz_1}{bx_1} \\ \Delta\Theta &= \arctang \frac{by_1}{bx_1} - \arctang \frac{0}{bx_0} = \arctang \frac{by_1}{bx_1}\end{aligned}\quad (6)$$

Rappelons qu'on peut imaginer un modèle de place $i = 0$ où les composantes de la base peuvent être arbitraires, mais restent constantes et sans erreurs à chaque répétition de la bande; en particulier on peut considérer ce qui suit:

$$\begin{aligned}bx_0 &= M(bx_{1j}) = \frac{\sum_{j=1}^n bx_{1j}}{n} \\ by_0 &= 0 \\ bz_0 &= 0\end{aligned}\quad (7)$$

Pour chacune de ces six valeurs des éléments d'orientation des modèles nous calculons:

- 8) la moyenne $M(O')_i$ de la valeur de l'élément O' obtenue pour chaque couple de photographes $i, i+1$, dans les n répétitions de la bande.
- 9) L'é. q. m. $\sigma(O')_i$ des valeurs de l'élément O' relatif à chaque couple de modèles. L'analyse statistique des éléments d'orientation est complétée par le calcul des corrélations existant entre les différents éléments d'orientation relatifs à chaque modèle. La formule employée est bien connue.

Ces valeurs de $r(O'O')_i$ permettent donc de calculer:

- 10) La moyenne $M(r)_g$ des coefficients de corrélation sur des groupes de modèles consécutifs (cinq ou six en général) pour pouvoir rendre significative la valeur aux fins d'une interprétation statistique.

Les points éventuellement restitués le long de la bande peuvent être utilisés pour étudier la propagation des erreurs accidentelles de restitution sur le parcours de la bande en question.

- 11) L'é. q. m. $\sigma(t)_h$ des coordonnées restituées relatif à chaque point dans les n répétitions de l'enchaînement.

Au cas où l'on connaîtrait les coordonnées de ces points, ils pourraient être utilisés pour étudier la composante systématique des erreurs d'enchaînement.

Pour cette étude il faut ajuster chaque bande, en utilisant tous les points connus, par une formule d'interpolation des erreurs du second ou du troisième degré, puis étudier le comportement des différents coefficients des termes des polynômes interpolateurs. Nous calculerons pour ces coefficients:

12) La moyenne $M(c)_i$ du coefficient c du polynôme dans les n répétitions de la bande pour chaque coordonnée.

13) L'é. q. m. $\sigma(c)_i$ du coefficient du polynôme relatif à chaque coordonnée.

3. Schéma d'interprétation des résultats

Tout en laissant à l'analyse *a posteriori* la plus vaste liberté d'interprétation des résultats des différents essais, nous donnons ci-après un schéma sommaire de la possibilité d'obtenir des indices statistiques énumérés au paragraphe 2d), des informations sur la nature et sur le comportement des erreurs:

- 1) Les valeurs moyennes des é. q. m. σ_{0ij} des parallaxes résiduelles, soit pour chaque modèle sur les différentes répétitions de la bande, soit pour chaque répétition le long de la bande, fournissent une information globale concernant:
 - l'homogénéité de comportement des modèles
 - l'uniformité d'exécution des mesures dans les différentes répétitions.
- 2) Les moyennes des parallaxes résiduelles dans les différents points d'orientation standard des modèles $M(P)_{ik}$ nous expliquent la présence d'erreurs systématiques d'orientation relative des modèles: si, en outre, ces valeurs moyennes se présentent avec une certaine stabilité le long de la bande, nous parvenons à la conclusion qu'il existe des erreurs systématiques dans le matériel photographique, erreurs probablement imputables à la caméra. Les tests statistiques de validité calculés en utilisant les é. q. m. $\sigma(P)_{ik}$, peuvent étayer l'analyse des moyens.
- 3) De même, les moyennes et les é. q. m. des cassures entre modèles consécutifs $M(R)_{ij}$, $\sigma(R)_{ij}$ permettent d'identifier la présence d'erreurs systématiques de formation des modèles, localisées entre les couples de modèles consécutifs ou communes à tous les modèles d'une bande. L'analyse quantitative de la composante systématique locale et globale peut parvenir parfois à l'identification de la cause de l'erreur correspondante.
- 4) La comparaison entre la courbe des moyennes des valeurs brutes des éléments d'orientation des photogrammes $M(O)_i$ le long de la bande et celle des é. q. m. de ces mêmes valeurs $\sigma(O)_i$ fournit une indication concernant l'influence des variations de la position de la chambre sur la précision d'enchaînement des photogrammes. La forme de la courbe des é. q. m. et les éventuelles variations brusques de la pente constituent autant d'indications qualitatives sur l'homogénéité de comportement de la bande.

Les mêmes courbes des é. q. m. pourront être comparées à l'allure déduite par voie théorique.

- 5) Le même type d'analyse quantitative et qualitative peut se dérouler avec une signification plus grande sur les moyennes et sur les é. q. m. des éléments d'orientation et d'enchaînement des modèles (O') obtenus par les calculs décrits en 2d). L'analyse de ces données, compte tenu de la valeur du test de Fischer pour la comparaison des é. q. m., nous fournit des indications quantitatives surtout sur l'homogénéité de formation des modèles et d'enchaînement de modèles consécutifs le long de la bande.

La présence d'hétérogénéités éventuelles (si la formation des modèles s'avère homogène sous le rapport des parallaxes comme sous le rapport des cassures) indique vraisemblablement la présence d'une configuration de la prise de vues susceptible d'affaiblir la rigidité de l'enchaînement. En outre, au sens absolu, ces données sont porteuses d'informations expérimentales sur la valeur de l'é. q. m. des différents types d'éléments d'orientation et d'enchaînement des modèles.

- 6) Le nombre trop peu élevé d'éléments dans l'échantillon analysé rend peu significatives les valeurs du coefficient de corrélation calculées pour les différents modèles. La moyenne par groupes de modèles, éventuellement choisis selon des critères particuliers, devient plus significative et permet soit la comparaison entre les valeurs obtenues expérimentalement et celles qu'on obtient analytiquement sur les modèles factices, soit le contrôle des variations éventuelles de valeur en fonction de la variation des conditions de prise de vues le long de la bande.
- 7) Si l'on dispose des é. q. m. des coordonnées restituées pour un certain nombre de points le long de la bande, on pourra comparer l'allure des é. q. m. expérimentales à celle qui provient des modèles théoriques.

Ces derniers se basent en général sur l'hypothèse d'un comportement homogène des modèles et des éléments d'orientation. Cette comparaison pourra donner bien à des suggestions pour la modification des modèles mathématiques sur la propagation des erreurs.

- 8) L'étude du comportement de la compensation polynômiale, peut constituer une conclusion utile de l'enquête sur chaque bande répétée afin de permettre un choix plus approprié du degré du polynôme à utiliser pour l'interpolation des erreurs systématiques sur chaque bande et pour vérifier si le degré du polynôme varie d'une bande à l'autre en fonction des paramètres de la prise de vues et des conditions du terrain.

4. Essais expérimentaux effectués

Au début de l'hiver 1971/72, lorsque on a accepté de participer au présent Symposium OEEPE, on nourrissait l'illusion de pouvoir présenter une moisson considérable de résultats.

Malheureusement la réalité actuelle est bien différente. On doit se borner à présenter les conclusions, d'ailleurs partielles, de deux seules bandes répétées; pour une troisième bande, les répétitions n'atteignent que la moitié du nombre prévu; des inconvénients de calcul ont empêché pour l'instant d'utiliser les résultats d'une quatrième bande.

Voici maintenant un rapide compte-rendu et une description résumée du matériel utilisé et des essais effectués.

Premier essai - Bande 6.2.1. Reichenbach

- Vol expérimental au-dessus du polygone de Reichenbach pour la Commission B de l'OEEPE.
- Hauteur de vol 1200 m au-dessus du sol, échelle moyenne 1 : 12 000.

- Caméra Wild RC7 à plaques 15 x 15, dist. focale 100 mm, 10 photographes.
- Terrain de collines, avec variations d'altitude atteignant au maximum 25 % de la hauteur de vol. Il a été procédé, en juin 1967, à 16 répétitions de l'enchaînement de cette bande.

Chaque enchaînement était complété en une seule journée (6 heures).

Deuxième essai — Bande 14/3 Oberschwaben

- Vol expérimental au-dessus du polygone d'Oberschwaben pour la Commission A/B de l'OEPE.
- Hauteur de vol 4300 m au-dessus du sol, échelle moyenne 1 : 28 000.
- Caméra Zeiss 23 x 23, dist. focale 150 mm, 25 photographes.
- Terrain de collines, avec variations maximales d'altitude de 10 %. Pendant les mois de novembre et décembre 1972, il a été procédé à la répétition de 16 enchaînements de la bande.

Durée de chaque enchaînement: 2 jours; les interruptions ont toujours correspondu au même photogramme.

Troisième essai — Bande L Vol Udine-Tarvisio

- But du vol: étude du tracé autoroutier.
- Hauteur de vol: 1500 m. Echelle moyenne 1 : 10 000.
- Caméra Wild RC5 23 x 23, dist. focale 150 mm, 6 photographes.
- Terrain montagneux, variations d'altitude parfois supérieures à 30 %, axe de la bande coïncidant avec le centre d'une vallée.

Il a été procédé, au mois de janvier 1973, à 16 répétitions de cette bande. La durée de chaque enchaînement était d'une journée.

Quatrième essai — Bande 6/1 — Vol Oberschwaben

Caractéristiques du vol identiques à celles du deuxième essai. En février et mars 1973 il a été procédé à 8 répétitions de l'enchaînement de la bande. La durée de l'enchaînement était de deux jours.

Sur tout ce matériel, les mesures ont été exécutées à l'aide du stéréocomparateur TA 3/P OMI à trois chariots. L'état de réglage de l'instrument était normal; en outre (ce qui est très important), la stabilité de cet instrument — constamment contrôlée au cours des essais — s'est révélée satisfaisante.

Les mesures ont été exécutées selon le procédé suivant:

On mesure sur chaque plaque, en plus des coordonnées des repères, les coordonnées instrumentales de 9 points disposés sur trois lignes normales à l'axe du vol. Chacun de ces 9 points est pointé sur trois plaques consécutives. Ces points ont été choisis chaque fois par l'opérateur dans la même zone standard, mais ce ne sont pas toujours les mêmes.

Au cours du premier essai il a été procédé à la restitution de 6 points de contrôle (respectivement deux dans le premier modèle, deux dans le cinquième et deux dans le neuvième).

Pendant le deuxième et le quatrième essai les points de contrôle restitués ont été au total 10 (deux pour chacun des cinq modèles dûment espacés le long de la bande).

Au cours des trois essais ci-dessus, les points restitués étaient signalisés sur le terrain et par conséquent parfaitement identifiés. Les points de contrôle appartenant à un même modèle sont situés sur les bords opposés de la bande.

Pendant le troisième essai les points restitués (approximativement situés au centre de la bande) ont été au total 6. Il s'agit de points naturels du terrain.

Le calcul des bandes a été effectué en utilisant un programme de calcul en langage Fortran. Il se compose de deux phases:

- a) transformation des coordonnées instrumentales en coordonnées-plaque;
- b) enchaînement analytique des bandes en utilisant 6 équations aux parallaxes transversales sur 6 points standard du modèle et une équation aux parallaxes longitudinales p_x sur le point nadiral de chaque photogramme.

L'orientation de la bande a été établie en imposant, dans le premier photogramme, des valeurs nulles pour les éléments d'orientation ($\varphi = \omega = \chi = 0$), et l'échelle en imposant la valeur de la hauteur du point nadiral du premier modèle.

L'orientation du nouveau photogramme sur le précédent est obtenue par un procédé itératif, en utilisant toujours la même matrice.

L'output final du calcul comprend:

- a) les indications générales sur la bande
- b) le numéro du modèle
- c) la suite des réitérations successives et, pour chaque réitération, les parallaxes résiduelles de départ (en mètres sur le terrain) et les éléments d'orientation de la plaque 2 du modèle
- d) les parallaxes résiduelles finales (en mètres sur le terrain) et le nombre de réitérations
- e) les éléments d'orientation terminaux
- f) les coordonnées-bande: — des six points d'orientation
— des points connus
— du point de prise de vue
- g) les cassures en X, Y, Z sur les trois points communs à deux modèles consécutifs (en mètres sur le terrain).

Le tableau 3 reproduit l'output du programme d'enchaînement.

5. Analyse des résultats des essais

Premier essai

La première analyse, exécutée sur le matériel des 16 répétitions de la bande 2.6.1, a permis la recherche d'un comportement systématique éventuel des parallaxes résiduelles des modèles.

Cette première analyse a révélé, dans l'enchaînement de la deuxième répétition de la bande, une anomalie de comportement de l'avant-dernier modèle, anomalie qui rend l'enchaînement inutilisable. Par conséquent, la deuxième répétition a été éliminée. Les répétitions considérées seront donc au nombre de 15.

Le résultat de cette analyse est contenu dans les tableaux 4a) et 4b).

Le tableau 4a) contient les moyennes des parallaxes résiduelles sur chacun des 6 points standard. Les parallaxes sont données en millimètres sur le terrain. Le tableau 4b) rassemble les é. q. m. des moyennes. Compte tenu que la valeur du test t de Student pour laquelle, dans l'hypothèse nulle, les valeurs sont significatives, atteint approximativement 2, nous voyons que la plupart des moyennes sont significatives. Deux faits valent la peine d'être soulignés:

- a) le comportement des moyennes des parallaxes résiduelles dans les 9 modèles ne présente aucun caractère systématique; dans les colonnes les signes positifs et les signes négatifs alternent d'une façon qui semble tout à fait casuelle;
- b) les moyennes les plus élevées en valeur absolue sont toujours concentrées sur les points nadiraux des photogrammes composant le modèle, c'est à dire sur les points 2 et 5. Sur ces deux points les moyennes des valeurs absolues des parallaxes atteignent le double des valeurs sur les quatre autres points.

L'analyse suivante concerne les cassures entre modèles consécutifs de la bande. Le tableau n. 5 rassemble les moyennes pour chacun des points situés sur la ligne nadirale, en centimètres sur le terrain. Les cassures des coordonnées X et Z sur le point nadirale N de photogramme commun à deux modèles sont toujours nulles du fait du calcul.

L'analyse de ce tableau donne lieu aux constatations suivantes:

- a) les cassures dans la coordonnée X sont très petites et, de ce fait, peu significatives; on ne remarque en outre aucun comportement systématique le long de la bande;
- b) les moyennes des cassures dans les deux autres coordonnées sont plus élevées. Les cassures dans la coordonnée Y révèlent une déformation systématique des modèles le long de la bande: l'échelle d'un demi-modèle est inférieure à l'échelle de l'autre. Cette déformation est caractéristique des erreurs d'orientation en ω ;
- c) en ce qui concerne la coordonnée Z , les moyennes des cassures décèlent une rotation systématique de tous les modèles autour d'un axe longitudinal. Cette déformation découle, elle aussi, d'une erreur systématique commise dans l'orientation ω des modèles, avec le résultat d'une torsion de la bande;
- d) pour ce qui est du comportement des coordonnées Y et Z , l'unité de comportement des modèles successifs le long de la bande est interrompue par le comportement anormal des modèles 7-8; le comportement du point b dans la zone de recouvrement

de ces deux modèles diffère du comportement de ce même point sur toutes les autres lignes de recouvrement. La valeur moyenne des cassures en Y et Z est négative au lieu d'être positive (remarquons que toutes ces moyennes sont statistiquement significatives). Ce fait est intéressant, car il décèle une variation imprévue de comportement du matériel photographique.

Le tableau 6a) nous donne les moyennes et les é. q. m. calculés sur les valeurs brutes des éléments d'orientation de chaque photogramme (O). Les é. q. m. augmentent le long de la bande du fait de l'accumulation des erreurs d'orientation des photogrammes successifs. La bande 2.6.1 est trop courte pour permettre toute déduction sur la loi d'accumulation des erreurs; il est cependant possible d'obtenir quelques indications qualitatives, savoir:

- a) des trois éléments d'orientation angulaire, φ accuse les valeurs les plus élevées, au début comme à la fin de la bande. Par contre, à l'élément χ correspond la dispersion initiale et finale la moins élevée;
- b) le rapport entre l'é. q. m. du deuxième photogramme (les é. q. m. pour le premier photogramme sont en effet toujours nuls) et celui du dernier atteint près de quatre fois et demie pour φ , un peu plus de trois fois pour ω et approximativement deux fois et demie pour χ ;
- c) la dispersion des trois éléments angulaires augmente considérablement dans les photogrammes initiaux. Cet accroissement devient négligeable dans les photogrammes de la seconde moitié de la bande (fig. 1);
- d) les composantes de la base de chaque modèle (b_x , b_y , b_z) ont des comportements très différents; les é. q. m. relatives à b_y sont toutes comprises dans une zone de 12 cm et n'accusent aucun accroissement systématique le long de la bande. La dispersion de b_x augmente le long de la bande dans la mesure totale de presque deux fois et demie, mais son comportement est très anormal dans les quatre premiers modèles. Enfin la dispersion de b_z augmente jusqu'à un maximum de 6 fois sa valeur initiale (au modèle 6) pour diminuer ensuite sensiblement (fig. 2).

Le tableau 6b) groupe les valeurs moyennes et les é. q. m. calculés sur les valeurs des éléments d'orientation relative de chaque modèle et des éléments de connexion entre modèles consécutifs (O'). Analysons ces données (fig. 3) pour rechercher des corrélations éventuelles entre la valeur de l'élément d'orientation considérée et son é. q. m.; il est assez évident qu'aucune corrélation n'existe.

En ce qui concerne les données du tableau nous remarquons:

- a) les é. q. m. relatifs à $\Delta\varphi$ sont en moyenne plus élevés que ceux qui se rapportent à $\Delta\omega$ et à $\Delta\chi$ et qu'en outre, le long de la bande, ils accusent la variation la plus considérable: entre le 3^e modèle et le 4^e, le rapport des é. q. m. en $\Delta\varphi$ est à peine inférieur à trois fois;
- b) les é. q. m. relatifs à $\Delta\omega$ sont en général moindres; le rapport entre la valeur la plus élevée et la valeur la plus faible est 2 : 1;
- c) les é. q. m. relatifs à $\Delta\chi$ sont encore plus petits, avec un rapport entre la valeur la plus grande et la valeur la plus petite inférieur à 2 : 1;

- d) les é. q. m. de R (transport d'échelle) sont comprises entre un minimum de près de 4 dixmillièmes et un maximum d'un millième;
- e) les é. q. m. d'orientation ($\Delta\theta$) des modèles sont comprises entre $2,3^\circ$ et $4,5^\circ$ [9]; sont généralement le double, en valeur absolue, des é. q. m. de l'inclinaison θ des modèles présentant la valeur maximale de $2,1^\circ$ et la valeur minimale de $0,9^\circ$.

Le tableau 7 reproduit, pour chaque modèle, les coefficients de corrélation entre tous les couples d'éléments d'orientation O' des modèles.

Les deux dernières lignes de ce tableau contiennent, dans l'ordre: les moyennes calculées sur les cinq premiers modèles, les moyennes calculées sur les cinq derniers, les moyennes calculées sur la totalité des 9 modèles. Le degré de significativité des valeurs moyennes accuse de ce fait une augmentation considérable.

Les valeurs théoriques des coefficients de corrélation entre les couples de valeurs des éléments d'orientation de n'importe quel modèle de la bande sont reproduites par le tableau 8. La comparaison entre ces valeurs et celles fournies par l'étude expérimentale n'est toutefois possible qu'à titre indicatif. En effet, les éléments utilisés pour le calcul expérimental sont des fonctions des éléments d'orientation de chaque modèle et du modèle qui le précède. Ce fait diminue nécessairement l'intensité de la corrélation, comme nous le voyons effectivement en comparant aux valeurs théoriques les moyennes constatées sur tous les modèles. On remarque des valeurs du coefficient significativement différentes de zéro là où cela doit également se produire en théorie, mais les valeurs expérimentales sont dans chaque cas inférieures, en valeur absolue, aux valeurs théoriques.

Dans le cas seulement du premier modèle et comme conséquence du type de calcul choisi, la comparaison entre valeurs expérimentales et valeurs théoriques s'avère possible, même quantitativement. Malgré l'étendue quantitative limitée de l'échantillon, et exception faite des valeurs qui auraient dû être théoriquement nulles, on remarque un excellent accord entre les deux séries de valeurs.

La comparaison entre les deux moyennes de groupes de modèles s'avère par contre significative. Dans trois cas (pour $\Delta\phi\Delta\Phi$, $\Delta\chi R$, $\Delta\Phi R$) les différences entre les deux groupes de modèles dépassent significativement l'intervalle dans lequel peuvent varier, pour des causes purement accidentelles, les valeurs des coefficients de corrélation.

Dans les trois cas, nous constatons pour le deuxième groupe de modèles des moyennes en valeur absolue supérieures à celles du premier groupe de modèles. Malgré l'ignorance des causes, cette constatation doit être soulignée et s'avère digne d'attention.

Deuxième essai

A l'instar du matériel photographique du premier essai, celui du deuxième est constitué par une bande appartenant à un bloc utilisé pour les recherches sur la triangulation aérienne. La bande 14/3 du bloc d'Oberschwaben fait en effet partie des recherches effectuées dans le cadre de la Commission A/B pour le programme 2A. Ce choix est motivé: associer les résultats de deux recherches approfondies effectuées sur un même matériel aboutit inévitablement à une amélioration des connaissances sur le comportement des erreurs.

L'analyse des résultats des seize répétitions de l'enchaînement a suivi à peu près complètement le schéma de la recherche précédente; elle devra être complétée en ce qui regarde aussi bien l'élaboration des données que l'analyse.

Considérons le premier des tableaux de rapportant à cet essai (tableau n. 9a et 9b):

- les moyennes des parallaxes résiduelles rarement dépassent la valeur d'un décimètre; elles sont inférieures — compte tenu de l'échelle des photogrammes — à celles des enchaînements du premier essai;
- le comportement est parfaitement systématique: les parallaxes moyennes sont toutes positives aux points pairs et toutes négatives aux points impairs, à la seule exception du modèle n. 11, où les signes sont invertis. Si nous tenons compte qu'à chaque répétition l'opérateur effectuait la collimation sur points différents dans la même zone, la présence de ce comportement systématique, non constaté au cours du premier essai, laisse supposer la présence d'une distorsion résiduelle de l'objectif;
- d'accord avec ce que nous avons déjà vu lors du premier essai, les parallaxes maximales sont situées aux points 2 et 5, autrement dit aux points nadiraux. Nous constatons également sur ces points le maximum de dispersion des valeurs. Cette coïncidence fait naître le supçon justifié que la cause de ce phénomène réside dans le schéma de calcul suivi pour l'orientation relative.

Le tableau n. 10 groupe les σ_0 des 24×16 modèles. Les valeurs moyennes par colonne, c'est-à-dire relatives à chacune des 16 répétitions de l'enchaînement, sont toutes très proches l'une de l'autre (valeur maximale 219 mm, valeur minimale 161 mm correspondant à peu près à une erreur de ± 6 microns sur la plaque). Cela signifie que, sous le rapport des mesures, les 16 répétitions de l'enchaînement ont toutes la même précision. Les moyennes par rangée présentent au contraire des variations très considérables (valeur maximale 317 mm, valeur minimale 98 mm*); cela prouve qu'il existe, dans la précision intrinsèque une disparité considérable de comportement entre les modèles d'une même bande.

L'analyse qualitative du tableau n. 11a contenant les moyennes (presque toutes statistiquement significatives: tab. 11b) des cassures entre modèles consécutifs nous permet d'établir ce qui suit:

- les cassures dans les coordonnées X sont entièrement négligeables. En général les moyennes sont de quelques centimètres;
- dans la coordonnée Y le comportement des cassures sur les deux points marginaux de la ligne nadirale n'est pas caractérisé par une allure systématique marquée. Les valeurs des moyennes sur le point a) sont variables tout en présentant une prédominance nette de valeurs positives. Par contre, sur le point b), les valeurs négatives ne prédominent que légèrement. On ne saurait rien relever de systématique en pareil cas. Pour la coordonnée Z les cassures sont principalement négatives sur le point a), en corrélation parfaite avec les valeurs positives, sur le même point, des cassures en Y ; nous avons par

*) c'est à dire respectivement 12 et 3 microns sur la plaque.

contre, sur le point b), une corrélation opposée, des cassures de même signe en Y et Z; lorsque sur les points a et b les cassures en Y sont de même signe, les cassures en Z sur ces mêmes points sont de signe contraire, et inversement. Les moyennes des cassures en Z sont toutefois d'importance généralement supérieure: elles dépassent même un mètre dans plusieurs modèles.

Nous constatons donc, dans la formation des modèles, types différents de déformations systématiques.

Les é. q. m. des éléments d'orientation O des photogrammes résultent du tableau n. 12a):

- a) l'accumulation des erreurs le long de la bande est évidente et très marquée, surtout pour les erreurs en φ et bz . Sur les 25 photogrammes ces erreurs augmentent de 6 à 7 fois. Pour ω et χ , l'augmentation est plus modique (3,6 fois environ);
- b) les composantes de la base b_x et b_y sont caractérisées par des erreurs qui augmentent beaucoup plus lentement (de 1,5 à 2 fois sur toute la bande). Un comportement analogue avait été remarqué lors du premier essai, compte tenu de la longueur moindre de la bande. En particulier b_y n'augmente que dans les quatre derniers modèles (fig. 4, 5).

Les é. q. m. des éléments angulaires d'orientation et d'enchaînement des modèles O' sont groupés dans le tableau n. 12b) (fig. 6):

- a) le comportement relatif entre les éléments $\Delta\varphi$, $\Delta\omega$, $\Delta\chi$ est comparable à celui que nous constatons dans le premier essai: les erreurs de $\Delta\varphi$ sont sensiblement supérieures à celles des deux autres éléments. Pour $\Delta\varphi$ et $\Delta\omega$ le rapport entre la valeur la plus élevée et la valeur la plus faible des é. q. m. est d'environ deux fois et demie; pour $\Delta\chi$ il n'est que deux fois;
- b) les é. q. m. de ces trois éléments angulaires d'orientation se rapportant à l'essai en question sont inférieurs à ceux que donne le premier essai; en particulier, pour $\Delta\omega$, la valeur moyenne est d'environ la moitié. Cette constatation doit être mise en corrélation avec le fait que la moyenne des éléments d'orientation angulaire des modèles est elle aussi très réduite. La valeur moyenne de $\Delta\omega$ ne dépasse jamais le degré centésimal; celle de $\Delta\varphi$ ne le dépasse que dans un modèle: celui pour lequel l'é. q. m. atteint son maximum. Cette constatation semble être très importante. La cause d'un pareil comportement peut résider dans le type de programme de calcul utilisé pour la formation du modèle;
- c) le comportement des trois éléments de l'enchaînement entre les modèles est très semblable à celui du premier essai, tout en présentant des moyennes de valeurs plus faibles sur tous les modèles. Le rapport entre la valeur la plus élevée et la valeur la plus faible se chiffre par 1 : 4 pour la composante $\Delta\Phi$. Notons que pour ces trois éléments les valeurs maximales sont toutes concentrées sur le dernier modèle.

Le dernier tableau (n. 13) relatif à cet essai est celui des coefficients de corrélation. Considérons uniquement la moyenne des groupes des cinq modèles consécutifs. Il n'y a rien à dire à leur sujet. On ne note variations justifiées de valeur entre différents groupes.

Troisième essai

Les mesures exécutées sur le matériel photographique de cet essai n'ont pu être utilisées, en l'absence de convergence du calcul de l'enchaînement, probablement par suite des variations considérables d'altitude du terrain, ainsi que des valeurs assez élevées des éléments d'orientation. Cette bande a été en effet choisie en considération du terrain très accidenté. On espère pouvoir parvenir également à utiliser ces mesures, en modifiant opportunément le programme de calcul.

Quatrième essai

Le choix du matériel photographique pour cet essai a été suggéré par le désir de comparer le comportement de deux bandes d'un même bloc. L'analyse des résultats, sur les huit répétitions de cette bande, aura donc comme but principal la comparaison entre les deux bandes.

Cependant, il faut rappeler que, se composant l'échantillon relatif au quatrième essai de huit répétitions seulement, pas de seize, les données que l'on obtient sont beaucoup moins significatives. Le centre de Milano compte poursuivre les mesures jusqu'à atteindre le total de seize, comme pour les autres essais.

Parallaxes résiduelles (tableaux 14a, 14b, 15):

- a) les valeurs moyennes sont généralement égales;
- b) le comportement systématique sur de points pairs et impairs, bien que évident, n'est pas si marqué que dans le deuxième essai;
- c) les moyennes par colonne du σ_0 sont presque du même ordre de grandeur et les moyennes par ligne ont des variations considérables (entre 105 et 463 mm).

Cassures sur les points communs (tableaux 16a, 16b):

- a) les valeurs des moyennes dans cet essai sont un peu plus petites surtout sur le point a) pour les coordonnées Y et Z;
- b) le comportement corrélié entre Y et Z est exactement le même.

Éléments d'orientation des photogrammes (tableau 17a):

- a) il y a une remarquable différence soit comme valeurs soit comme comportement; les valeurs sont en général plus élevées;
- b) l'é. q. m. de b_y ne subit aucune variation le long de la bande;
- c) l'é. q. m. de b_x semble même diminuer.

Éléments d'orientation et d'enchaînement des modèles (tableau 17b):

- a) aussi pour ces éléments les valeurs des é. q. m. sont plus élevées (ordre de grandeur de l'accroissement de 30 à 50 %);
- b) étant l'échantillon peu nombreux et, par conséquent, les é. q. m. peu significatifs, on ne peut pas tirer des conclusions moins générales.

Impossible, évidemment, déduire des informations des coefficients de corrélation linéaire.

Conclusions

Il n'y a pas lieu de s'étonner que les conclusions de cette étude sur un travail, entre autres, incomplet, se réduisent à bien peu de chose. Nous avons donné, dans tout ce qui précède, une énumération de cas où l'on avait pu noter des déviations significatives vis-à-vis du comportement uniforme et signalé telle ou telle valeur pour mettre en évidence un comportement qui se distinguait des autres.

Si cet aperçu d'ensemble a suffi à mettre en lumière que toutes les hypothèses théoriques sur l'homogénéité de comportement des éléments constituant une bande sont partiellement inexactes sur le plan expérimental et à convaincre de la nécessité d'approfondir cette question et de mieux souligner son aspect quantitatif, ainsi que les causes du phénomène à l'étude, nous nous estimons satisfaits.

Le temps restreint, qui a considérablement limité les recherches, les difficultés intrinsèques du problème et l'exiguité de l'échantillon statistique examiné, ont empêché à coup sûr d'apporter des hypothèses toutes formulées sur les causes des anomalies de comportement. Une recherche de ce type exigerait un effort d'imagination que nous ne sommes pas en mesure d'accomplir.

On estime cependant que l'effort d'imagination devient toujours moindre au fur et à mesure que les données s'accumulent et que l'hétérogénéité des conditions d'opérations s'accroît.

Certains doutes, par exemple, pourraient être éliminés si les mêmes données de mesure étaient traitées selon d'autres programmes de calcul.

Tout en souhaitant que quelqu'un parvienne à découvrir la cause de ces déviations, même à partir de données aussi peu nombreuses, on exprime le vœu que d'autres Centres s'unissent à celui de Milano pour poursuivre les recherches. Le Centre milanais, qui s'engage pour sa part à compléter avant tout l'élaboration et l'étude de ce premier bloc de matériel, se tient à la disposition de tous les intéressés pour fournir des données sur ses propres mesures aux fins de toute élaboration différente possible.

ABSTRACT:

The aim and justification of the research can be summed up thus to document as widely as possible the behaviour of bridging errors, their variability inside the strip, their experimentally determined entity, their eventual correlation to ambiental conditions of exposure and plotting. Our present knowledge of the behaviour of the numerous phenomena connected with the operation of photograph orientation is in fact elementary and rough. Experimental analysis therefore aims at supplying the future perceptions with more detailed information from which to start theoretical developments and the construction of mathematical models of photogrammetry problems.

The operative methodology thus consists in homogeneously repeating the measurements and calculations for the bridging of a strip, for a sufficient number of times to enable valid deductions to be made from analysis of the results. An exploratory investigation

conducted with statistical means established that at least fifteen repetitions were necessary to give significant results, which consist of the statistical indices: average, root-mean-square deviation, coefficients of correlation, calculated on the following numerical bridging results: residual parallaxes on the orientation points of the models, the discrepancies between the coordinates of points common to consecutive models along the strip, the orientation elements of the photographs and, derived from these, the orientation and bridging elements of the models, the coordinates of the plotted points, the coefficients of the error interpolator polynomials.

The analyses are intended to single out the entity of both the accidental and the systematic errors that may be present in the photographic material.

The operative criteria described at the beginning of this report were applied by the Milan centre on four tests, each of which was composed of a series of repetitions of the same strip. Only the first two tests, each composed of 16 repetitions of two distinct strips, were able to be almost completely worked out and the results examined.

These were analytical bridgings of two strips belonging respectively to the experimental Test-fields of Reichenbach and Oberschwaben. It has not yet been possible to calculate the third test, while the fourth is made up of only eight repetitions of another strip from the Oberschwaben Test-field.

Analysis, for the time being only partial, of the two completed tests has brought to light how the behaviour of the different models inside the strip is quite different both with regard to the parallax and discrepancies between models, and with regard to the orientation elements of the photographs. The hypothesis of the intrinsic homogeneity of the strip, on which all theoretical constructions are based, was experimentally verified as being not always acceptable. Furthermore the presence of systematic errors of formation, orientation and bridging of the models was brought to light. However statistical research cannot be over after such a small number of tests; if one wishes to attain significant results, above all in the search for the causes of behavioural anomalies discovered, it is necessary to increase the number of tests while varying the exposure conditions and the variety of the lands photographed in every way possible.

One goal has however been reached by this research: that of having proved that some hypothesis on the internal homogeneity of the strips have no experimental foundation. The Milan centre will continue along this line both by completing analysis of the data gathered in this first phase of work and by extending research to other strips; it is however indispensable that other centres, with other systems of operating and calculating, join in the research in order to extend its significance and make it more meaningful from the statistical point of view.

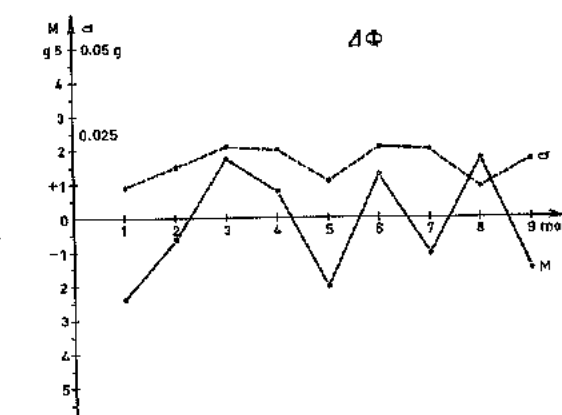
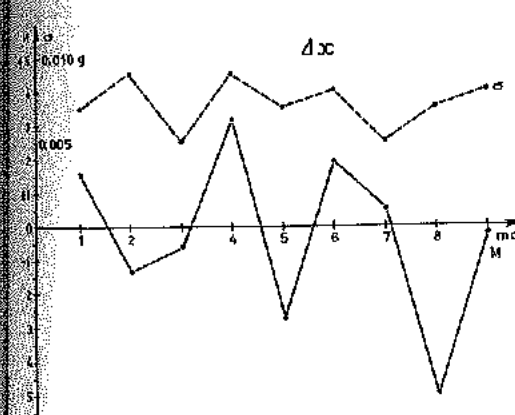
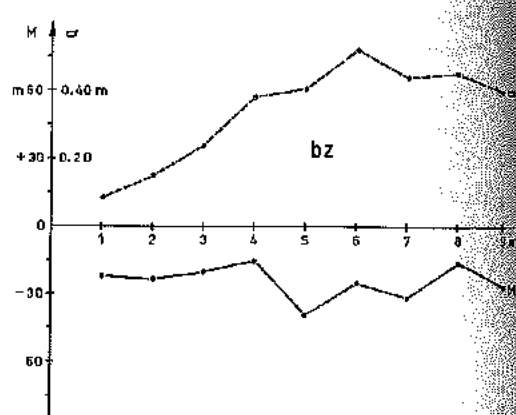
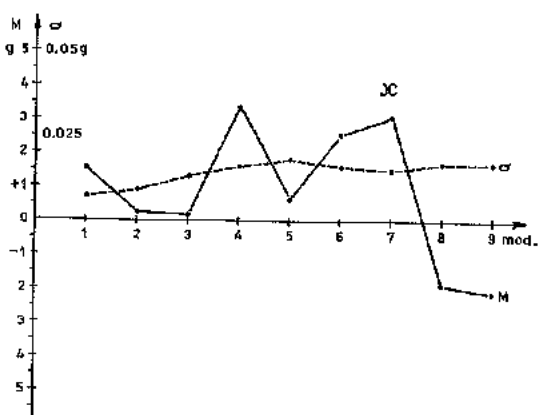
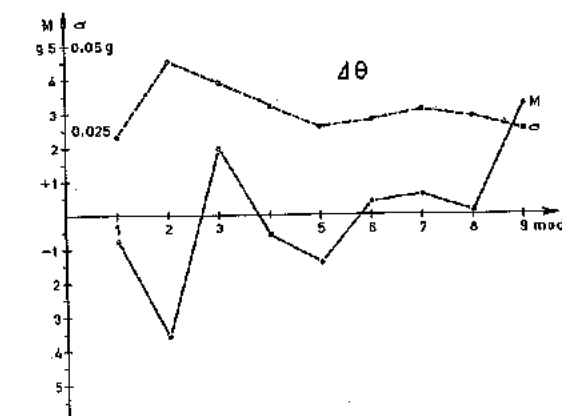
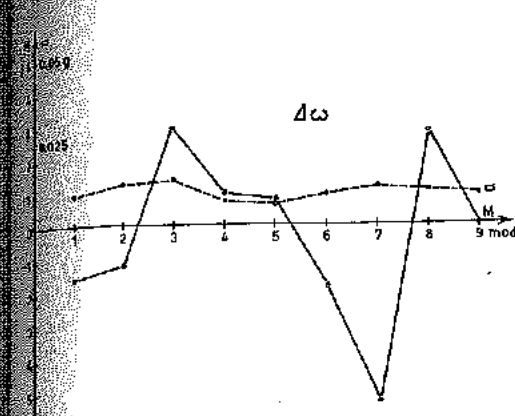
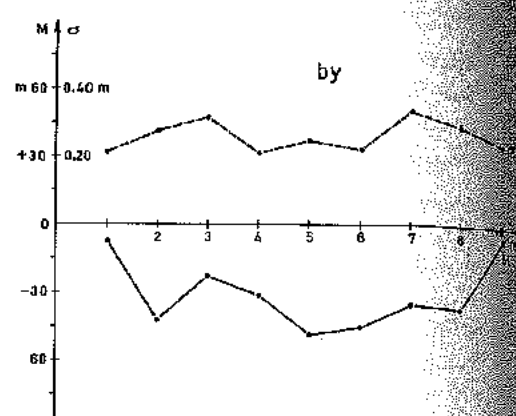
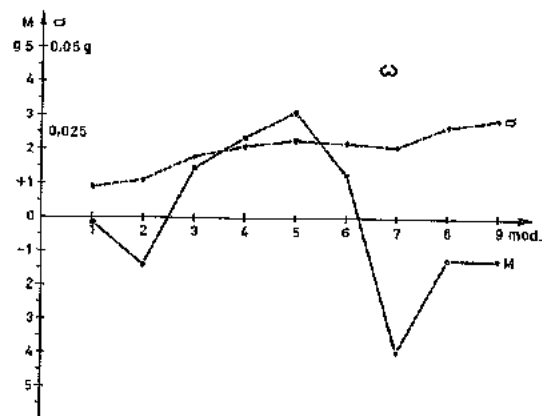
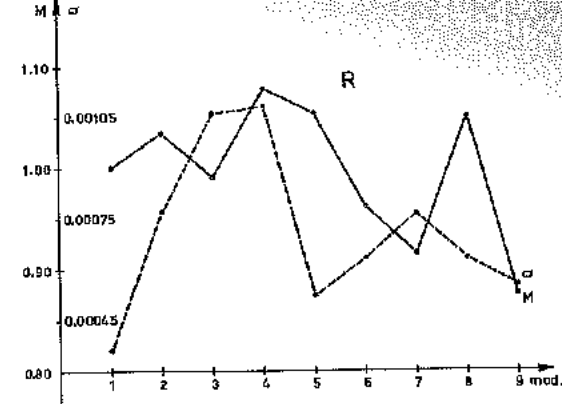
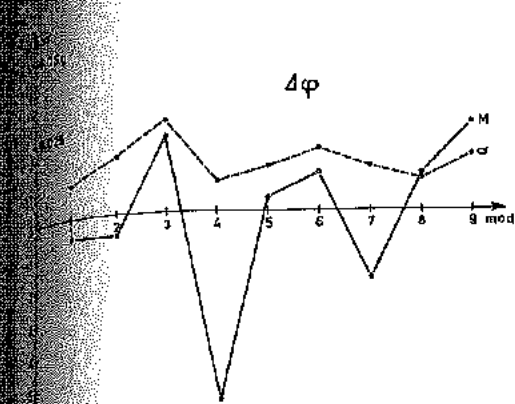
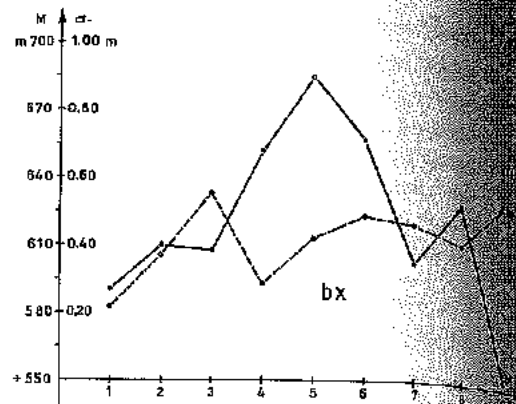
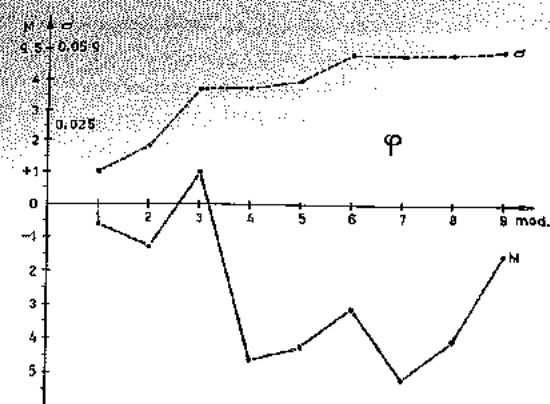


Figure 1

Figure 2

Figure 3

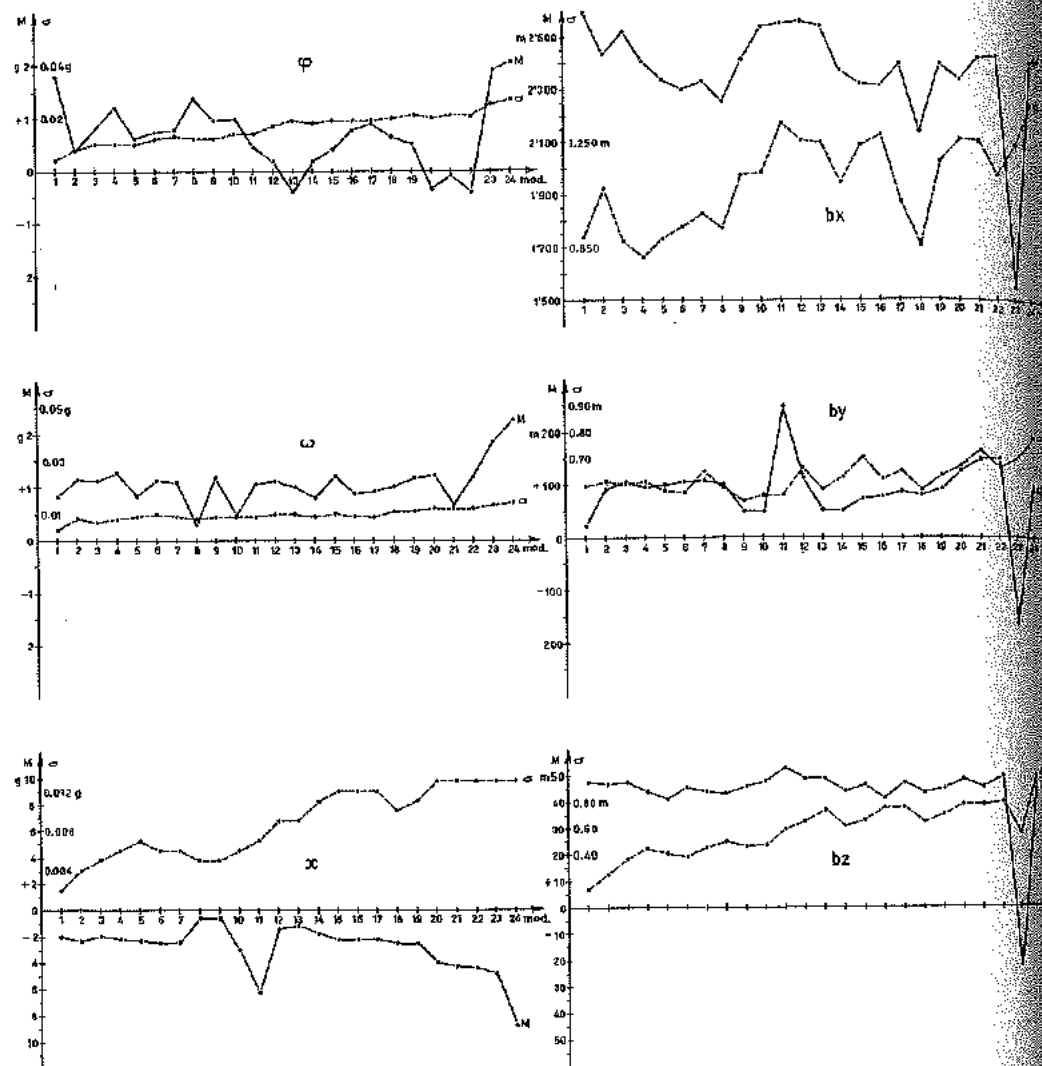


Figure 4

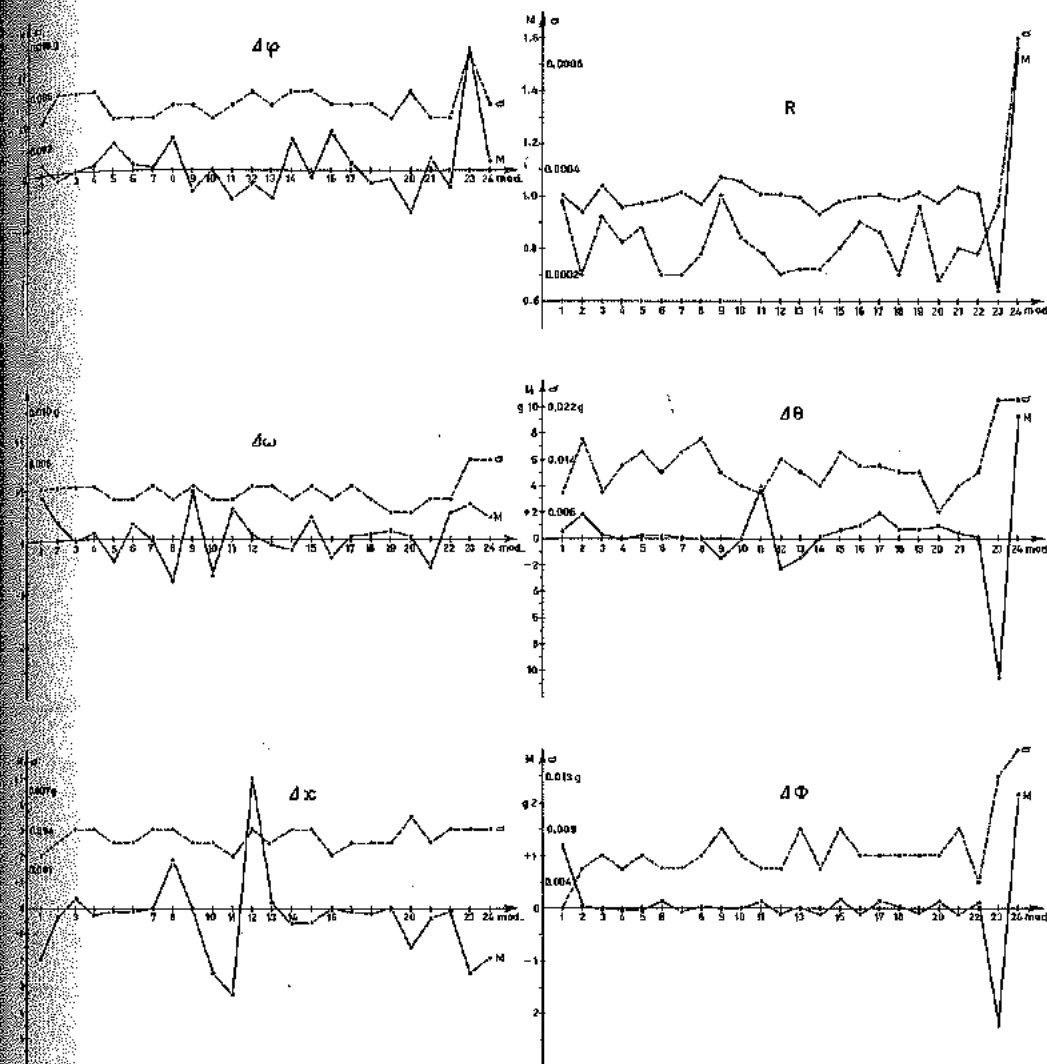


Figure 5

Figure 6

Tableau 1

Nombre répétitions	limites	intervalle	amplitude intervalle
15	- 0,52	+ 0,52	1,04
20	- 0,45	+ 0,45	0,90
25	- 0,40	+ 0,40	0,80
30	- 0,375	+ 0,375	0,75
50	- 0,275	+ 0,275	0,55
100	- 0,20	+ 0,20	0,40

Tableau 2

r_y théorique	limites zone non critique		intervalle
0	- 0,52	+ 0,52	1,04
- 0,17	- 0,63	+ 0,37	1,00
- 0,58	- 0,85	- 0,12	0,73
+ 0,71	+ 0,34	+ 0,90	0,56
- 0,97	- 0,99	- 0,90	0,09 très incertain

Tableau 3

POLIGONO REICHENBACH PROVA N° 10							
STRISCIA N° 1 MODALITA				6-7			
MODELLO N° 5							
PARALLASSI RESIDUE INTERMEDIE (METRI SUL TERRENO)							
ITERAZIONE N° 1							
-48.09348	-10.41257	-24.79141	-5.84461	-5.81902	-60.19137	-76.34898	
-3.71675	2.73776	0.57327	680.83414	-46.43727	-31.23436		
ITERAZIONE N° 2							
-7.14964	-4.23970	-7.13748	-3.01091	-4.34989	-11.59710	-7.01220	
-4.12472	3.03895	0.60921	683.68614	-48.02463	-36.90656		
ITERAZIONE N° 3							
-1.67196	-0.97995	-1.52332	-0.88306	-1.07845	-2.62051	-1.64532	
-4.21412	3.10593	0.60707	684.31215	-48.24941	-38.05857		
ITERAZIONE N° 4							
-0.39619	-0.23672	-0.32508	-0.21465	-0.23581	-0.58682	-0.38605	
-4.23551	3.12110	0.60669	684.48543	-48.29827	-38.30098		
ITERAZIONE N° 5							
-0.08472	-0.06413	-0.07040	-0.05023	-0.04326	-0.13764	-0.08839	
-4.24041	3.12454	0.60675	684.52428	-48.31077	-38.35757		
ITERAZIONE N° 6							
-0.01459	-0.02495	-0.01180	-0.01556	-0.00102	-0.03628	-0.02039	
-4.24152	3.12534	0.60676	684.53271	-48.31365	-38.37084		
ITERAZIONE N° 7							
0.00137	-0.01592	0.00200	-0.00766	0.00874	-0.01272	-0.00474	
-4.24178	3.12553	0.60677	684.53468	-48.31430	-38.37388		
ITERAZIONE N° 8							
0.00509	-0.01382	0.00517	-0.00581	0.01102	-0.00726	-0.00111	
-4.24184	3.12557	0.60677	684.53506	-48.31444	-38.37457		
PARALLASSI RESIDUE FINALI (METRI SUL TERRENO)				ITERAZIONE N° 9			
0.00593	-0.01334	0.00594	-0.00535	0.01155	-0.00599	-0.00025	
ELEMENTI DI ORIENTAMENTO RELATIVO (ANGOLI IN GR. CENT-COMP. BASE METRI SUL TERRENO)							
PI= -4.24184	OM= 3.12557	CA= 0.60677	BC= 684.535	BY= -48.314	BZ= -38.375		
COORDINATE GEODETICHE DI STRISCIA (METRI SUL TERRENO)							
NUMERO	U	V	Q				
106010000	33497.400	9828.667	466.413				
106020000	33501.840	9116.293	583.255				
106030000	33367.261	8408.228	479.387				
107010000	34252.778	9734.070	424.298				
107020000	34157.525	8989.062	584.762				
107030000	34098.634	8300.260	430.319				
106048715	33907.749	8438.852	467.886				
106058745	34064.924	9614.780	423.462				
PO. PRESEA	34216.919	9010.922	1628.370				
NOTIZIE							
NUMERO	DU	DV	DQ				
10601	-0.023	0.506	-0.803				
10602	0.000	0.018	0.000				
10603	0.050	0.185	0.351				

Tableau 4

Mod.	a)						b) en mm sur le terrain					
	1	2	3	4	5	6	1	2	3	4	5	6
1	-18,5	+36,4	-18,3	+21,5	-38,3	+17,2	± 4,2	± 8,0	± 4,1	± 4,7	± 8,2	± 3,8
2	-20,9	+44,2	-15,6	+24,3	-45,0	+16,9	9,8	18,9	6,8	8,8	18,5	7,5
3	+13,7	-20,0	+12,7	-17,2	+29,9	-19,1	4,2	6,2	3,9	5,3	9,2	5,8
4	-24,0	+45,9	-23,5	+13,7	-25,4	+8,7	6,2	12,1	6,1	3,6	6,5	2,5
5	-13,5	+26,2	-19,3	+11,2	-24,9	+10,9	4,7	9,8	4,7	3,5	8,8	4,3
6	+13,5	-28,3	+15,5	-33,1	+53,0	-20,8	3,9	7,5	4,8	9,7	14,1	5,6
7	+19,0	-24,5	+9,5	-11,7	+21,3	-7,3	6,7	9,1	3,0	4,5	7,7	2,9
8	-23,1	+38,7	-16,9	+24,1	-35,5	+15,7	5,3	9,1	4,1	5,5	8,2	3,6
9	+11,3	-17,1	+7,3	-3,8	+9,7	-3,7	6,1	10,3	4,2	2,9	6,0	2,4
M	159,5	281,3	132,6	160,6	283,0	120,3						

Tableau 5

en cm sur le terrain											
Coord.	Points	1.2	2.3	3.4	4.5	5.6	6.7	7.8	8.9	M	
x	a	- 0,5	+ 1,3	- 0,8	- 1,0	+ 0,6	- 1,0	- 1,0	- 1,9	- 0,8	
	b	+ 0,4	- 2,7	+ 2,1	+ 4,7	+ 4,4	+ 1,0	- 6,0	+ 6,1	+ 1,2	
y	a	+12,0	+13,2	+14,7	+11,9	+23,3	+11,4	+11,2	+10,8	+13,6	
	N	0,0	+ 3,2	- 3,7	- 0,4	+ 2,8	- 1,4	- 2,7	+ 6,0	+ 0,5	
z	b	+ 5,0	+29,4	+16,3	+20,1	+23,7	+ 4,8	-28,2	+35,4	+13,3	
	a	-22,8	-28,0	-21,1	-19,1	-39,9	-23,1	-20,8	-28,2	-25,4	
	b	+ 8,4	+42,5	+27,0	-36,3	-43,4	-16,9	-46,6	+56,6	+23,1	

a)

Éléments	1	2	3	4	5	6	7	8	9
φ^g	M - 0,613 0,010	- 1,260 0,018	+ 0,984 0,036	- 4,593 0,036	- 4,212 0,038	- 3,109 0,046	- 5,129 0,045	- 4,043 0,045	- 1,499 0,046
ω^g	M - 0,160 0,009	- 1,370 0,011	+ 1,471 0,018	+ 2,385 0,021	+ 3,121 0,023	+ 1,289 0,022	- 3,947 0,021	- 1,220 0,027	- 1,230 0,029
κ^g	M + 1,543 0,007	+ 0,242 0,009	+ 0,184 0,013	+ 3,348 0,016	+ 0,630 0,018	+ 2,565 0,016	+ 3,085 0,015	- 1,863 0,017	- 2,100 0,017
$b x_m$	M 589,615 0,214	609,353 0,368	603,316 0,552	650,173 0,286	684,761 0,418	658,277 0,486	602,571 0,467	633,056 0,414	553,592 0,547
$b y_m$	M - 7,400 0,212	-42,071 0,275	-23,152 0,316	-31,159 0,210	-48,244 0,245	-45,970 0,220	-36,432 0,337	-37,320 0,292	- 4,242 0,245
$b z_m$	M -22,234 0,085	-23,590 0,150	-21,682 0,238	-15,315 0,382	-38,011 0,403	-23,614 0,519	-31,726 0,438	-15,779 0,453	-26,902 0,398

b)

$\Delta \varphi^g$	M - 0,613 0,010	- 0,647 0,017	+ 2,245 0,027	- 5,578 0,009	+ 0,380 0,013	+ 1,102 0,018	- 2,019 0,013	+ 1,085 0,009	+ 2,553 0,016
$\Delta \omega^g$	M - 0,160 0,009	- 1,210 0,012	+ 2,842 0,013	0,913 0,007	0,736 0,006	- 1,832 0,009	- 5,236 0,011	+ 2,726 0,010	- 0,010 0,009
$\Delta \kappa^g$	M + 1,543 0,007	- 1,301 0,009	- 0,058 0,005	+ 3,164 0,009	- 2,717 0,007	+ 1,935 0,008	+ 0,519 0,005	- 4,948 0,007	- 0,237 0,008
R	M 1,00000 0,00036	1,03347 0,00077	0,99009 0,00106	1,07766 0,00108	1,05319 0,00052	0,96132 0,00063	0,91537 0,00076	1,05059 0,00063	0,87447 0,00055
$\Delta \theta^g$	M - 0,798 0,023	- 3,589 0,045	+ 1,946 0,039	- 0,606 0,032	- 1,429 0,026	+ 0,039 0,028	+ 0,594 0,031	+ 0,095 0,029	+ 3,260 0,025
$\Delta \Phi^g$	M - 2,399 0,009	- 0,063 0,015	+ 0,176 0,021	+ 0,787 0,020	- 2,030 0,011	+ 1,247 0,021	- 1,066 0,020	+ 1,762 0,009	- 1,504 0,017

Tableau 7

Node	$\Delta\varphi\Delta\theta$	$\Delta\varphi\Delta\phi$	$\Delta\varphi R$	$\Delta\varphi\Delta x$	$\Delta\varphi\Delta y$	$\Delta\omega\Delta\theta$	$\Delta\omega R$	$\Delta\omega\Delta x$	$\Delta x\Delta\theta$	$\Delta x R$	$R\Delta\phi$	$\theta\Delta\phi$
1	+0,11	-0,36	-0,91	+0,05	+0,44	+0,37	-0,37	+0,04	-0,20	-0,11	-0,56	-0,23
2	+0,10	-0,33	-0,82	-0,46	+0,09	+0,58	-0,04	+0,18	+0,03	+0,47	+0,12	-0,44
3	-0,51	+0,47	-0,85	-0,05	-0,48	-0,43	+0,41	+0,03	-0,17	-0,03	-0,17	+0,33
4	+0,08	+0,36	-0,38	-0,00	-0,14	-0,35	-0,08	-0,06	-0,38	-0,30	+0,76	-0,19
5	+0,76	+0,30	-0,67	+0,06	+0,31	-0,23	+0,43	+0,47	-0,17	+0,42	-0,36	+0,54
6	+0,74	-0,40	-0,62	+0,35	+0,13	+0,24	-0,05	-0,20	-0,50	+0,34	-0,14	-0,58
7	+0,72	-0,43	-0,62	-0,51	+0,52	+0,34	-0,16	-0,31	+0,12	+0,63	-0,30	-0,86
8	+0,60	-0,08	-0,61	-0,33	-0,11	-0,27	+0,00	+0,13	+0,01	+0,35	-0,76	-0,20
9	+0,82	-0,36	-0,79	-0,03	+0,32	+0,40	-0,54	-0,06	-0,10	+0,28	-0,38	-0,42
M (1-5)	+0,25	+0,09	-0,73	-0,11	-0,08	-0,03	+0,07	-0,13	-0,11	+0,09	-0,04	0,00
M (5-9)	-0,73	-0,19	-0,66	-0,11	+0,11	+0,07	+0,05	-0,01	-0,13	+0,40	-0,43	-0,13
(1-9)	+0,47	-0,09	-0,69	-0,12	+0,05	+0,11	+0,02	-0,03	-0,11	+0,21	-0,20	-0,14

Tableau 8

	φ_{bz}	φ_{by}	φ_{bx}	φ_{θ}	ω_{bz}	ω_{by}	ω_{bx}	ω_{θ}	κ_{bz}	κ_{by}	κ_{bx}	κ_{θ}	trby	trbz
Edwards														
Walewska	+0.707	0	-0.966	0	0	-0.971	0	0	0	-0.168	0	-0.576	0	0

Tableau 9a

en sur le terrain

MOD.	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆
1	- 58,5	+ 120,0	- 54,8	+ 58,3	- 118,8	+ 54,8
2	- 31,8	+ 57,2	- 34,0	+ 35,6	- 58,9	+ 32,0
3	- 28,6	+ 47,5	- 27,5	+ 29,6	- 50,3	+ 27,8
4	- 9,4	+ 18,8	- 9,2	+ 8,2	- 18,0	+ 9,3
5	- 3,0	+ 7,3	- 3,3	+ 3,6	- 8,9	+ 4,2
6	- 32,8	+ 91,3	- 47,5	+ 38,9	- 79,3	+ 29,2
7	- 76,4	+ 136,7	- 69,7	+ 84,3	- 164,5	+ 89,1
8	- 57,2	+ 112,9	- 49,4	+ 54,3	- 101,8	+ 39,3
9	- 85,6	+ 168,9	- 68,2	+ 86,0	- 175,1	+ 73,6
10	- 2,9	+ 4,9	- 2,3	+ 2,8	- 4,6	+ 3,0
11	+ 13,8	- 29,8	+ 12,7	- 13,7	+ 32,3	- 16,1
12	- 54,1	+ 122,0	- 60,7	+ 54,8	- 121,6	+ 58,8
13	- 9,5	+ 18,0	- 9,9	+ 9,0	- 17,4	+ 9,3
14	- 10,6	+ 19,5	- 10,5	+ 10,5	- 20,1	+ 11,1
15	- 26,0	+ 66,0	- 29,1	+ 31,8	- 67,9	+ 24,9
16	- 31,8	+ 54,2	- 28,1	+ 32,4	- 56,3	+ 29,4
17	- 12,3	+ 20,0	- 11,1	+ 11,8	- 19,2	+ 11,2
18	- 23,6	+ 54,3	- 23,8	+ 25,0	- 56,3	+ 22,9
19	- 30,7	+ 62,0	- 30,3	+ 29,6	- 62,6	+ 32,4
20	- 11,7	+ 24,1	- 12,5	+ 13,8	- 23,8	+ 9,9
21	- 26,5	+ 55,1	- 23,5	+ 28,2	- 60,8	+ 26,2
22	- 10,7	+ 16,5	- 9,1	+ 11,7	- 18,8	+ 10,3
23	- 86,1	+ 166,6	- 99,3	+ 65,2	- 161,1	+ 107,7
24	- 38,6	+ 121,8	- 63,8	+ 59,5	- 128,8	+ 52,7
M	32,2	66,5	32,9	33,3	67,8	32,7

Tableau 9b

en mm sur le terrain

MOD.	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆
1	± 10,4	± 21,3	± 9,8	± 10,4	± 21,2	± 9,9
2	11,5	20,7	12,2	12,5	21,2	11,7
3	12,9	21,4	12,3	13,6	22,6	12,6
4	7,2	14,4	6,8	6,1	13,7	7,2
5	7,8	21,3	9,5	8,8	26,4	13,5
6	4,5	13,0	6,7	5,4	11,3	4,1
7	7,7	13,5	6,9	8,5	16,3	8,8
8	10,9	21,6	9,4	10,5	19,4	7,5
9	9,9	19,3	7,8	9,9	20,1	8,5
10	10,2	16,7	8,7	9,8	15,9	8,4
11	9,3	20,3	9,0	9,5	22,0	10,6
12	9,3	21,0	10,4	9,4	20,8	10,0
13	10,9	21,0	11,6	10,9	20,2	10,8
14	9,7	18,8	9,6	9,5	19,3	10,3
15	9,6	25,1	10,8	11,7	25,7	9,3
16	9,7	16,6	8,6	10,0	17,3	9,0
17	10,3	17,6	9,7	9,7	16,9	9,7
18	6,6	15,4	6,7	7,1	15,9	6,5
19	7,0	14,3	7,0	6,8	14,4	7,5
20	10,0	21,3	10,7	11,4	21,1	9,0
21	9,2	19,1	8,1	9,7	20,9	9,1
22	8,1	13,1	6,9	8,8	14,7	7,9
23	8,8	15,6	9,6	6,1	15,3	10,0
24	4,3	13,2	6,8	6,2	14,0	5,9

Tableau 10

en mm. sur le terrain

MOD.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	σ ₆
1	± 583	± 193	± 894	± 187	± 16	± 234	± 74	± 200	± 43	± 246	± 174	± 240	± 296	± 276	± 253	± 30	± 247
2	365	95	39	134	137	263	224	51	307	187	176	9	187	141	96	29	181
3	11	301	225	232	80	181	303	95	163	76	252	6	23	143	234	118	180
4	56	67	45	125	104	34	137	153	50	109	93	56	156	150	61	9	98
5	169	165	98	52	130	238	264	121	11	229	30	129	238	97	9	130	153
6	106	80	71	42	226	196	189	89	166	96	341	211	82	210	63	111	162
7	210	238	241	356	237	94	273	223	155	207	128	390	282	434	390	462	286
8	201	348	156	31	149	242	24	192	153	324	120	355	213	258	303	219	227
9	216	329	416	188	298	236	495	451	217	40	204	313	219	311	478	222	317
10	96	50	117	121	60	152	94	174	163	34	187	68	158	46	109	24	115
11	29	210	19	222	20	20	61	295	7	61	304	18	20	43	202	151	147
12	56	159	345	470	45	269	168	79	254	276	33	218	284	388	289	63	248
13	187	18	119	96	9	111	48	61	12	58	192	267	189	1	90	324	145
14	33	31	197	131	225	144	49	27	42	171	77	49	68	317	93	2	133
15	301	30	72	163	35	9	19	246	132	165	507	93	219	180	14	180	195
16	23	43	89	50	18	152	9	2	322	311	163	32	117	278	126	97	154
17	214	16	14	17	177	187	45	101	92	203	67	153	168	156	21	54	127
18	91	221	56	40	217	174	185	73	47	158	30	5	92	120	127	223	135
19	345	89	119	14	97	108	135	212	242	68	93	27	32	166	59	25	143
20	327	98	27	46	100	151	224	272	18	160	143	64	34	70	54	103	147
21	122	220	26	214	99	269	13	224	80	22	30	106	37	142	313	31	162
22	132	70	96	162	124	5	119	82	34	194	33	98	169	46	51	11	138
23	88	315	108	356	321	286	141	372	379	379	283	200	394	314	280	500	314
24	219	187	236	105	267	241	227	349	101	195	300	71	353	68	260	190	218
σ ₆	± 219	181	171	187	161	187	186	206	169	191	206	173	197	214	209	190	

Tableau 11a

en cm sur le terrain

MOD.	x		y			z	
	a	b	a	b	b	a	b
1	- 0,3	+ 0,4	+107,3	+ 3,1	- 6,8	-149,9	- 8,4
2	- 5,2	+ 0,3	+117,5	+ 0,6	- 37,8	-160,7	- 55,1
3	- 0,7	+ 0,5	+ 17,9	+ 1,6	- 16,0	- 26,3	- 22,1
4	+ 0,2	+ 1,6	- 5,1	+ 0,5	- 31,4	+ 6,5	- 45,8
5	- 2,4	- 0,2	+ 84,4	- 4,1	+ 6,0	-118,1	+ 6,7
6	- 0,5	+ 2,9	+ 30,0	- 2,9	- 54,4	- 38,7	- 84,9
7	+ 0,1	- 0,9	+ 18,8	+ 2,6	- 41,2	- 28,9	- 66,7
8	+ 1,9	- 2,8	+ 89,4	- 3,3	+ 52,8	-125,6	+ 75,6
9	- 0,3	+ 1,9	- 24,5	+ 8,5	- 62,9	+ 28,9	- 87,8
10	- 1,6	- 2,6	+102,9	+ 1,7	+ 26,6	-145,6	+ 39,2
11	+ 0,2	+ 5,9	- 1,6	- 7,7	- 41,4	+ 7,3	- 69,0
12	+ 0,8	- 0,3	- 17,3	+ 5,2	+ 4,2	+ 20,8	+ 10,2
13	- 2,4	- 0,5	+ 91,5	- 0,1	+ 49,4	-125,1	+ 71,9
14	- 0,2	+ 0,9	+ 67,7	- 2,3	- 41,1	- 94,5	- 61,7
15	- 0,1	- 0,6	+ 2,4	+ 0,7	+ 35,6	- 3,4	+ 52,7
16	- 0,6	- 1,3	+ 71,6	+ 1,8	+ 37,7	-102,6	+ 57,5
17	+ 1,8	+ 3,4	+ 61,1	- 1,7	- 50,4	- 83,2	- 76,0
18	- 1,2	+ 0,3	+ 33,4	- 0,3	- 27,4	- 46,8	- 41,3
19	- 1,0	- 0,2	+ 53,7	+ 1,9	+ 34,5	- 75,2	+ 52,6
20	- 1,9	+ 3,1	+ 24,9	- 1,6	- 67,0	- 32,7	-100,4
21	+ 1,7	+ 0,2	- 24,1	+ 2,2	- 5,2	+ 34,3	- 6,4
22	+ 4,5	+ 6,4	- 28,0	- 7,5	- 56,9	+ 47,8	- 95,9
23	- 4,4	+19,3	+ 73,0	+ 2,0	-114,1	-103,0	-179,3
M	- 0,5	+ 1,6	+ 41,2	0,0	- 17,7	- 57,2	- 27,6

Tableau 11b

en cm sur le terrain

MOD.	x		y			z	
	a	b	a	c	b	a	b
1	0,1	2,8	35,2	5,7	37,8	50,0	57,0
2	3,4	1,1	75,7	5,9	66,5	103,8	98,0
3	1,9	1,3	46,7	4,8	48,1	65,2	70,2
4	3,2	1,5	56,9	4,8	30,7	76,1	44,4
5	1,5	1,4	47,1	5,2	30,8	67,3	52,0
6	1,0	2,8	44,6	4,0	47,2	61,4	71,5
7	0,2	1,0	36,9	4,7	44,8	53,0	75,5
8	0,6	3,6	32,2	5,6	62,5	44,1	90,7
9	0,5	1,0	30,0	5,3	33,0	42,3	49,5
10	0,8	3,9	39,9	6,1	40,7	57,9	59,5
11	2,1	4,5	45,1	5,6	33,5	64,5	51,9
12	2,0	1,6	39,6	6,0	37,6	54,3	55,3
13	1,3	0,5	46,0	5,1	46,2	62,4	66,8
14	0,2	0,7	44,3	5,9	31,9	64,6	48,5
15	1,7	0,7	41,8	5,0	37,8	60,2	57,0
16	0,7	1,2	36,3	4,6	35,6	51,7	53,5
17	1,4	3,6	47,8	4,1	50,7	66,6	77,5
18	0,9	0,4	24,6	4,8	38,8	34,9	58,3
19	0,5	0,4	28,0	3,4	39,5	37,6	57,6
20	2,8	2,0	35,9	5,5	43,3	48,4	63,7
21	2,2	2,1	29,3	4,4	32,2	44,0	47,1
22	6,6	4,2	46,6	4,0	39,7	70,7	62,8
23	2,9	8,5	47,8	4,1	48,6	66,5	78,8

Tableau 12a

MOD.	φ^a	ω^a	κ^a	bx_m	by_m	bz_m
1	+ 0,18 0,004	+ 0,84 0,004	- 1,98 0,002	2592,499 0,891	23,158 0,385	47,592 0,140
2	+ 0,04 0,008	+ 1,16 0,004	- 2,30 0,004	2435,339 1,072	91,743 0,427	46,190 0,255
3	+ 0,01 0,010	+ 1,14 0,007	- 1,95 0,005	2520,449 0,872	105,627 0,411	47,400 0,371
4	+ 0,12 0,010	+ 1,27 0,008	- 2,20 0,006	2404,313 0,811	96,102 0,424	44,075 0,447
5	+ 0,63 0,010	+ 0,84 0,009	- 2,33 0,007	2336,559 0,884	99,694 0,355	41,176 0,411
6	+ 0,74 0,012	+ 1,12 0,010	- 2,46 0,006	2298,093 0,926	104,711 0,345	45,410 0,389
7	+ 0,78 0,013	+ 1,09 0,009	- 2,44 0,006	2329,167 0,976	108,272 0,500	43,870 0,456
8	+ 1,38 0,012	+ 0,29 0,008	- 0,61 0,005	2253,105 0,920	100,476 0,376	43,354 0,497
9	+ 0,98 0,012	+ 1,19 0,009	- 0,62 0,005	2410,339 1,121	49,251 0,280	45,798 0,461
10	+ 0,99 0,014	+ 0,48 0,009	- 3,09 0,006	2536,741 1,133	48,710 0,318	47,917 0,471
11	+ 0,44 0,014	+ 1,06 0,009	- 6,36 0,007	2547,291 1,317	204,755 0,317	52,775 0,588
12	+ 0,18 0,017	+ 1,13 0,010	- 1,43 0,009	2554,964 1,251	111,156 0,520	48,631 0,647
13	- 0,40 0,019	+ 1,00 0,010	- 1,21 0,009	2534,425 1,246	50,382 0,365	48,634 0,729
14	+ 0,18 0,018	+ 0,79 0,009	- 1,78 0,011	2365,984 1,096	50,682 0,457	40,374 0,613
15	+ 0,04 0,019	+ 1,20 0,010	- 2,23 0,012	2319,187 1,235	72,901 0,599	46,023 0,657
16	+ 0,78 0,019	+ 0,87 0,009	- 2,20 0,012	2310,417 1,269	76,110 0,438	41,112 0,748
17	+ 0,90 0,019	+ 0,92 0,009	- 2,33 0,012	2390,577 1,019	85,873 0,497	47,402 0,756
18	+ 0,65 0,020	+ 1,02 0,011	- 2,54 0,010	2135,009 0,852	78,910 0,355	43,382 0,645
19	+ 0,52 0,021	+ 1,18 0,011	- 2,55 0,011	2392,510 1,171	90,914 0,458	45,116 0,701
20	- 0,29 0,020	+ 1,23 0,012	- 4,03 0,013	2334,751 1,254	124,916 0,535	48,466 0,774
21	- 0,07 0,021	+ 0,68 0,012	- 4,32 0,013	2419,271 1,251	146,983 0,648	45,637 0,777
22	- 0,39 0,021	+ 1,19 0,012	- 4,40 0,013	2429,176 1,125	148,925 0,527	49,735 0,786
23	+ 1,92 0,025	+ 1,85 0,013	- 6,87 0,013	1549,386 1,244	-163,512 0,616	-22,667 0,563
24	+ 2,07 0,027	+ 2,26 0,014	- 8,76 0,013	2417,305 1,401	97,384 0,762	46,825 0,995

Tableau 12b

MOD.	$\Delta\varphi^a$	$\Delta\omega^a$	$\Delta\kappa^a$	R	$\Delta\theta^a$	$\Delta\Phi^a$
1	+ 0,18 0,004	+ 0,84 0,004	- 1,98 0,002	1,000 0,34 · 10 ⁻³	+ 0,57 0,009	+ 1,17 0,003
2	- 0,14 0,006	+ 0,33 0,004	- 0,33 0,003	0,939 0,20	+ 1,83 0,017	+ 0,06 0,006
3	- 0,03 0,006	- 0,03 0,004	+ 0,36 0,004	1,035 0,31	+ 0,27 0,009	- 0,03 0,007
4	+ 0,10 0,006	+ 0,13 0,004	- 0,25 0,004	0,954 0,26	- 0,12 0,013	- 0,03 0,006
5	+ 0,51 0,004	- 0,43 0,003	- 0,12 0,003	0,971 0,29	+ 0,17 0,015	- 0,04 0,007
6	+ 0,11 0,004	+ 0,28 0,003	- 0,13 0,003	0,984 0,20	+ 0,18 0,012	+ 0,14 0,006
7	+ 0,04 0,004	- 0,03 0,004	+ 0,02 0,004	1,013 0,20	+ 0,06 0,015	- 0,06 0,006
8	+ 0,60 0,005	- 0,81 0,003	+ 1,82 0,004	0,967 0,24	- 0,12 0,017	+ 0,03 0,007
9	- 0,40 0,005	+ 0,90 0,004	- 0,01 0,003	1,070 0,35	- 1,54 0,012	- 0,01 0,009
10	+ 0,01 0,004	- 0,71 0,003	- 2,47 0,003	1,052 0,27	- 0,08 0,010	- 0,01 0,007
11	- 0,55 0,005	+ 0,58 0,003	- 3,27 0,002	1,004 0,24	+ 3,88 0,009	+ 0,12 0,006
12	- 0,26 0,006	+ 0,07 0,004	+ 4,93 0,004	1,003 0,20	- 2,34 0,014	- 0,11 0,006
13	- 0,57 0,005	- 0,13 0,004	+ 0,22 0,003	0,992 0,21	- 1,50 0,012	+ 0,01 0,009
14	+ 0,58 0,006	- 0,21 0,003	- 0,57 0,004	0,933 0,21	+ 0,10 0,010	- 0,13 0,006
15	- 0,14 0,006	+ 0,41 0,004	- 0,45 0,004	0,980 0,25	+ 0,64 0,015	+ 0,18 0,009
16	+ 0,74 0,005	- 0,34 0,003	+ 0,03 0,002	0,996 0,30	+ 0,10 0,013	- 0,13 0,007
17	+ 0,11 0,005	+ 0,06 0,004	- 0,13 0,003	1,035 0,28	+ 0,19 0,013	+ 0,13 0,007
18	- 0,25 0,005	+ 0,10 0,003	- 0,20 0,003	0,983 0,20	+ 0,07 0,012	+ 0,03 0,007
19	- 0,13 0,004	+ 0,16 0,002	- 0,01 0,003	1,121 0,33	+ 0,07 0,012	- 0,09 0,007
20	- 0,81 0,006	+ 0,04 0,002	- 1,48 0,005	0,976 0,19	+ 0,98 0,006	+ 0,12 0,007
21	+ 0,22 0,004	- 0,54 0,003	- 0,29 0,003	1,036 0,25	+ 0,46 0,010	- 0,12 0,009
22	- 0,32 0,004	+ 0,50 0,003	- 0,08 0,004	1,004 0,24	+ 0,03 0,012	+ 0,10 0,005
23	+ 2,31 0,009	+ 0,66 0,006	- 2,47 0,004	0,638 0,33	-30,59 0,023	- 2,23 0,013
24	+ 0,16 0,005	+ 0,41 0,006	- 1,89 0,004	1,560 0,65	+ 9,25 0,023	+ 2,16 0,015
V. Max	- 0,020	- 0,014	- 0,011	0,94	- 0,051	- 0,027
Mod.	23	24	3	1	24	24

Tableau 13

no.	$\Delta\varphi\Delta\theta$	$\Delta\varphi\Delta\omega$	$\Delta\varphi\Delta\kappa$	$\Delta\varphi\Delta\theta$	$\Delta\varphi\Delta\omega$	$\Delta\varphi\Delta\kappa$	$\Delta\omega\Delta\theta$	$\Delta\omega\Delta\omega$	$\Delta\omega\Delta\kappa$	$\Delta\kappa\Delta\theta$	$\Delta\kappa\Delta\omega$	$\Delta\kappa\Delta\kappa$	$R\Delta\theta$	$R\Delta\omega$	$R\Delta\kappa$	$\Delta\Phi\Delta\theta$
1	+ 0,51	+ 0,30	- 0,26	- 0,40	+ 0,40	- 0,21	+ 0,31	- 0,89	+ 0,31	- 0,14	+ 0,01	- 0,04	- 0,27	- 0,17	- 0,04	- 0,43
2	+ 0,54	- 0,37	- 0,16	- 0,73	- 0,49	- 0,06	- 0,04	- 0,81	- 0,04	+ 0,06	+ 0,14	+ 0,30	- 0,05	- 0,20	+ 0,30	+ 0,45
3	+ 0,58	+ 0,32	- 0,44	- 0,64	- 0,01	- 0,69	- 0,43	- 0,30	- 0,43	+ 0,17	+ 0,25	+ 0,58	- 0,10	- 0,09	+ 0,58	+ 0,10
4	+ 0,58	- 0,34	- 0,10	- 0,62	- 0,04	- 0,61	- 0,17	- 0,61	- 0,17	+ 0,35	+ 0,22	+ 0,01	+ 0,01	- 0,41	+ 0,01	+ 0,17
5	+ 0,48	- 0,08	+ 0,06	- 0,49	- 0,08	- 0,40	+ 0,04	- 0,34	+ 0,04	+ 0,40	- 0,10	+ 0,46	- 0,18	- 0,18	+ 0,46	- 0,10
6	+ 0,46	- 0,09	+ 0,16	- 0,74	- 0,06	- 0,81	- 0,13	- 0,81	- 0,13	+ 0,07	- 0,05	+ 0,32	- 0,10	- 0,10	- 0,07	- 0,06
7	+ 0,38	+ 0,04	+ 0,17	- 0,43	- 0,20	- 0,84	- 0,21	- 0,84	- 0,21	+ 0,13	+ 0,20	- 0,08	- 0,06	- 0,20	- 0,06	+ 0,20
8	+ 0,03	+ 0,39	+ 0,39	- 0,83	- 0,07	- 0,62	- 0,21	- 0,62	- 0,21	- 0,28	+ 0,57	- 0,42	+ 0,02	- 0,02	+ 0,42	+ 0,51
9	+ 0,06	+ 0,26	- 0,10	- 0,69	- 0,31	- 0,68	- 0,22	- 0,68	- 0,22	+ 0,09	- 0,27	+ 0,29	+ 0,29	+ 0,29	+ 0,27	+ 0,39
10	+ 0,47	+ 0,31	+ 0,08	- 0,82	- 0,51	- 0,71	+ 0,59	- 0,71	+ 0,59	+ 0,22	+ 0,09	- 0,05	- 0,13	- 0,05	- 0,50	+ 0,16
11	+ 0,39	+ 0,43	+ 0,19	- 0,84	- 0,11	- 0,74	+ 0,11	- 0,74	+ 0,11	+ 0,38	+ 0,08	+ 0,03	- 0,06	- 0,06	- 0,60	+ 0,06
12	+ 0,31	- 0,50	+ 0,39	- 0,50	- 0,15	- 0,75	- 0,33	- 0,75	- 0,33	+ 0,62	- 0,45	0	- 0,09	- 0,09	0	+ 0,15
13	+ 0,60	- 0,18	+ 0,01	- 0,65	- 0,01	- 0,56	- 0,32	- 0,56	- 0,32	- 0,33	- 0,49	- 0,14	- 0,19	- 0,19	- 0,36	+ 0,39
14	+ 0,85	+ 0,30	+ 0,16	- 0,71	- 0,19	- 0,46	- 0,35	- 0,46	- 0,35	- 0,18	+ 0,26	- 0,09	+ 0,33	- 0,33	+ 0,36	+ 0,04
15	+ 0,31	- 0,34	+ 0,40	- 0,61	- 0,50	- 0,51	- 0,22	- 0,51	- 0,22	- 0,04	- 0,31	- 0,25	- 0,03	+ 0,08	- 0,03	+ 0,38
16	+ 0,36	- 0,06	- 0,30	- 0,64	- 0,24	- 0,37	- 0,20	- 0,37	- 0,20	+ 0,21	- 0,38	+ 0,15	+ 0,02	+ 0,15	+ 0,16	+ 0,24
17	+ 0,25	- 0,43	+ 0,32	- 0,54	- 0,55	- 0,82	+ 0,40	- 0,82	+ 0,40	- 0,43	- 0,31	- 0,19	- 0,22	- 0,22	+ 0,16	+ 0,57
18	+ 0,14	- 0,11	+ 0,17	- 0,68	- 0,16	- 0,44	+ 0,15	- 0,44	+ 0,15	+ 0,01	+ 0,56	- 0,19	- 0,10	- 0,10	- 0,16	+ 0,19
19	+ 0,24	- 0,13	+ 0,23	- 0,67	0	- 0,64	- 0,25	- 0,64	- 0,25	+ 0,02	- 0,59	- 0,25	+ 0,21	+ 0,21	+ 0,06	+ 0,04
20	+ 0,32	+ 0,13	+ 0,12	- 0,68	- 0,19	- 0,64	+ 0,12	- 0,64	+ 0,12	- 0,15	- 0,05	- 0,05	- 0,21	- 0,21	- 0,16	+ 0,16
21	+ 0,69	- 0,29	+ 0,32	- 0,21	+ 0,62	- 0,60	- 0,60	- 0,60	- 0,60	+ 0,43	+ 0,04	+ 0,28	+ 0,29	+ 0,29	+ 0,46	- 0,28
22	+ 0,19	- 0,06	- 0,22	- 0,60	- 0,01	- 0,40	- 0,41	- 0,40	- 0,41	+ 0,05	- 0,57	+ 0,09	- 0,08	- 0,08	+ 0,47	0
23	+ 0,01	- 0,20	- 0,48	- 0,95	- 0,08	- 0,88	- 0,07	- 0,88	- 0,07	- 0,46	+ 0,08	+ 0,46	+ 0,16	+ 0,16	+ 0,16	+ 0,10
24	+ 0,68	+ 0,37	- 0,67	- 0,08	+ 0,33	- 0,38	+ 0,34	- 0,38	+ 0,34	- 0,57	- 0,39	- 0,01	- 0,27	- 0,27	- 0,04	+ 0,14
M(1-5)	+ 0,53	- 0,03	- 0,18	- 0,59	- 0,04	- 0,59	+ 0,01	- 0,59	+ 0,01	+ 0,16	+ 0,10	+ 0,26	- 0,21	- 0,21	+ 0,26	+ 0,03
M(6-10)	+ 0,29	+ 0,12	+ 0,14	- 0,58	- 0,23	- 0,73	- 0,02	- 0,73	- 0,02	- 0,01	- 0,12	+ 0,01	- 0,03	- 0,03	+ 0,01	+ 0,26
M(11-15)	+ 0,37	+ 0,09	+ 0,23	- 0,66	- 0,20	- 0,63	- 0,12	- 0,63	- 0,12	+ 0,22	- 0,18	- 0,05	- 0,19	- 0,19	- 0,20	+ 0,20
M(16-20)	+ 0,30	+ 0,02	+ 0,10	- 0,64	- 0,22	- 0,62	+ 0,04	- 0,62	+ 0,04	- 0,07	- 0,15	- 0,15	- 0,20	- 0,20	- 0,20	+ 0,20
M(20-24)	+ 0,51	- 0,06	- 0,50	- 0,50	+ 0,33	- 0,58	- 0,38	- 0,58	- 0,38	- 0,57	- 0,39	- 0,01	- 0,27	- 0,27	- 0,04	+ 0,14

Tableau 14a

en mm sur le terrain

no.	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆
1	- 0,9	- 0,2	- 1,6	+ 0,1	+ 5,3	- 1,0
2	- 45,7	+ 88,6	- 46,4	+ 47,6	- 91,0	+ 44,8
3	- 15,4	+ 27,3	- 12,7	+ 15,5	- 25,9	+ 10,5
4	- 73,1	+ 138,7	- 69,8	+ 69,7	- 140,6	+ 78,1
5	- 28,6	+ 56,9	- 28,7	+ 28,0	- 54,7	+ 29,3
6	- 0,9	+ 0,7	+ 0,1	+ 0,6	+ 1,3	- 0,3
7	- 102,5	+ 169,2	- 71,6	+ 95,2	- 162,8	+ 68,4
8	+ 2,0	- 0,5	- 0,1	+ 0,1	- 1,5	- 0,3
9	- 13,4	+ 33,1	- 13,1	+ 15,0	- 29,8	+ 9,1
10	+ 13,1	- 25,1	+ 13,5	- 12,2	+ 22,0	- 11,4
11	+ 5,7	- 16,2	+ 7,7	- 7,5	+ 15,1	- 6,8
12	- 25,7	+ 70,3	- 32,9	+ 26,1	- 69,2	+ 30,7
13	- 58,6	+ 116,1	- 52,5	+ 51,9	- 114,9	+ 57,6
14	+ 16,9	- 30,7	+ 18,0	- 18,9	+ 30,8	- 16,1
15	- 34,4	+ 63,4	- 22,9	+ 32,6	- 59,3	+ 21,8
16	- 36,4	+ 60,5	- 26,5	+ 44,6	- 73,1	+ 30,9
17	+ 31,6	- 57,9	+ 20,6	- 24,7	+ 50,3	- 23,4
18	- 29,2	+ 72,9	- 33,2	+ 31,6	- 71,2	+ 30,0
19	- 145,4	+ 232,1	- 127,9	+ 129,9	- 232,8	+ 129,6
20	- 19,5	+ 41,7	- 20,6	+ 21,9	- 41,7	+ 18,4
21	+ 19,5	- 35,3	+ 16,0	- 18,8	+ 33,6	- 14,9
22	- 31,6	+ 60,5	- 28,5	+ 35,8	- 64,2	+ 29,6
23	- 28,4	+ 49,9	- 24,4	+ 22,9	- 42,0	+ 21,8
M	33,9	63,0	30,0	32,7	62,3	29,8

Tableau 14b

en mm sur le terrain

MOD	P ₁	P ₂	P ₃	P ₄	P ₅	P ₆
1	± 25,0	± 41,3	± 17,6	± 21,5	± 38,3	± 17,0
2	16,0	31,1	16,1	16,1	30,4	15,4
3	16,7	38,9	18,8	18,3	39,0	17,3
4	19,9	38,3	19,7	19,4	39,6	22,2
5	22,7	40,6	21,9	21,6	39,2	21,6
6	17,9	34,9	16,9	22,0	40,4	18,2
7	26,8	44,0	20,5	28,3	44,7	19,3
8	21,7	38,4	12,8	20,1	37,0	13,1
9	12,6	28,1	12,4	13,8	27,4	10,6
10	12,1	21,2	11,2	11,2	19,2	10,6
11	19,3	43,5	21,3	18,3	41,4	21,1
12	16,9	46,6	20,6	16,4	45,4	20,1
13	19,6	38,2	17,5	16,8	38,1	19,9
14	10,9	19,4	11,1	11,7	19,2	10,1
15	18,2	33,3	13,0	17,7	31,0	12,0
16	15,0	25,1	10,9	18,9	30,5	12,5
17	16,1	29,9	10,7	12,7	25,6	11,7
18	16,6	41,7	19,1	17,4	39,9	16,1
19	25,0	39,0	22,6	21,2	37,9	21,6
20	24,3	49,2	23,4	25,4	51,2	24,2
21	17,2	32,1	13,7	16,2	31,1	13,8
22	18,6	34,6	16,2	21,5	37,3	17,4
23	27,3	49,3	23,0	22,8	42,0	20,2

Tableau 15

en mm sur le terrain

MOD.	1	2	3	4	5	6	7	8	σ ₀
1	± 420	± 106	± 39	65	± 55	± 264	± 70	54	± 185
2	76	426	8	53	155	271	164	211	212
3	156	41	295	194	198	190	79	160	179
4	451	62	152	161	69	421	452	322	305
5	154	233	215	42	315	171	69	321	213
6	171	231	93	244	24	91	156	234	173
7	167	81	475	268	291	466	611	134	358
8	237	70	322	25	39	2	68	235	168
9	68	90	67	12	98	89	262	196	133
10	70	194	200	25	65	9	5	26	105
11	109	422	182	52	113	225	77	46	193
12	37	31	360	192	191	119	447	1	229
13	502	218	430	0	13	164	180	74	262
14	27	11	24	56	67	80	166	225	108
15	72	191	344	135	25	186	152	141	179
16	131	57	2	32	121	297	340	85	176
17	212	22	171	303	144	29	36	50	155
18	465	28	202	198	231	11	131	8	214
19	159	426	589	638	446	92	568	474	463
20	37	391	450	15	7	88	273	130	239
21	223	22	133	169	186	167	132	130	155
22	89	371	333	152	3	134	129	38	199
23	275	493	13	64	236	157	33	84	227
σ ₀	± 235	224	276	193	175	201	261	187	

Tableau 16a

en cm sur le terrain

MOD.	x		y			z	
	a	b	a	o	b	a	b
1	- 1,1	+ 0,3	+ 26,2	- 4,7	- 4,3	- 37,1	- 10,9
2	+ 1,2	+ 2,4	- 18,6	+ 3,2	- 35,8	+ 25,3	- 50,2
3	+ 2,7	+ 1,9	- 38,2	- 5,7	- 25,8	+ 63,5	- 43,2
4	- 1,8	- 3,4	+ 24,0	+ 4,2	+ 4,1	- 40,9	+ 66,5
5	- 5,8	+ 0,3	+ 80,0	+ 2,7	- 36,5	- 126,9	- 49,9
6	+ 0,4	- 1,8	+ 7,5	- 8,5	- 17,7	- 2,8	- 31,7
7	- 3,0	- 8,7	- 24,3	+ 8,2	- 51,0	+ 32,4	- 73,7
8	- 1,5	- 0,3	- 16,2	- 1,6	- 84,0	+ 28,5	- 126,9
9	0	- 0,1	+ 2,8	+ 2,7	+ 14,0	- 6,9	+ 21,0
10	+ 0,4	- 2,0	+ 0,9	- 0,3	+ 16,6	- 1,1	+ 25,5
11	- 0,7	+ 10,9	+ 5,7	- 4,3	- 76,1	- 6,5	- 117,1
12	- 0,1	+ 0,4	+ 22,0	- 2,3	+ 14,4	- 30,6	+ 21,2
13	+ 0,4	+ 0,1	+ 16,8	+ 7,3	- 6,3	- 30,8	- 3,4
14	- 4,7	- 3,1	- 42,1	- 4,7	- 75,9	+ 75,7	- 110,0
15	- 0,3	- 0,9	- 17,1	- 0,1	- 27,8	+ 27,2	- 40,4
16	+ 0,9	+ 1,2	- 11,7	+ 6,5	- 61,3	+ 13,3	- 89,1
17	- 0,9	+ 8,4	+ 17,0	- 6,2	- 54,6	- 23,6	- 84,4
18	+ 0,7	+ 5,6	- 2,5	- 8,1	- 46,2	+ 13,5	- 78,9
19	- 0,2	+ 1,3	- 2,6	+ 9,5	- 26,0	- 4,6	- 31,5
20	- 0,7	+ 1,0	- 25,4	+ 3,8	- 54,2	+ 38,3	- 80,9
21	- 0,2	- 0,7	- 15,3	- 4,7	- 14,8	+ 27,3	- 25,6
22	+ 0,5	0	+ 19,5	+ 0,7	- 9,3	- 33,7	- 13,6
M	- 0,6	+ 0,6	+ 0,4	- 0,1	- 28,2	0	- 42,1

Tableau 16b

en cm sur le terrain

MOD.	x		y			z	
	a	b	a	o	b	a	b
1	1,9	2,5	37,6	7,2	59,2	56,1	89,4
2	2,6	3,5	42,1	6,5	48,4	64,5	73,9
3	3,7	3,5	40,3	6,6	68,1	63,5	107,2
4	3,1	3,6	44,8	8,9	31,1	77,5	51,1
5	4,3	3,2	51,6	9,4	54,2	80,4	80,0
6	2,7	4,3	64,2	7,0	52,5	106,4	77,8
7	2,5	8,6	19,2	10,0	48,3	27,5	74,2
8	1,9	5,0	17,0	6,1	57,5	29,2	87,2
9	0,1	2,0	40,1	4,1	40,1	64,4	58,9
10	5,6	4,4	39,2	4,8	32,8	60,2	47,3
11	8,6	9,4	63,5	9,2	71,4	94,3	99,7
12	0,2	1,4	33,9	7,5	38,3	48,4	59,2
13	0,4	1,0	17,2	5,3	50,0	27,3	73,7
14	6,8	4,3	59,8	6,0	50,5	97,9	69,7
15	1,3	1,7	72,8	5,6	53,7	114,7	75,2
16	1,9	0,8	30,0	4,4	58,3	53,4	88,3
17	4,5	8,0	52,5	9,0	44,5	84,4	71,1
18	3,9	8,2	50,2	8,9	63,7	78,4	100,3
19	5,5	2,8	55,0	9,0	37,6	85,7	55,2
20	1,7	1,6	53,8	6,4	95,8	90,3	151,5
21	1,0	3,5	49,4	5,1	66,8	77,1	97,7
22	0,6	1,6	28,7	6,6	24,5	53,6	37,0

Tableau 17a

MOD.	φ^0	ω^0	κ^0	$b x_m$	$b y_m$	$b z_m$
1	+ 0,28 0,009	- 0,61 0,005	- 2,20 0,004	2586,978 2,723	- 190,243 0,593	+ 9,744 0,314
2	+ 0,10 0,012	- 0,09 0,009	- 4,40 0,004	2584,328 2,814	- 77,589 0,570	+ 7,260 0,496
3	+ 0,17 0,017	- 0,13 0,010	- 5,46 0,002	2508,252 2,527	+ 7,595 0,647	+ 6,315 0,500
4	+ 0,02 0,016	- 0,82 0,010	- 4,85 0,003	2545,629 2,855	+ 5,897 0,664	+ 3,834 0,793
5	- 0,16 0,020	- 1,08 0,007	- 1,91 0,003	2521,182 2,891	- 23,536 0,530	+ 7,844 0,828
6	- 0,13 0,017	- 0,99 0,011	+ 4,49 0,006	2411,499 3,004	- 184,351 0,555	+ 8,090 0,633
7	- 0,25 0,017	- 1,99 0,014	+ 9,55 0,007	2397,596 2,187	- 457,878 0,600	+ 11,577 0,725
8	- 0,15 0,021	- 1,29 0,015	+ 3,08 0,008	2516,517 2,768	- 459,939 0,554	+ 15,818 0,740
9	+ 0,18 0,021	- 1,64 0,014	- 2,62 0,010	2536,459 2,852	- 232,877 0,621	+ 9,099 0,930
10	+ 0,42 0,022	- 0,41 0,014	- 7,71 0,011	2592,136 2,760	- 39,814 0,502	+ 6,088 0,875
11	+ 0,45 0,019	- 0,87 0,015	- 8,17 0,011	2545,149 2,588	+ 149,805 0,756	+ 4,057 0,764
12	+ 0,36 0,018	- 0,80 0,019	- 1,12 0,012	2633,706 2,991	- 22,771 0,506	+ 11,370 0,787
13	+ 0,38 0,019	- 0,96 0,020	+ 3,02 0,012	2576,376 2,756	- 257,006 0,720	+ 9,366 0,715
14	+ 0,46 0,019	- 1,27 0,023	+ 3,02 0,012	2567,100 2,652	- 310,958 0,777	+ 10,437 0,867
15	+ 0,61 0,021	- 1,27 0,021	+ 0,34 0,011	2526,958 2,386	- 253,272 0,529	+ 11,491 0,820
16	+ 1,81 0,020	- 0,65 0,022	+ 1,36 0,013	2608,693 2,329	- 228,304 0,848	+ 11,513 0,910
17	+ 0,85 0,015	- 1,89 0,022	- 6,73 0,013	2615,900 2,928	- 189,943 0,551	+ 16,412 0,710
18	+ 0,60 0,019	- 0,83 0,022	- 8,77 0,014	2743,941 2,328	+ 89,696 0,840	+ 4,438 0,720
19	+ 0,63 0,018	- 0,08 0,019	- 4,38 0,016	2646,518 2,172	+ 33,503 0,929	+ 9,104 0,791
20	+ 0,34 0,017	- 0,70 0,019	+ 2,18 0,019	2690,354 1,655	- 216,232 0,884	+ 13,715 0,885
21	+ 0,41 0,027	- 0,77 0,020	+ 2,04 0,021	2591,288 1,993	- 338,984 1,242	+ 13,812 0,959
22	+ 0,18 0,024	- 1,32 0,020	+ 1,52 0,024	2644,733 2,026	- 305,965 1,209	+ 9,906 1,085
23	+ 0,44 0,027	- 0,91 0,025	- 1,54 0,021	2627,746 1,943	- 186,978 0,638	+ 11,651 1,148

Tableau 17b

MOD.	$\Delta\varphi^0$	$\Delta\omega^0$	$\Delta\kappa^0$	R	$\Delta\theta^0$	$\Delta\Phi^0$
1	+ 0,28 0,009	- 0,61 0,005	- 2,20 0,004	1,000 $1,05 \cdot 10^{-3}$	- 4,67 0,011	+ 0,22 0,008
2	- 0,18 0,008	- 0,52 0,007	- 2,20 0,004	0,999 0,50	+ 2,76 0,020	- 0,05 0,008
3	+ 0,07 0,008	- 0,05 0,007	- 1,06 0,004	0,971 0,27	+ 2,10 0,018	- 0,02 0,007
4	- 0,16 0,006	- 0,69 0,007	+ 0,60 0,004	1,015 0,36	- 0,34 0,030	- 0,06 0,011
5	- 0,18 0,008	- 0,26 0,006	+ 2,94 0,002	0,991 0,30	- 0,45 0,024	+ 0,10 0,009
6	+ 0,04 0,011	+ 0,09 0,005	+ 6,40 0,004	0,957 0,51	- 4,26 0,023	+ 0,01 0,009
7	- 0,12 0,009	- 1,00 0,004	+ 5,06 0,005	0,994 0,51	- 7,16 0,011	+ 0,09 0,007
8	+ 0,11 0,006	+ 0,70 0,003	- 6,47 0,003	1,050 0,34	+ 0,51 0,010	+ 0,09 0,005
9	+ 0,32 0,006	- 0,35 0,005	- 5,69 0,005	1,008 0,33	+ 5,68 0,016	- 0,17 0,006
10	+ 0,25 0,005	+ 1,24 0,004	- 5,10 0,003	1,022 0,21	+ 4,85 0,014	- 0,08 0,009
11	+ 0,03 0,010	- 0,46 0,005	- 0,46 0,005	0,982 0,29	+ 4,72 0,017	- 0,05 0,006
12	- 0,09 0,007	- 0,06 0,006	+ 7,06 0,004	1,035 0,51	- 4,29 0,017	+ 0,17 0,010
13	+ 0,02 0,005	- 0,15 0,006	+ 4,13 0,004	0,978 0,35	- 5,78 0,017	- 0,04 0,010
14	+ 0,08 0,008	- 0,31 0,007	+ 0 0,003	0,996 0,36	- 1,34 0,017	+ 0,03 0,010
15	+ 0,15 0,006	+ 0,01 0,008	- 2,67 0,005	0,984 0,39	+ 1,31 0,024	+ 0,03 0,008
16	+ 1,20 0,003	+ 0,62 0,006	+ 1,02 0,004	1,032 0,22	+ 0,81 0,025	- 0,01 0,012
17	- 0,96 0,012	- 1,24 0,005	- 8,09 0,004	1,003 0,32	+ 0,94 0,016	+ 0,12 0,014
18	- 0,25 0,009	+ 1,06 0,006	- 2,04 0,003	1,049 0,53	+ 6,69 0,014	- 0,30 0,007
19	+ 0,03 0,006	+ 0,75 0,005	+ 4,38 0,007	1,965 0,42	- 1,27 0,015	+ 0,12 0,011
20	- 0,29 0,010	- 0,62 0,005	+ 6,56 0,006	1,017 0,44	- 5,91 0,011	+ 0,10 0,013
21	+ 0,07 0,013	- 0,07 0,007	- 0,14 0,004	0,964 0,42	- 3,17 0,019	+ 0,01 0,011
22	- 0,23 0,005	- 0,55 0,005	- 0,52 0,004	1,021 0,40	+ 0,95 0,011	- 0,10 0,008
23	+ 0,25 0,008	- 0,41 0,007	- 3,06 0,004	0,994 0,28	+ 2,81 0,019	+ 0,04 0,011
V. MAX	- 0,020	+ 0,015	- 0,012	$-1,79 \cdot 10^{-3}$	+ 0,049	- 0,026
MOD.	18	2	19	1	5	19

Discussion about the Subject 8 under the Chairmanship of Prof. W. K. Bachmann

Chairman: Even if Prof. *Cunietti's* results are only provisional he has certainly provided us with sufficient material for discussion and so to whom may I give the floor?

Wiser: Strip 14 of the Oberschwaben block has been duplicated in both your and our investigation. Can we now not compare the variations you found in the orientation elements with those we found in the coordinates?

Cunietti: A large number of comparisons are certainly possible. We also intend to perform space resections in order to obtain the absolute orientation elements from the model coordinates.

Wiser: The effort involved is considerable but I certainly think worthwhile. May I ask you whether the preliminary results indicate a degree of correlation with factors such as the photographic conditions and the type of terrain?

Cunietti: The results of the Reichenbach test were worse than those of Oberschwaben, but I believe that the larger φ and ω tilts might be an influencing factor.

Wiser: The geometric regularity is certainly important.

Chairman: I think Prof. *Cunietti* has demonstrated the influence of both the observation and calculating procedure on the results obtained. We must bear in mind that we are dealing with a physical process and so must be very much aware of the mathematical models used.

Van der Weele: Prof. *Cunietti's* comparison of the experimental and theoretical correlation coefficients refers to differences in the orientation elements of successive models and I would like to know whether the same comparison has been made for the orientation elements of individual models.

Cunietti: We do intend to compute a covariance matrix for each model but our initial intention was to see whether there was any significant difference in the results for various types of terrain and types of photography.

Chairman: Research of this type should be done on photography of flat countries like Holland since different results will be obtained in hilly terrain.

Van der Weele: I would say that it should be done for all types of terrain.

Ackermann: In trying to relate Prof. *Cunietti's* results with those obtained by Dr. *Stark* I come to the conclusion that, even with comparator measurements, the influence of the observational procedure is larger than was expected. Furthermore, Dr. *Stark's* results indicate smaller transfer errors than those obtained in this investigation, a possible explanation being the extra degrees of freedom present in the independent model method.

Cunietti: I cannot offer any explanation for the differences found.

Wiser: I agree with the conclusions drawn by Prof. *Ackermann*.

Chairman: Since there appears to be no more discussion, I would like to close this session after thanking the speakers for their presentations.

DK 528.711.18-187(430-2.442):778.151:528.735.2

(9)

Image Geometry as Obtained from Rheidt Test Area Photography
(with 17 Figures, 5 Tables)

By *G. Kupfer, Bonn*

RÉSUMÉ see p. 267

The Test Area and its Purpose

The Rheidt Area near Bonn has been established in 1965. Its main purpose was to get a better understanding of the photogrammetric process. There seemed to be a strong demand for a closer insight especially into the fundamentals of the survey flight and the processing of films and plates in the darkroom. Following the experiences made with the correction of systematic errors of heights in topographic mapping at the State Survey Office of Northrhine-Westphalia it seemed to be justified to use the results of test area flights for similar purposes in analytical photogrammetry. From the very beginning it was understood that the image coordinates were best suited for evaluation of systematic errors and for dealing with other tasks in connection with the photogrammetric process.

Since then several test flights have been performed on a mutual base, i. e. the results of evaluations at the Institute of Photogrammetry, University of Bonn were handed over to the initiating offices of the test flights without fee. A certain amount of typical results has been published, see ref. [3] and [4].

The test area is situated in the almost level Rhine valley. The area of approximately 2 km by 2 km was initially covered by an array of 5 by 5 point nests (Fig. 1), each nest containing three aligned points with mostly three meters distances between adjacent points. This net was densified in the spring 1969 by another 16 nests after very short negotiations with Prof. *Ackermann*. The field work, i. e. triangulation and levelling, for this densification, was performed by Dr. *Stark*, very ably assisted by his wife.

The whole net, including also electronic distance measurements, was adjusted in two steps, the first dealing with the basic net only (Fig. 2), the second for the densification (Fig. 3).

Rms errors of the point coordinates after adjustment were less than ± 10 mm. These geodetic errors are thought negligible in this context.

25 points of the basic net were marked by pillars, the remaining points as a rule by plastic tubes. All points (one nest excluded) were signalized by plates of plastic with a square size of 30 cm by 30 cm. These signals were centered by force. The remaining signals of one nest have been painted equally sized on the foundation of a high voltage pole.

Evaluation of Test Photographs

The main procedure, after printing diapositives on glass and monoscopic measurement of coordinates on a ZEISS PSK was a spatial resection of each photograph, using nine points

of the test area as control. For further investigation the residual error vectors of all points observed on one print after spatial resection were used.

For completeness sake the formulae considered for transformation of comparator coordinates into image coordinates as defined by fiducial marks may be recalled (cf. e. g. [4]).

Linear conformal transformation,

$$x = a_0 + a_1 x' - a_2 y' \quad (1)$$

$$y = a_3 + a_2 x' + a_1 y'.$$

Affine transformation,

$$x = a_{10} + a_{11} x' + a_{12} y' \quad (2)$$

$$y = a_{20} + a_{21} x' + a_{22} y'.$$

Pseudo-affine transformation,

$$x = a_{10} + a_{11} x' + a_{12} x' y' + a_{13} y' \quad (3)$$

$$y = a_{20} + a_{21} x' + a_{22} x' y' + a_{23} y'.$$

Perspective transformation,

$$x = a_0 + a_1 x' + a_2 y' + a_4 x' y' - a_3 x'^2 \quad (4)$$

$$y = a_5 + a_6 x' + a_7 y' - a_3 x' y' + a_4 y'^2.$$

Deformational transformation,

$$x = a_0 + a_1 x' + a_2 y' + a_4 x' y' + a_3 y'^2 \quad (5)$$

$$y = a_5 + a_6 x' + a_7 y' + a_3 x' y' + a_4 x'^2.$$

All transformations have been applied to each photograph.

Corrections for earth curvature and photogrammetric refraction were made applying a simplified version of *Leyonhufvud's* formulae [6], bearing in mind as well the smallness of the test area as the small height of this area above sea level (appr. 56 meters).

The average lens distortion was taken account of according to the latest inspection slip of the manufacturer.

Rms errors of unit weight σ_0 and rms errors of the check-point coordinates have been derived after each spatial resection.

Regression Polynomials

As shown before (cf. [3], [4]), special attention was paid to systematics or deterministic trends in the field of residual error vectors of each photograph. They result from the photogrammetric process which in its entirety is not correctly described by the above mentioned functional model. Despite other formulae or procedures existing these systematics have been described up till now using third order polynomials (8):

$$dx = a_{10} + a_{11}x + a_{12}y + a_{13}xy + a_{14}x^2 + a_{15}y^2 + a_{16}x^2y + a_{17}xy^2 + a_{18}x^3 + a_{19}y^3 \quad (6)$$

$$dy = a_{20} + a_{21}x + \dots$$

These regression polynomials were applied to eliminate systematic errors from image coordinates without knowledge of their origin proper, subsequently followed by another spatial resection after regression. Rms errors as obtained before and after regression can be used as a means for judging the effectiveness of this procedure. Beyond this such polynomials have been applied to following flight missions with the same photogrammetric system which partly resulted in considerable increase of overall accuracy. This holds true not only for spatial resection of single photographs but also for pairs of photographs and smaller bundle triangulations.

Least Squares Interpolations

As had been pointed out in ref. [4], considerable correlation exists between residual error vectors before regression. By the regression procedure it was possible to reduce this correlation, but there still existed a somewhat smaller remainder of it. Following the significant success with prediction by the method of least squares in various fields of geodesy, which has been introduced to photogrammetric tasks by K. Kraus (cf. e. g. [2]), it seemed worthwhile to give the statistic properties of residual error vectors after space resection a closer look. This has been done for one example in a diploma thesis of Dipl.-Ing. H. Ellenbeck [1], using results of two flights already described e. g. in ref. [4]. Tests on normal distribution and homogeneity of covariances did not yield positively, which could be obtained however, when testing on isotropy. This held true as well for residuals before as after regression. In spite of these facts a *Taylor-Karman* structure was accepted, for dealing with the two dimensional vector signals. Therefore longitudinal and lateral correlations were computed for discrete classes, depending only on distances between two points respectively. Thus covariance functions could be derived. Almost all correlations of residuals before and after regression appeared to be significant on at least a 95 percent level. The same did not hold true for cross correlations. Hence longitudinal and lateral correlations were independent. The differences between these correlations however appeared to be significant at least for some classes of distances. Thus assuming a *Taylor-Karman* structure seemed to be justified.

The whole lot of 41 nests of points have been used for derivation of covariance functions. Hence no check points exist. A final check of the prediction procedure could only involve filtering. For a first experiment four different filters have been used:

1) Applying longitudinal and lateral covariances as obtained *before regression*. As there exists only a limited number of classes of distances at the Test Area, it was possible to use the computed discrete correlation numbers without smoothing.

2) Applying covariances as obtained *after regression* in the same way as explained before.

The above mentioned covariances were applied up to a maximum radial distance of 55 mm in photo scale, called *filter 1*. The same up to a maximum radial distance of

110 mm: filter 2. As this system tended to become ill-conditioned during matrix inversion the number of reference points applied at one time had to be limited to a maximum of twenty.

- 3) For filter 3 a smoothed Gaussian as derived from longitudinal covariances has been applied before regression using reference points up to a maximum radial distance of 110 mm (Fig. 12a).
- 4) The same procedure using a bisected filter 3, has been applied after regression, called filter 4 (Fig. 12b).

Briefly the findings of *Ellenbeck* were as follows:

- 1) Application of filters 1 and 2 before regression resulted in larger remaining error vectors than applying regression without filtering. The difference appeared to be at the verge of significance.
- 2) Application of filter 3, as advocated by *Kraus* [2] gave significantly larger residual rms errors than all procedures mentioned in paragraph 1). It may be conceded that perhaps the Gaussian applied in this experiment was not chosen optimally. This has not been checked up till now.
- 3) Significantly better results as compared with paragraph 1) and 2) were obtained when applying filters 1, 2 and 4 after regression, with resulting rms errors being practically equal in each case.

Bearing in mind that these findings only are related to mere filtering, the significance of them has yet to be proven in a real interpolation procedure. It seems to be of some importance that the more complicated filters 1 and 2 did not yield better than a scalar filter 4 comparable to that described by *Kraus* but applied after regression.

As could be expected, all filters yielded normally distributed residual error vectors with only insignificant correlations of any kind between groups of points.

Comment: After centering and normalization the covariances of residual error vectors of any photograph are computed as follows:

$$\begin{aligned} l_i &= dx_i \sin a_{jk} + dy_i \cos a_{jk} \\ q_i &= dx_i \cos a_{jk} - dy_i \sin a_{jk} \end{aligned} \quad (7)$$

with $dx_i, dy_i, \dots$ components of error vector at the point P_i

$a_{jk}, \dots$ azimuth of the distance $\overline{P_i P_k}$ in e. g. image coordinates

Hence for each class s of distances

$$L_s = \frac{1}{n_s} [l_i l_k] \text{ longitudinal correlation,} \quad (8)$$

$$Q_s = \frac{1}{n_s} [q_i q_k] \text{ lateral correlation,} \quad (9)$$

$$K_s = \frac{1}{n_s} [l_i q_k] \text{ cross correlation,} \quad (10)$$

where $n_s, \dots$ number of distances in that class.

Test Area Flights in Connection with Oberschwaben Mission

All cameras used at the Oberschwaben mission should be flown over the test area before and after performing the Oberschwaben flights using the same plane. There should be four strips per flight and camera with an image scale of 1 : 10 500. So three photographs of each strip should cover the whole test area stereoscopically.

Diapositive prints on glass from these flights were delivered by Prof. *Ackermann* in March 1971 according to the following table.

Table 1: Rheidt Test Area Flights

Flight No.	R M K	Day of Flight	Number of Strips	Image Scale Factor
W 1	15/23 No. 111680	5. 4. 69	4	10040 10800
W 2		12. 6. 69	4	10320 11060
S 1	8.5/23 No. 111164	5. 4. 69	1	9520 9670
S 2		13. 5. 69	4	9950 10500

In addition 12 prints on glass, from photographs taken with the RMK AR 15/23 No. 21 196 were purchased in August 1972. Coordinates of these plates have only been measured on a ZEISS PSK up till now. No computations have been carried out. The following chapters therefore deal only with the material listed in Table 1.

In October 1969 another WA test area flight has been performed, [3], [4]. The then used ZEISS RMK A 15/23 seems to be of the same series of fabrication. As the same arrangement of signalized points could be used, it might be justified to add some results from this flight for the sake of comparison.

Coordinate Measurements

All measurements were carried out on a ZEISS PSK. One of the operators, Dipl.-Ing. *Wegener*, had only slight experience, the other one, Ing.(grad.) *Schröter*, is a skilled operator. Each point has been measured twice. Pointing accuracies are given in table 2.

The pointing accuracy is better for fiducial marks than for signalized points. The more skilled operator pointed with higher accuracy. Judging from pointing accuracy point definition seems to be better in the earlier flights and least favourable in flight S 2. Different pointing accuracies for both operators mentioned before may be readily seen from Fig. 4 and 5. There histograms of pointing differences are plotted against normal distribution.

Table 2: Pointing Accuracy [μm]

Flight No. Operator	Number of Photos Points	Fiducial Marks		Signalized Points	
		x	y	x	y
W 1 W	12 1030	1.1	1.1	1.8	1.8
W 2 W	12 1054	1.2	1.3	2.1	2.0
S 1 S	3 236	0.8	0.7	1.2	1.4
S 2 S/W	12 1027	1.3	1.2	2.2	2.3
10.69 K	6 720	1.0	1.3	0.9	1.0

Spatial Resections for Single Photographs

Spatial resections have been performed with all photographs available. In Table 3 the results are given only for photographs covering the whole test area. The figures are quadratic means of all photos concerned in one flight. As transformation equations (2) and (3) resulted in almost equal accuracies, only the simpler one is presented here. Equations (4) resulted in all cases in largest residuals, hence these results are omitted. Standard deviations after fitting to fiducial marks according to equations (1) and (2) vary to a great extent from flight to flight, although the photographic definition of the marks was good. Scale factors, scale differences and deviation of rectangularity of the axes did not only amount to considerable values unequal to one or zero respectively. They also altered within one flight to a reasonable amount, especially within flight S 2. If one assumes that during flight there were no discontinuities between successive exposures, these erroneous values may only stem from darkroom processing, presumably from the printing procedure. These errors cannot be uncovered by computation, except one uses a reseau. Thus the errors will take full effect on the computed results. Altering film shrinkage etc. may therefore be indicative of unfavourable results.

Table 3: Results of Spatial Resections for Single Photographs
Quadratic means of photographs covering the whole test area

Flight No. or Day of Flight	Fitting to Fiducial Marks		Rms Error at Image Scale (μm)				
	Equat.	σ_0	Within Point Nest	Before Regress.		After Regress.	
				σ_0	σ_{Check}	σ_0	σ_{Check}
W 1	(1)	5.2	4.0	10.0	8.7	3.7	5.1
	(2)	6.4		9.1	8.1	(5.5)	(6.0)
	(5)	—		* 8.2	7.5		
W 2	(1)	8.9	4.2	9.4	9.2	4.7	6.3
	(2)	7.8		5.8	6.9	(5.7)	(9.0)
	(5)	—		* 5.8	6.5		
S 1	(1)	4.5	4.6	9.5	10.0	5.1	6.4
	(2)	5.9		10.5	10.4	(10.3)	(9.0)
	(5)	—		* 9.6	9.5		
S 2	(1)	15.0	5.8	* 7.7	12.4	5.9	10.0
	(2)	18.3		9.4	13.6	(6.8)	(11.8)
	(5)	—		11.5	16.9		
10.69	(1)	7.5	3.3	6.1	6.7	3.0	5.0
	(2)	7.1		5.0	6.1		
	(5)	—		* 4.6	5.8		

Rms errors in brackets were obtained after exchanging regression polynomials, i. e. applying polynomials from W 1 to W 2 etc.

* Fitting to fiducials that resulted in least residual rms errors

Errors within point nests have been computed against the mean error vector of each nest concerned. Rms errors then are quadratic means of these local vectors. It may be noticed that the latest flight in October 1969 yielded better results in this respect than the OEEPE flights. The geometric condition of the test area must have been better during these flights. Only partly covered signals (by grass etc.) could have lead to larger errors. As the test area has been kept tidy during the whole season one may suspect poor definition in the original film negatives. This could not be checked. Besides that better pointing accuracy did not necessarily result in smaller point error vectors.

From Table 3 it may also be seen that fitting equations (5) gave least rms errors after spatial resection before regression, flight S 2 excepted. Results of WA flights are considerably better than of SWA flights.

Regression polynomials (6) have been computed from residual error vectors from all fully covered photos of each flight. The given rms errors after regression have been obtained after linear conformal transformations (1) of fiducial marks. The results are by far better than the best results obtained before regression. The usefulness of regression polynomials may be demonstrated by applying them after being exchanged. The polynomials deduced from flight W 1 were applied to flight W 2 and vice versa. The same happened to the SWA flights. The results are shown in brackets in the last two columns of Tab. 3. They are also depicted in Fig. 6. It can easily be checked, that regression by exchanged polynomials did not yield as good as by what one may call auto regression. But results as obtained with exchanged polynomials are at least at the bulk of check points significantly better than without regression at all. Only for flight W 2 one gets adverse results. For this flight however one gets only slightest improvements even after auto regression.

Some regression polynomials are graphically presented in Fig. 7 through Fig. 9, the first and the last one showing polynomials after fitting to fiducial marks by transformation (1) for wide and super wide angle flights respectively. The resemblance of the polynomials for each camera from different flights seems to be obvious. Fig. 8 shows the effect of using transformation (5) in place of transformation (1) by reducing the overall amount of additional regression considerably. It is hoped, that regression polynomials can be applied soon to the Oberschwaben block. As the Oberschwaben flights have been performed between related Test Area flights, application of (weighted) means of the polynomials shown in Fig. 7 and 9 seems to be recommendable.

Residual error vectors of some examples have been plotted, see Fig. 10a) through d) and Fig. 11b) and d). Whereas the initial vector fields of both photos after fitting by equations (1) seem to be of comparable structure, (Fig. 10b) and Fig. 11b) respectively) the influence of regression results in appreciable higher accuracy for the photo chosen from flight W 1.

Table 4: Rms Error Vectors After Spatial Resections for Single Photographs [μm]
Mean Error Vectors of Point Nests

Regression	Before			After	
	(1)	(5)	(1)	(1)	(1)
Fitting to Fiducials	(1)	(5)	(1)	(1)	(1)
Applied Filter	None	None	2	None	2/4
Flight W 1 + W 2	7.6	5.7	6.6	4.1	3.7
Flight S 2	11.9	16.6	9.5	8.4	7.6
Flight 10.69	6.7	—	5.3	4.9	4.5

Linear Filtering

Analyses of residual error vectors after spatial resections for single photographs have been made, cf. page 257. Results of OEBPE flights are widely comparable with the findings in ref. [1]. Table 4 shows selected results. As filtering up till now was only applied to mean error vectors of point nests, results of other computations had to be adjusted for the photographs involved. Hence all data of Table 4 look more favourable as far as comparable to Table 3. From Fig. 10e) and f) as well as from Fig. 11e) and f) the effect of filtering can be seen in comparison to resulting error vectors without filtering.

Fig. 13 and 14 may suggest, that a standard filter 4 (Fig. 12) should be recommended, cf. page 258, if wanted at all. Filtering may fail however after regression, if a high noise level has to be anticipated, cf. Fig. 15d) and Table 3.

Generally it may be concluded from Table 4 that a combined application of regression and filtering promises least rms errors. This only confirms results in other branches of applied statistics. If this procedure seems to be too expensive, mere regression may be recommended. Up till now no exchanged filters have been applied as has been done with regression polynomials. On the other hand it could be checked that filters 2 and 4 yielded equal results after regression. Therefore it seems possible to reject the more sophisticated filter 2. This should however be finally decided after application under working conditions (i. e. block adjustment etc.).

Double Point Resections in Space

It has been shown in the previous chapters that the photo coordinates have been prepared by different procedures. For all Test Area flights subsequently double point resections in space have been computed using the bundle method. In each case six evenly distributed points with full control and nine further intersection points were used for establishing the model, cf. Fig. 16. For the remaining points model coordinates have been computed defined as the center of the line of shortest distance between homologous rays. These latter points have served as check points.

Nearly all afore-mentioned different sets of prepared photo coordinates have been used for model computations up till now, the more interesting results being compiled in Tab. 5 and Fig. 17. Comparing Tab. 3 and Tab. 5, one may state for each flight that similar tendencies in alteration of accuracies occur when procedures are changed. This holds true in most cases for standard errors of unit weight σ_0 and also for errors in planimetry σ_p at check points. As the model area only covers about half the photo area all errors mentioned are smaller after double resection in space. The height errors σ_z in the models seem to be independent in amount and tendency from other error values. This may however partly be due to choice of control.

Rms errors within point nests are not invariant against alteration of the procedure. As control points, intersection points, and check points, are differently defined, this might be the source of variations.

Table 5: Rms Errors After Double Resection in Space.

All errors in μm in photo scale except σ_z^* which are given in 10^{-6} of flying height.
Filtering has been performed after taking the means of the coordinates within point nests.
Without this procedure rms errors would come out somewhat larger.

Flight (No. of Modells)	Fitting to Fiducials	Regres- sion	Filter	σ_0	Check Points					Within Point Nests				
					σ_x	σ_y	σ_p	σ_z	σ_z^*	σ_x	σ_y	σ_p	σ_z	σ_z^*
W 1 (8)	(1)	—	—	5.6	5.1	5.6	7.6	10.4	68	2.8	3.7	4.7	6.5	42
	(5)	—	—	5.2	4.7	5.5	7.2	11.3	74	2.7	3.7	4.6	6.7	44
	(1)	Auto Exch.	—	3.2 4.3	3.5 3.8	4.1 4.6	5.4 6.0	9.1 9.5	59 62	2.6 2.7	3.3 3.4	4.2 4.2	5.9 6.2	39 41
W 2 (8)	(1)	Auto	4	2.6	2.6	2.8	3.8	6.5	42	—	—	—	—	—
	(1)	—	—	6.7	5.5	6.2	8.3	11.2	73	3.2	3.9	5.0	6.4	42
	(5)	—	—	5.1	4.2	5.7	7.1	11.8	77	2.9	3.8	4.8	6.6	43
S 1 (2)	(1)	Auto Exch.	—	4.1 4.6	3.6 3.7	4.8 4.9	6.0 6.2	10.5 11.8	69 77	2.7 2.8	3.5 3.6	4.4 4.5	6.2 6.2	41 41
	(1)	Auto	4	3.3	3.1	4.1	5.2	10.6	69	—	—	—	—	—
	(1)	—	—	6.2	4.8	5.3	7.2	10.6	124	2.5	2.9	3.8	3.7	43
S 2 (8)	(1)	Auto Exch.	—	5.8	5.2	4.8	7.0	10.3	120	2.5	2.7	3.7	3.7	43
	(1)	Auto	—	4.4	3.5	4.9	6.0	6.3	74	2.2	2.9	3.7	3.4	40
	(1)	—	—	5.1	4.1	5.6	7.0	5.3	62	2.4	3.0	3.8	3.3	39
S 2 (8)	(1)	Auto	4	3.7	2.1	4.6	5.0	4.5	53	—	—	—	—	—
	(1)	—	—	6.7	5.9	9.2	10.9	12.5	146	3.6	4.2	5.5	4.8	56
	(5)	—	—	9.3	8.8	12.1	15.9	13.0	152	4.3	4.8	6.5	5.5	64
(8)	(1)	Auto Exch.	—	5.4 5.9	5.4 6.9	6.5 7.7	8.4 10.4	7.9 8.2	92 96	3.7 3.7	3.5 3.7	5.1 5.4	4.5 4.6	53 54
	(1)	—	—	—	—	—	—	—	—	—	—	—	—	—
	(1)	Auto	4	4.6	4.6	6.3	8.0	7.5	88	—	—	—	—	—

Further findings are, that the more sophisticated procedures give in general better results. Most impressing is the gain in height accuracy after regression for SWA flights which holds also true when applying exchanged polynomials. In terms of fractions of flying heights the rms errors are only slightly larger than those of WA flights. From these results one might conclude that application of regression should result in an appreciable gain of height accuracy also in block adjustment of SWA photography. Within small areas of the model (point nests) the height accuracies σ_z show the same relation between WA and SWA flights without significant changes for different procedures. Finally there is also a better accuracy in planimetry for SWA flights as compared to Tab. 3 within point nests. Adverse results are obtained for WA photography.

Tab. 5 shows most favourable results for double resections after filtering. These however were obtained using mean coordinates of point nests. One may expect rms errors comparable to those after regression without filtering when single point coordinates are used.

Summary and Conclusions

Image geometry as obtained by the photogrammetric systems used during the Oberschwaben missions has been investigated. To this purpose photography of two flights of the Rheidt Test Area with WA and SWA camera respectively executed before and after performing the Oberschwaben missions were at hand.

The Test Area offers full control with forty-one point nests with three signalized points each. Thus different analytical approaches aiming at an improvement of image geometry could be tested. Steps along this line were

- different transformations of comparator coordinates fitting them to fiducial marks;
- regression by third order polynomials;
- least squares interpolations.

These different procedures have been applied up till now before spatial resections for single photographs and double point resections in space.

The accuracy of fitting to fiducial marks by other than similarity transformations seems to be limited by geometrical definition of the marks. Therefore at least in some cases a worsening effect on the residual errors is to be anticipated.

In general regression by polynomials reduces systematic errors considerably. This effect is also clearly visible if exchanged polynomials are applied. Height errors after regression behaved highly different in WA and SWA models against double point resection before regression. Being of the same magnitude metrically before regression they became rather comparable by fractions of flying height after regression.

Least squares interpolations might result in higher accuracy than other procedures, but only after regression. A scalar filter seems to be sufficient.

Findings for spatial resections for single photographs agree widely with those for double point resections, at least tendentially and as far as comparable.

If one aims at application of regression polynomials in block adjustments or simpler, in double point resections, differences in definition of points or error have to be born in mind. The overall computational procedure (i. e. using bundle method, method of independent models etc.) should as far as possible also be used for evaluation of Test Area photography. The same holds true for distribution of control, intersection points etc. This has not been taken in consideration up till now. So the rms errors of double point resection might be slightly larger than optimum.

In spite of these minor shortcomings at the time being, application of regression polynomials to the Oberschwaben photography is highly recommended. Besides this seems worth while to investigate also other types of regression procedures.

Acknowledgements

The ZEISS PSK mentioned is on loan of the Deutsche Forschungsgemeinschaft. All computations were conducted on an IBM 370/165 of the Gesellschaft für Mathematik und Datenverarbeitung, Birlinghoven-Bonn.

For all computations, existing programs from the Institute of Photogrammetry had to be rewritten in FORTRAN IV, which in the phase being has been done by Dipl.-Ing. Ellenbeck. This involved also recurrence to programs and findings given by Seeber [7] and a paper from Lauer and Wrobel [5].

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- [6] *Leijonhufvud, A.*: On astronomic, photogrammetric and trigonometric refraction. — Rikets almäna Kartverk, Meddelande Nr. 13, Stockholm, 1950.
- [7] *Seeber, G.*: Über das stochastische Verhalten von photographisch bestimmten Stern- und Satellitenkoordinaten. — German Geodetic Commission, Series C, No. 178, Munich, 1972.
- [8] *Ziemann, H.*: Réseau photography in photogrammetry — a review. — National Research Council of Canada, NRC 10408, AP-PR 39, Ottawa, 1968.

RÉSUMÉ:

Le champ d'essai de Rheidt près de Bonn a été aménagé en 1965. On a disposé 41 groupes de points sur une surface de 2 km x 2 km (Fig. 1). En général, chaque groupe comprend 4 points alignés espacés de 3 m. Les coordonnées ont été déterminées par arpentage avec une précision supérieure à ± 10 mm. En vue des clichés aériens, on a signalisé les points à l'aide de plaques blanches de 30 cm x 30 cm au centrage forcé.

Les publications [3] et [4] donnent un compte-rendu des résultats du survol du champ d'essai obtenus jusqu'à présent. Lors de la mission «Oberschwaben», le champ d'essai a été survolé avant et après le vol photographique «Oberschwaben» avec les mêmes systèmes de prise de vues. Il était prévu de recouvrir lors de chaque vol le champ d'essai stéréoscopiquement (4fois) à l'aide de 4 bandes de clichés à l'échelle de 1 : 10 500. Cependant, le vol avant la mission «Oberschwaben» n'a donné qu'une bande de clichés pour la chambre métrique 8,5/23. L'échelle effective des clichés était en partie supérieure à 1 : 10 000, ce qui fait que le champ d'essai n'a pas été recouvert entièrement dans toutes les photos.

Comme on l'a décrit dans les publications citées plus haut, on a effectué les mesures monoscopiques dans des diapositives en verre des prises de vues aériennes à l'aide du PSK de ZEISS. Les résultats quant à la précision après relèvements dans l'espace des clichés pris séparément sont indiqués dans des tables pour chaque vol photographique.

Afin de déterminer et de tenir compte des parts systématiques des vecteurs des erreurs résiduelles, on a calculé des polynômes de régression du 3^e degré pour chaque vol photographique. Les vecteurs des erreurs résiduelles obtenues après la régression montrent le succès de ces efforts.

Le calcul des variances et des covariances à partir des champs des vecteurs d'erreurs donne une indication supplémentaire quant à la géométrie de l'image et l'efficacité de prédictions (interpolation selon la méthode des moindres carrés) au sujet du matériel photographique de la mission «Oberschwaben». Le but du calcul des polynômes de régression et des covariances obtenus à partir des survols du champ d'essai de Rheidt est d'essayer d'améliorer la géométrie de l'image de la mission «Oberschwaben».

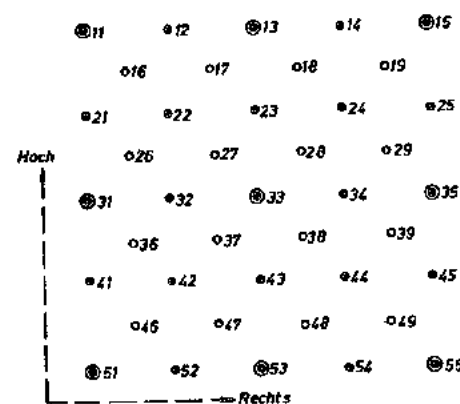


Figure 1 - The Points of the Test Area

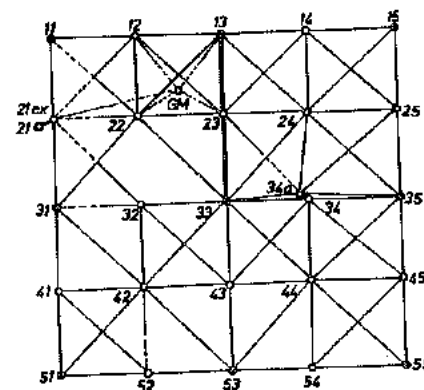


Figure 2 - Trigonometric Observation of the Test Area 1965/66

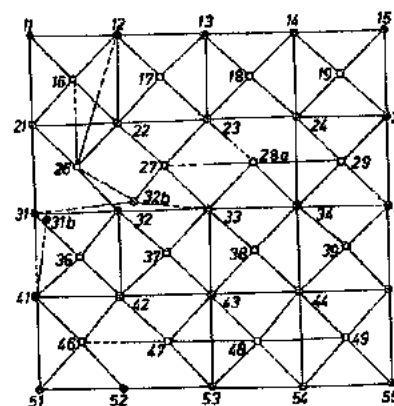


Figure 3 - Densification of the Trigonometric Net 1969

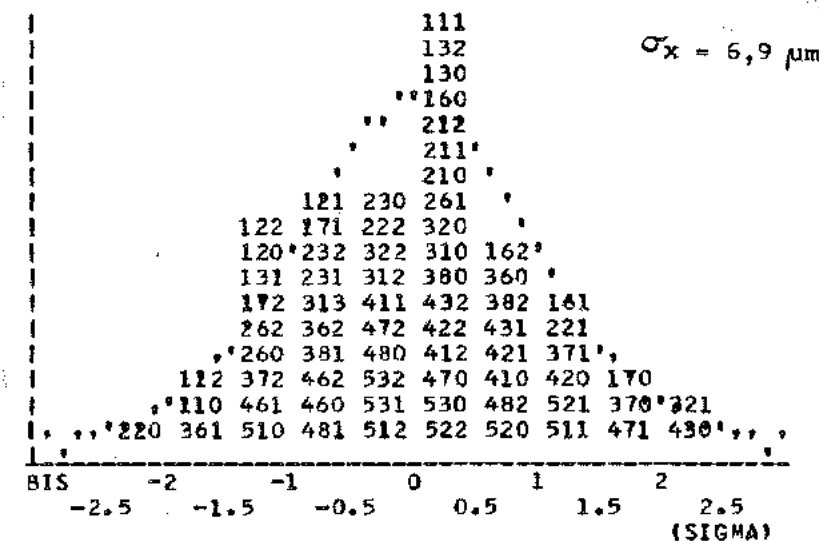


Figure 4 - Histogram of Pointing Differences in x and Related Normal Distribution. Operator W, Photo 249, Flight S 2

RANDVERTEILUNG X

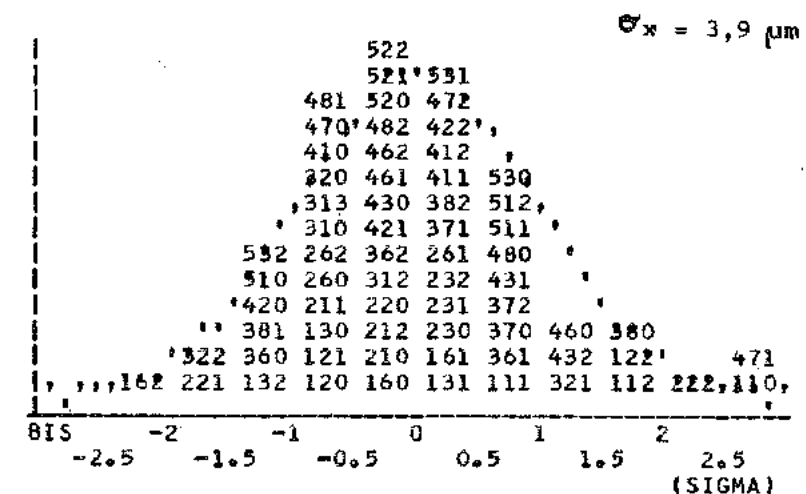


Figure 5 - Histogram of Pointing Differences in x and Related Normal Distribution. Operator S, Photo 249, Flight S 2

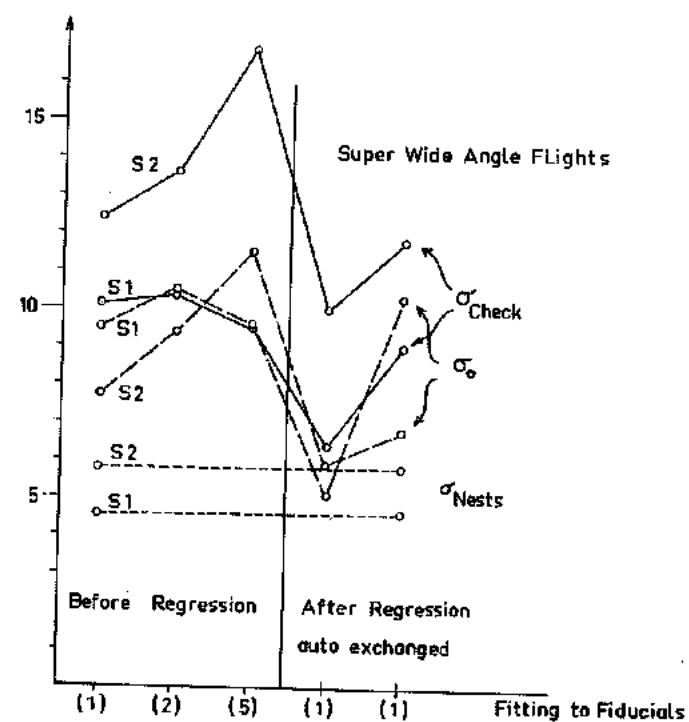
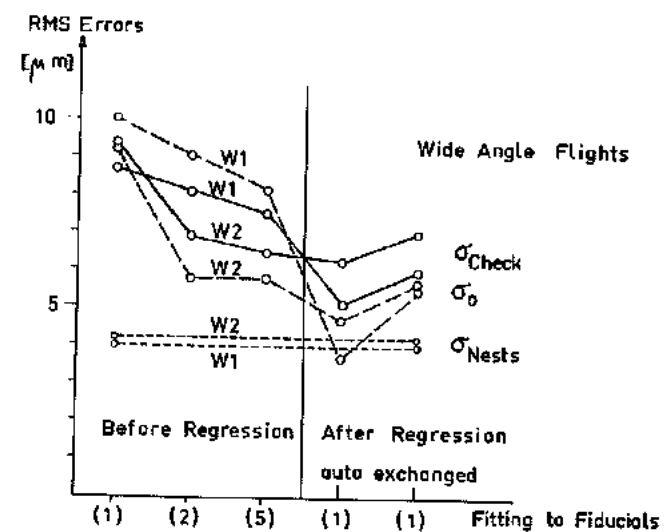


Figure 6 — Results of spatial Resections for Single Photographs

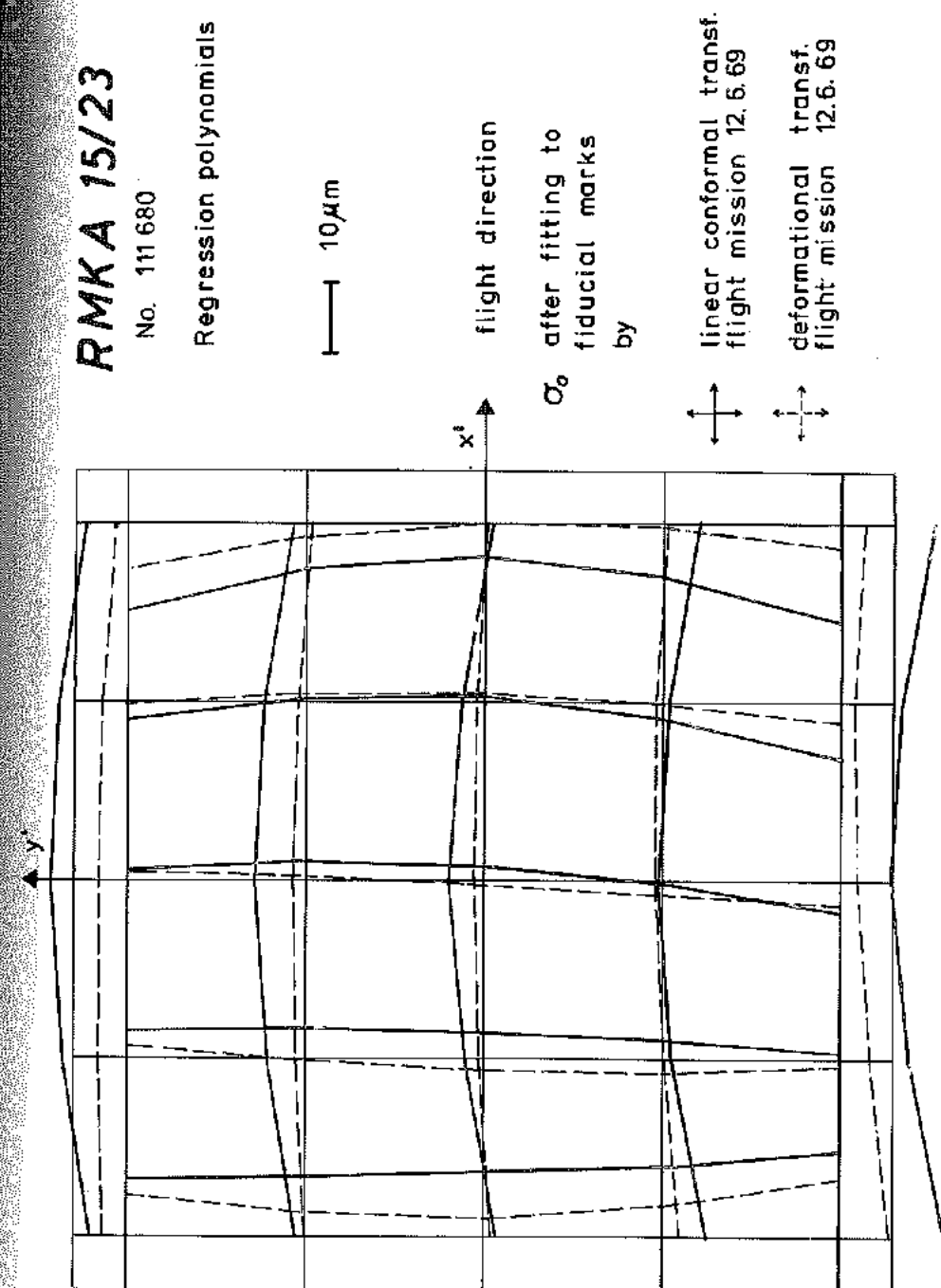


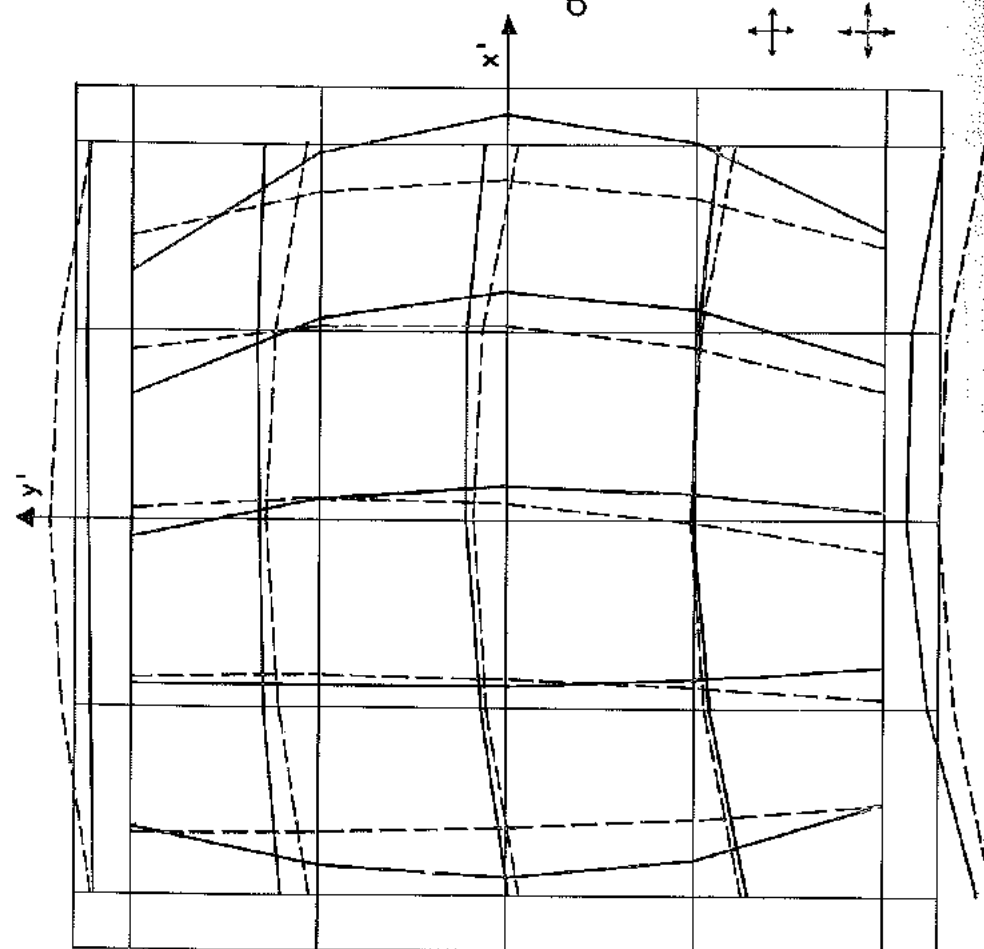
Figure 7

RMKA 15/23

No. 111680

Regression polynomials

10 μ m



σ_0

flight mission 5.4.69

flight mission 12.6.69

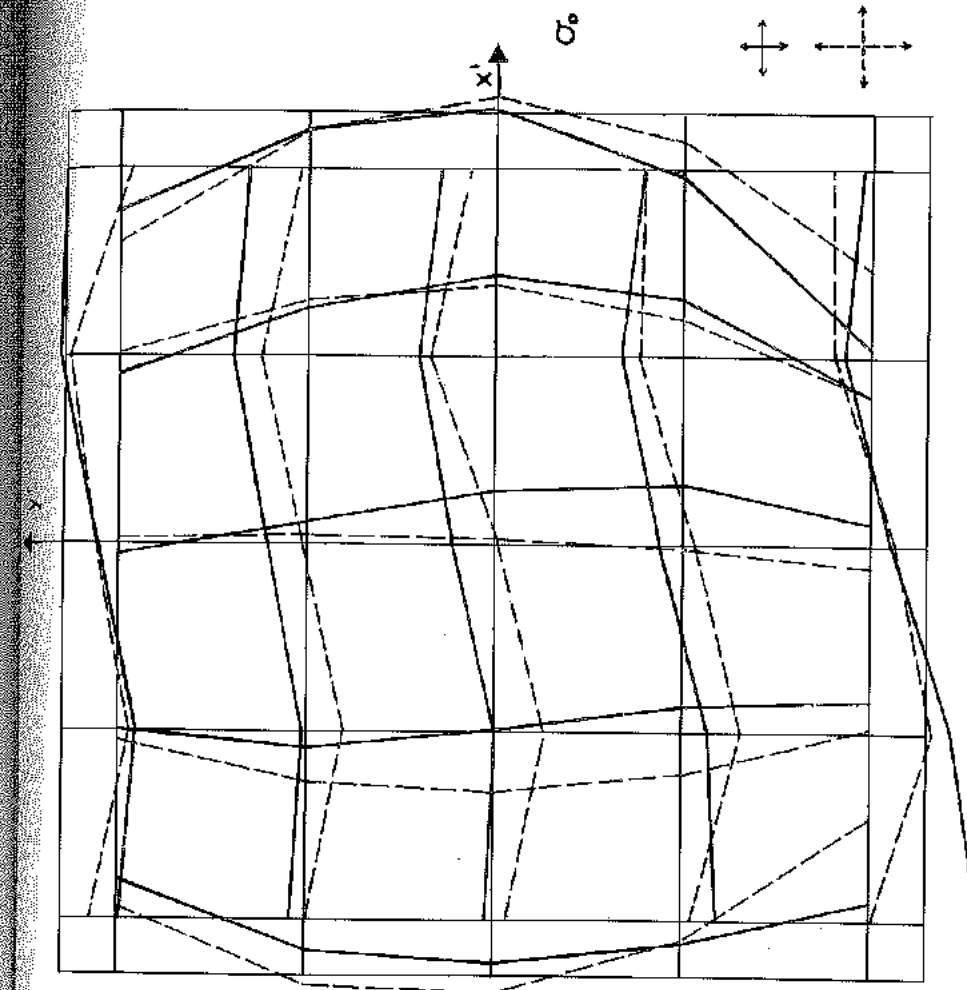
Figure 8

RMKA 8,5/23

No. 111164

Regression polynomials

10 μ m



σ_0

flight mission 5.4.69

flight mission 13.5.69

Figure 9

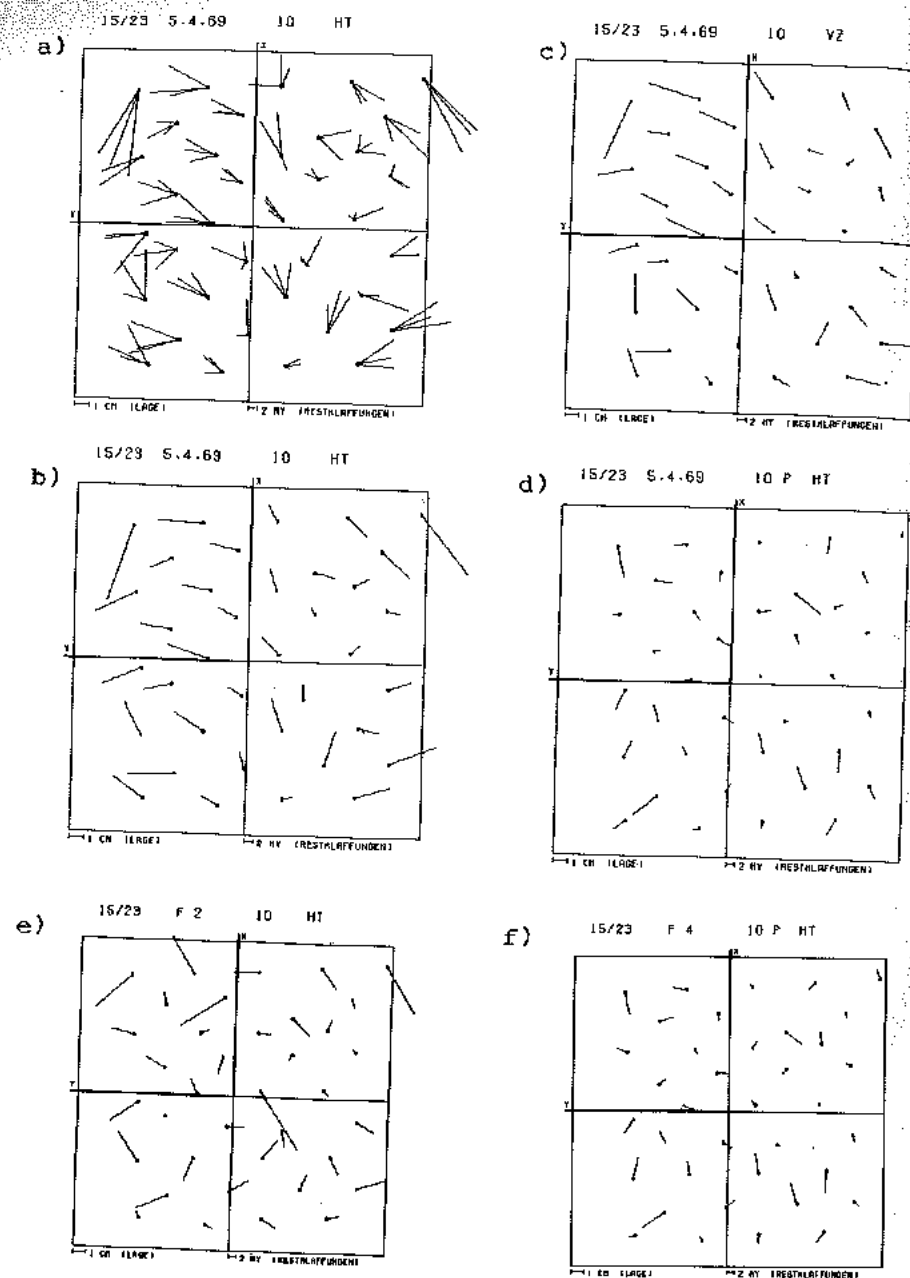


Figure 10 – Residual Error Vectors, Photo 10, Flight W 1
(Definitions see next page)

Definitions:

Only in Fig. a) residual error vectors are shown of all points, Fig. b) through f) only show means for each point nest.

Fitting to Fiducials: Regression: Filter:

a)	(1)	none	—
b)	(1)	none	—
c)	(5)	none	—
d)	(1)	yes	—
e)	(1)	none	2
f)	(1)	yes	4

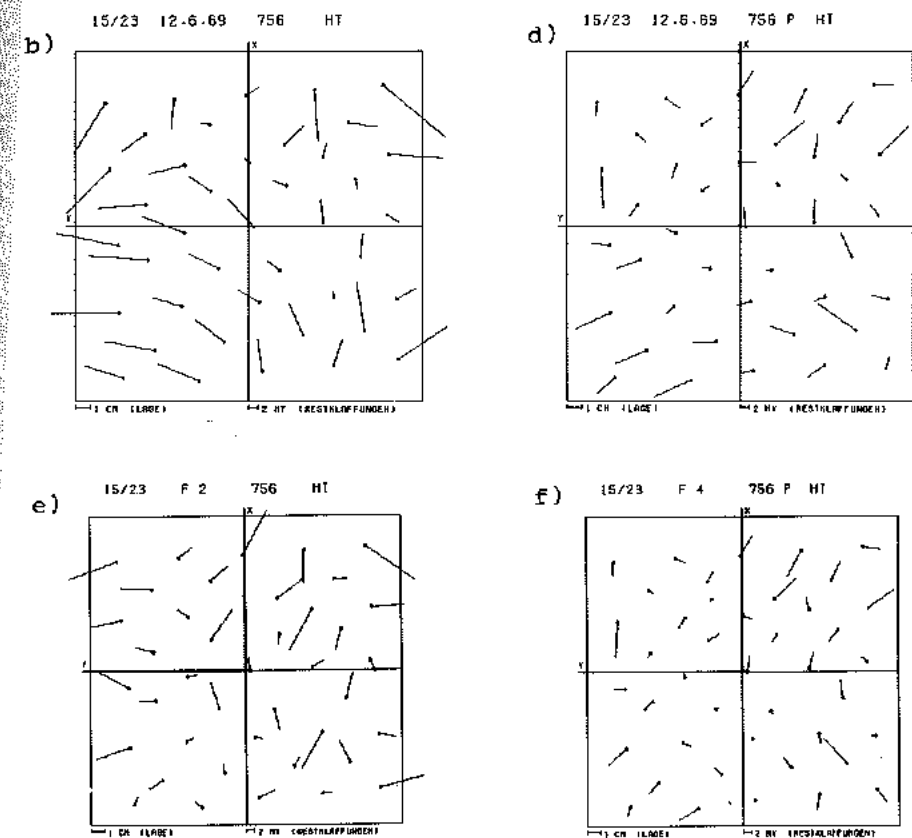


Figure 11 – Residual Error Vectors, Photo 756, Flight W 2
For details cf. Fig. 10
(Fig. 11a) and 11c) have been omitted)

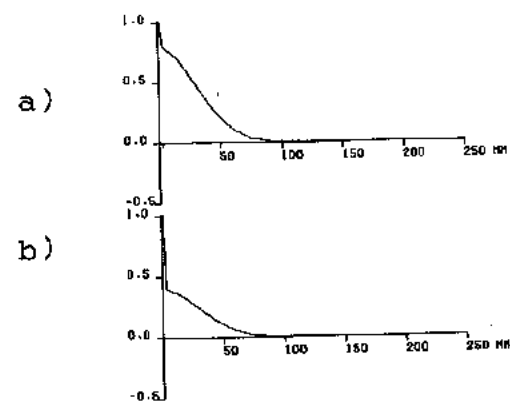


Figure 12 — Scalar Filters for Least Squares Interpolation
 a) Filter 3, as applied before regression
 b) Filter 4, as applied after regression

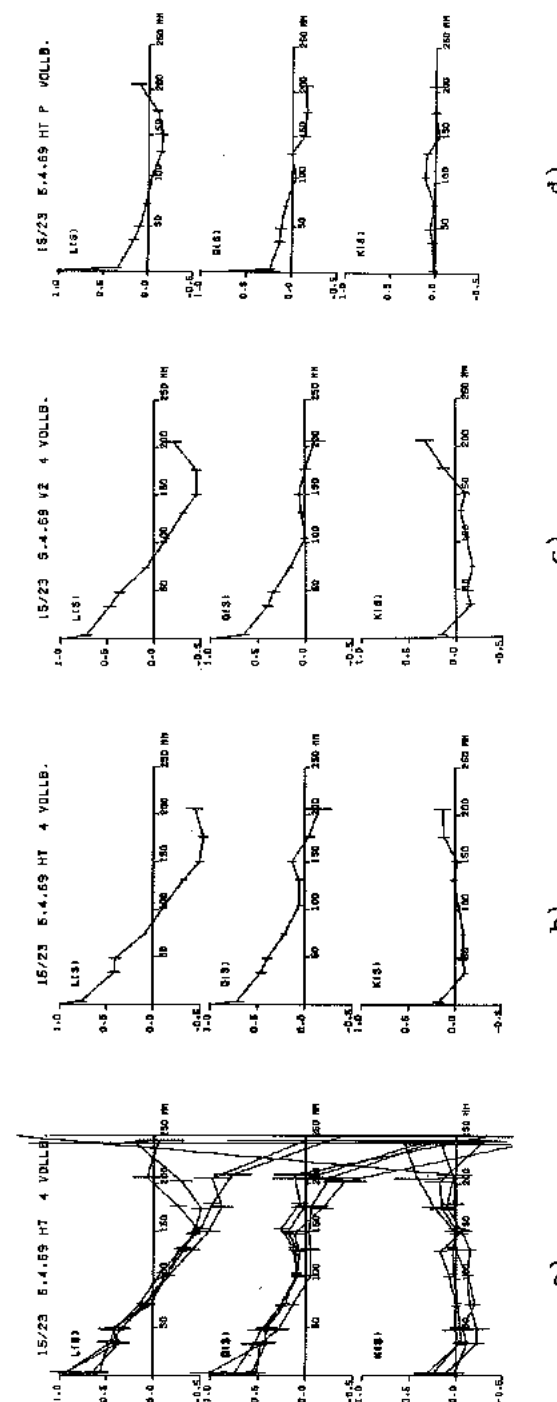
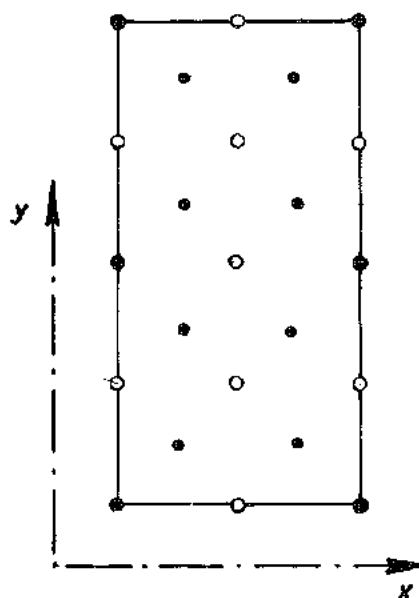


Figure 13 — Covariances, Flight W 1

Fitting to fiducials:

- a) equations (1); four fully covered photographs
- b) equations (1); means from a); before regression
- c) equations (5); before regression
- d) equations (1); after regression

$L(S)$ = Longitudinal Cov.; $Q(S)$ = Lateral Cov.; $K(S)$ = Cross Cov.



Definitions:

- one point within the nest is
- control point
- intersection point
- nest with check points only

Figure 16 — Distribution of Points for Double Resection in Space

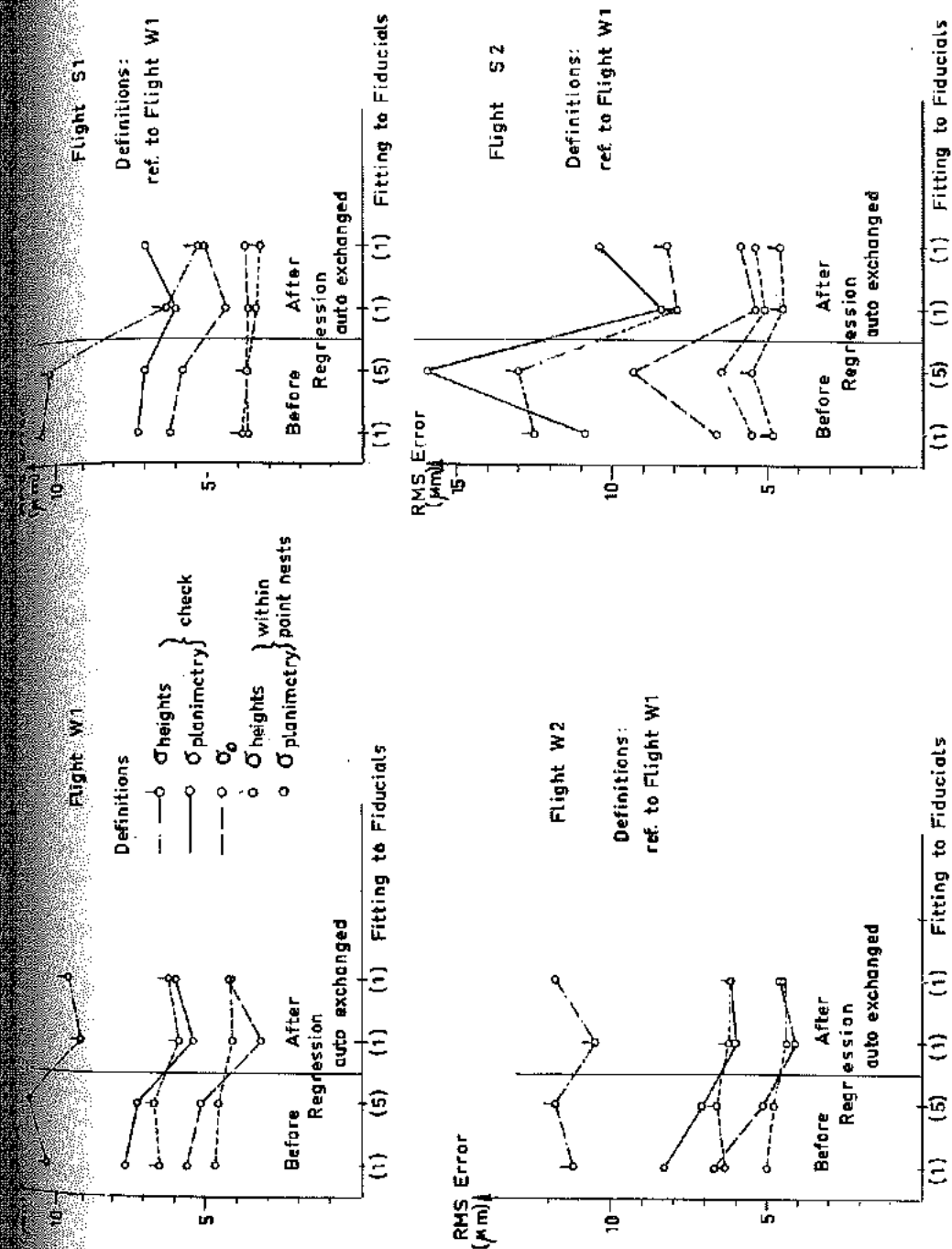


Figure 17 — RMS Errors after Double Resection in Space

Discussion about the Subject 9 under the Chairmanship of Prof. E. Gotthardt

Chairman: Who would like to open the discussion on the interesting paper presented by Prof. Kupfer?

Ackermann: I have a number of questions which perhaps I should pose successively. First of all, examining table 3 in your paper, it appears that there is very little difference between applying an affine or a conformal transformation onto the fiducial marks. We, however, believe that if the affinity is not corrected this should have a considerable influence and I wonder if you have any explanation for the small differences found here?

Kupfer: Bearing in mind that we used a spatial resection for a single photo so that our results are not directly comparable, we have discovered two different types of behaviour. In one case there was close agreement between the results after linear and affine transformations and in the other there was an improvement with the affine fitting.

Ackermann: My second question concerns the fact that you apply the regression polynomials to the same photographs from which these were derived. Can these regression polynomials now be applied to all the photographs of a strip?

Kupfer: If you examine the figures which should be added to table 3*) you will see that there is very little difference in the results when regression polynomials derived from the post-flight measurements were applied to the pre-flight observations.

Ackermann: The similarity between the post- and pre-flight polynomials shows that one can expect this. If I may now proceed to my final question, do you have any explanation for the fact that the longitudinal correlation is stronger than the lateral?

Kupfer: This is only in this case and may vary from flight to flight.

Jerie: It would appear that there is a lot of affinity in the photographs and I would suggest that the fiducial marks are not capable of accurately defining this.

Van der Weele: Table 4*) in your paper only presents the results for WA photography. Have you since been able to compare these results with that for SWA photography?

Kupfer: The results do compare favourably in planimetry as far as an improvement by regression is concerned. However, these results are derived from the special array of 32 groups of check points as given in the test area. Results may differ if one considers different point distributions, especially the distribution of tie points in a block.

Jerie: Going back to the application of regression polynomials, perhaps one could interpolate between the pre- and post-flight polynomials and apply an additional least squares interpolation for local deformations.

Kupfer: I favour the application of polynomials although these might have to be rewritten.

*) The figures and results in question are part of the tables in this completed version of the paper.

Konecny: I consider that Prof. Kupfer has presented us with a very nice approach using regression analysis and would like to ask him whether this approach is simpler than the models of Dr. Torlegard and Duane Brown?

Kupfer: Very good results have been achieved with star photography using simple regression techniques but I admit that we might have to go to more sophisticated least squares techniques for our applications.

Chairman: Thank you Prof. Kupfer for your interesting presentation.

OK 528.738.016.4—187(430—313.5)

The Signal-Field of Baustetten

A Short Report

By R. Förstner, Frankfurt a. M.

SUMMARY:

In the signal-field of Baustetten 64 points have been marked on a marshy meadow at 8 m intervals in form of a grid. The points have been signalized at the same height. The signals were partly horizontal, partly tilted, partly cylindrically curved. Some points have, in addition, been provided with stone dummies. From the existing aerial photographs which had also been used for the test "Oberschwaben", 9 wide-angle and 9 super-wide-angle photographs have been selected. The diapositives have been restituted in a Stereocomparator PSK 1. The test should demonstrate, whether or not the position of the points on the photographs depends on the type of signalization or on the position of the sun and of the camera, respectively. No systematic influences of the parameters mentioned above could be stated. The mean error of a point measured four times amounted to $2.5 \mu\text{m}$ or $4.2 \mu\text{m}$ respectively. Consequently, errors of such an order of magnitude must also be expected in the test of Oberschwaben.

RÉSUMÉ see p. 287

In the test field "Oberschwaben" trigonometric points of the national survey have been used as check points. For signalization, the stone pillars have been painted in white and in addition marked by white plastic plates 80 cm by 80 cm in size. In the centre of the plates there was an opening of 30 cm by 30 cm. About one sixth of the 544 stone pillars was higher than 25 cm (maximum 52 cm). If we take for example a stone pillar of 40 cm in height which is irradiated by the sun from southwest at 50° and which is photographed from southeast also at 50° , then the centre point of the signal surface irradiated by the sun can be displaced by about 12 cm or — with an image scale of 1 : 28 000 — by $4 \mu\text{m}$ in the image, in case the shadows do not contribute to the photographic projection.

The tie points — mostly double points at a distance of 30 m — have been marked with white plastic plates in the size of 60 cm by 60 cm. In order to accelerate the signalization, the plates have mostly renounced levelling the plates, partly they have been bent cylindrically. For this reason it may be imagined that the exact position of the check points on the aerial photograph particularly depends on the azimuth and on the height of the sun as well as on the direction of taking the photographs, and that the exact position of the points with tilted or curved signals depends on the direction of the line of greatest slope or on the curvature of the signals.

In order to determine such influences, a field of signals has been established on a marshy meadow at tie-point No. 604 — i. e. 2 km west of Baustetten — and covering an area of 56 m by 56 m. It comprises a total of $8 \times 8 = 64$ points which have been marked at 8 m distances in the form of a square grid. In conformity with the check points,

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In order to determine such influences, a field of signals has been established on a marshy meadow at tie-point No. 604 — i. e. 2 km west of Baustetten — and covering an area of 56 m by 56 m. It comprises a total of $8 \times 8 = 64$ points which have been marked at exactly 8 m distances in the form of a square grid. In conformity with the check points,

four points each have been provided with stone dummies of 20 cm, 30 cm or 40 cm in height and marked by signals of 80 cm by 80 cm in size. The remaining points, however, have been marked in conformity with the tie-points by 20 horizontal, 16 tilted, 8 slightly curved and 8 strongly curved signals of 60 cm by 60 cm in size. The centres of all plates were located in a horizontal plane. Due to the marshy ground, a good contrast on the aerial photographs could be expected.

From the existing aerial photographs which had also been used for the block test, 9 wide-angle photographs and 9 super-wide-angle photographs have been selected. Thus, the field of signals was shown either in the centre of the image, in a corner, or in the middle of an image border. The diapositives have been restituted in a Stereocomparator PSK 1 in four sets with one run each. Between two sets the images have been turned by 100°. The image coordinates have then been transformed — without change of scale — set by set onto each other, the transformation being linear and conformal. Afterwards the mean values of the coordinates of each point have been computed. All further calculations are based on these mean values.

From the differences between the single values and the arithmetic mean we have computed the mean error of pointing m_0 of a point measured four times. From all aerial photographs we have found on an average:

for wide-angle photographs $m_0' = 1.4 \mu\text{m}$,

for super-wide-angle photographs $m_0'' = 2.2 \mu\text{m}$.

As the field of signals has only a size of 2 mm by 2 mm on the photographs, no systematic errors due to refraction and lens distortion can be expected. In order to eliminate the influence of image tilt, we then performed a projective transformation of the image coordinates into the theoretical grid (terrestrial coordinates). This had been necessary because we have referred the image coordinates to the centre of gravity of the field of points, in order to obtain a greater certainty of calculation. However, we thus have renounced the exact perspective association of both groups of points. From all 9 aerial photographs we have found an average:

for wide-angle photographs $m_1' = 2.5 \mu\text{m}$,

for super-wide-angle photographs $m_1'' = 4.1 \mu\text{m}$.

With the super-wide-angle aerial photographs, the mean error m_1'' increases with rising distance of the field of points from the centre of the image. When we group the errors by type of points, it becomes apparent that the errors resulting with the strongly curved signals exceed the total mean value, but they are smaller with the stone dummies. This may depend on the greater dimensions of the signal plates. From additional studies of the errors for the different kinds of signals we obtained the following results:

- The mean displacement due to the shadow of the stone dummies only amounts to about $1.2 \mu\text{m}$ and $1.5 \mu\text{m}$, respectively. It thus remains within the measuring accuracy.
- Tilted signals are on an average displaced by 1 to $2 \mu\text{m}$ in the direction of the line of greatest slope.
- The slightly and strongly curved signals are, on an average, displaced by 1 to $2 \mu\text{m}$ in the direction of the photographic camera.

With the small extent of the field of signals on the photograph, one is inclined to assume that the errors m_1 are approximately equal to the errors of pointing m_0 . But this is not the case. Repeated measurements made by the same operator, as well as by a second operator, confirmed the values m_0 . But they also confirmed the irregular displacements of different signals by up to $8 \mu\text{m}$, which are no more explicable by the parameters mentioned above, as e. g. tilt or curvature of the signals, altitude of the sun, and the like. Their causes are still unknown. They may be due to other kinds of deformations of the signals or other phenomena during the photographic process.

Anyhow the studies of the field of signals near Baustetten show that errors in the order of magnitude m_1 will appear in the test of Oberschwaben. Thereby one should take into account that the above-mentioned values m_1' and m_1'' refer to the mean taken from 4 individual measurements.

(For further details see the original publication entitled "Das Signalfeld Baustetten" by H. G. Neubauer.)

RÉSUMÉ:

Sur le champ de signaux de Baustetten on a repéré sur un pré marécageux 64 points à une distance de 8 m sous forme d'un réseau, les points étant signalisés à la même hauteur. Les signaux ont été soit horizontaux, soit inclinés, soit de courbure cylindrique. En outre, quelques points ont été munis d'attrapes de pierres. Parmi les photographies aériennes existantes, qui ont été utilisées pour l'essai d'Oberschwaben également, on a choisi 9 prises de vues grand-angulaires et 9 prises de vues super-grand-angulaires. Les diapositives ont été restituées sur un stéréocomparateur PSK 1. L'essai devait montrer si la position des points sur les photos dépend du mode de la signalisation ou de la position du soleil ou de celle de la chambre de prise de vues. On n'a pas constaté des influences systématiques des paramètres mentionnés. L'erreur moyenne d'un point, mesuré quatre fois, s'élève à $2,5 \mu\text{m}$ ou $4,2 \mu\text{m}$, respectivement. Par conséquent, il nous faut compter sur des erreurs du même ordre de grandeur dans l'essai d'Oberschwaben.

OEEPE Oberschwaben Reseau Investigations

(with 10 figures, 13 tables)

By J. Visser, F. Leberl and J. Kure, Enschede

RÉSUMÉ see p. 316

1. Introduction

In the framework of the larger experiment on aerial triangulation, the OEEPE wished to include an additional experiment with reseau photography. To that end, the five alternate strips of the non-reseau wide-angle block Oberschwaben, that had been photographed with 90 % forward overlap, viz. strips 6, 8, 10, 12 and 14, were re-photographed with the Zeiss Reseau camera RMK AR 15/23, also at the scale 1 : 28,000 and again with 90 % forward overlap, and with 20 % side lap.

At ITC, which acted as a pilot centre for Commission F of the OEEPE (i.e. the commission that deals with fundamental problems in photogrammetry), two investigations have been carried out with the above reseau photography. The first investigation concerned monocular reseau observations in some 37 glass positives, the second investigation concerned reseau observations during stereo-restitution on the Wild Stereocomparator of glass positives of some 100 stereomodels. The results of these two investigations, as carried out in 1970-71, were published in *Photogrammetria*, 27 (1971), pages 169-199. The main findings of these investigations are given in section 2.

Meanwhile, at the Photogrammetric Institute of Stuttgart University, West Germany, K. Kraus investigated the method of linear least squares interpolation, as originally developed some 30 years ago, and further developed by H. Moritz for application in gravimetry (1970). Kraus applied this method to photogrammetric interpolation problems, including the problem of correction for film deformation using reseau photography.

To that end, he used the ITC monocular observations of three OEEPE reseau plates (nos. 302, 358 and 412 of strip 10) and the ITC stereo observations on two stereomodels, formed by another three OEEPE reseau plates, covering the test field Rheidt (nos. 39, 43 and 48).

The results of the linear least squares interpolations were published in *Photogrammetric Engineering*, May 1972, pages 487-493 (as for the monocular observations) and in *Photogrammetria*, 28 (Nov. 1972), pages 179 and 180 (as for the stereo observations). The main findings of these two investigations are given in section 3.

At ITC, subsequently, the authors of the present paper rounded off the investigations by including somewhat less superior photo material in their research. So far, the OEEPE Oberschwaben photography had shown to be "too good", from the geometrical point of view, to improve it significantly by the use of the reseau. To obtain less superior material, ITC ordered, at the end of 1972, besides new glass diapositives of the original negatives (then taken 3 $\frac{1}{2}$ years ago), also positives on (polyester-)film.

After observing the film positives, the transparencies were "mistreated" by placing them on a heated glass plate and subsequently were re-observed. The results of these observations are presented in section 4.

2. The ITC Reseau Investigations 1970-1971

2.1. Monocular Reseau Investigation

In the central strip, run 10, 25 plates were selected thus forming a strip with approximately 60 % forward overlap. Three plates were selected at the beginning, middle and end of each of the other four strips.

The monocular reseau measurements were carried out mainly on the Wild STK 1 Stereo-comparator, using a black circular measuring mark with a diameter of 60 microns.

All 524 reseau crosses were observed in 5 plates of strip 10. In the other 32 plates an irregular 2cm- and 3cm-grid of 81 points was measured, in three instances both backwards and forwards in order to estimate the pointing accuracy. The distribution of these 81 points can be seen in the vector diagrams, figures 4-6. The standard deviation of a single pointing appeared to be:

$$\sigma_x = 2.0\mu; \sigma_y = 3.1\mu.$$

The monocular reseau measurements were transformed, by linear conformal transformation, onto the ideal grid of the 81 points. The 37 vector diagrams of residual distortions, obtained after conformal transformation of each of the 37 plates showed:

- that the residual distortions were fairly small: very small in the central square of 12 x 12 cm; outside this square the order of magnitude of the vectors is some 7 microns, with maximum values of 20 to 24 microns along the edge of the photographs. See figures 1-6*).
- that the distortion pattern was fairly systematic throughout a strip. In order to show this in numerical form, average distortion values were computed per point (81 points), for each strip. In the case of strip 10, the averages could thus be determined from 25 plates (and in the other strips from 3 plates each). Comparing now, per point, the distortion values in each of the 25 plates with the corresponding average values, a "noise" of 2.6 μ in x , and 2.8 μ in y , is found. For the other four strips, similar values were found. Bearing in mind the accuracy of the pointing error ($\sigma_x = 2.0\mu$; $\sigma_y = 3.1\mu$), it is clear that the variation of the distortion pattern within a strip is insignificantly small.
- that, although the distortion patterns remained fairly constant throughout a strip, and to some extent between adjacent strips, there was a marked difference between the patterns of the two outer strips 6 and 14.

In order to find the minimum number of reseau points, and the optimum distribution of these points, as required for correcting the systematic components of the film deformation

*) In the present paper, only some of the figures as published in Photogrammetria 27 (1971) are included.

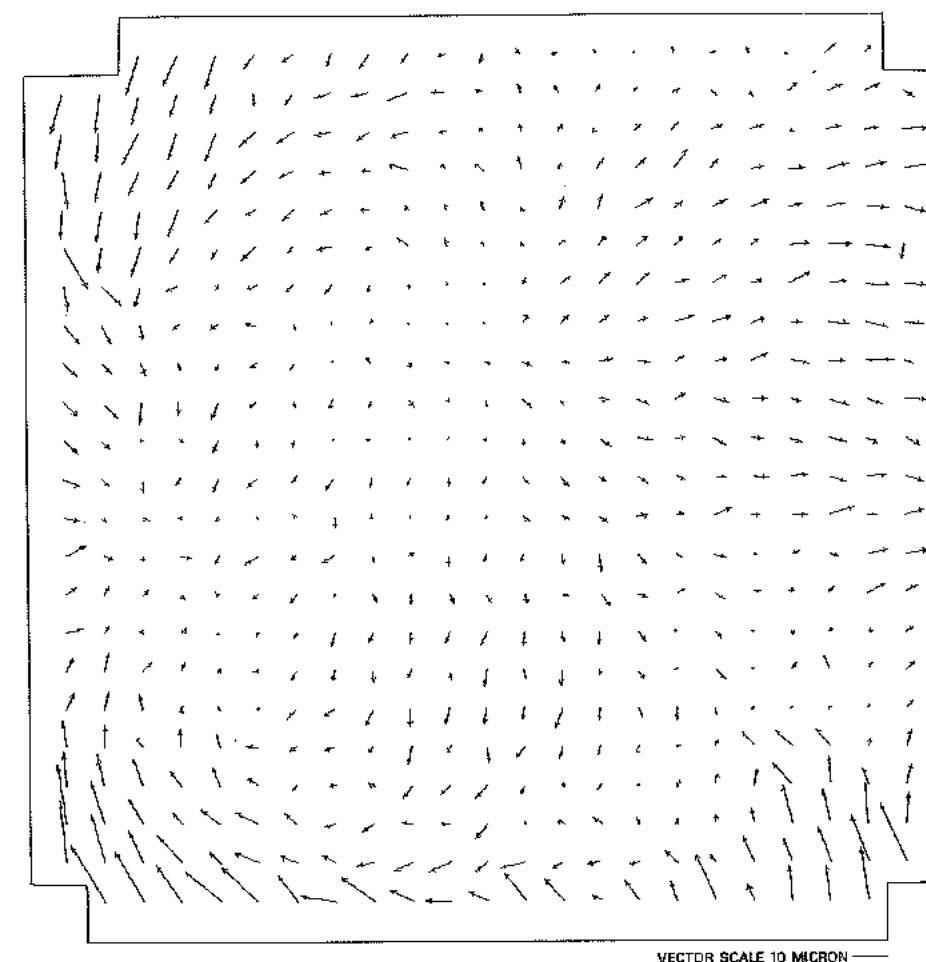


Figure 1 - Residual distortion-plate 302

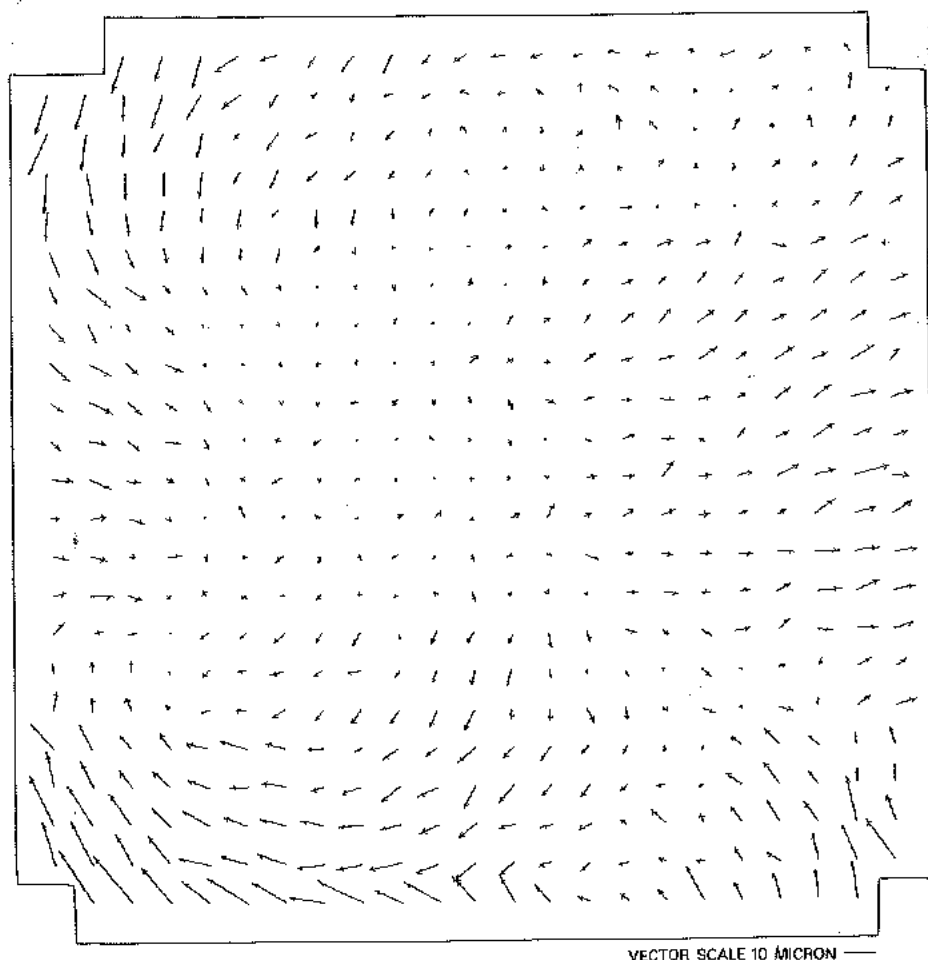


Figure 2 — Residual distortion-plate 358

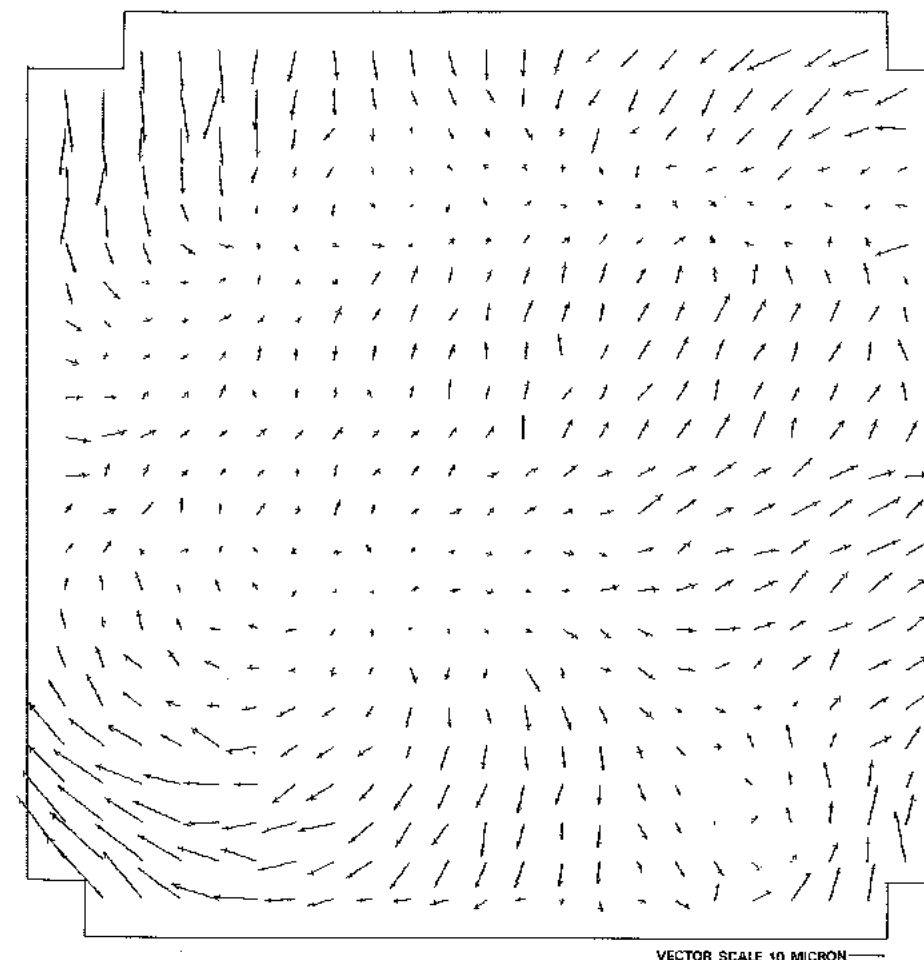
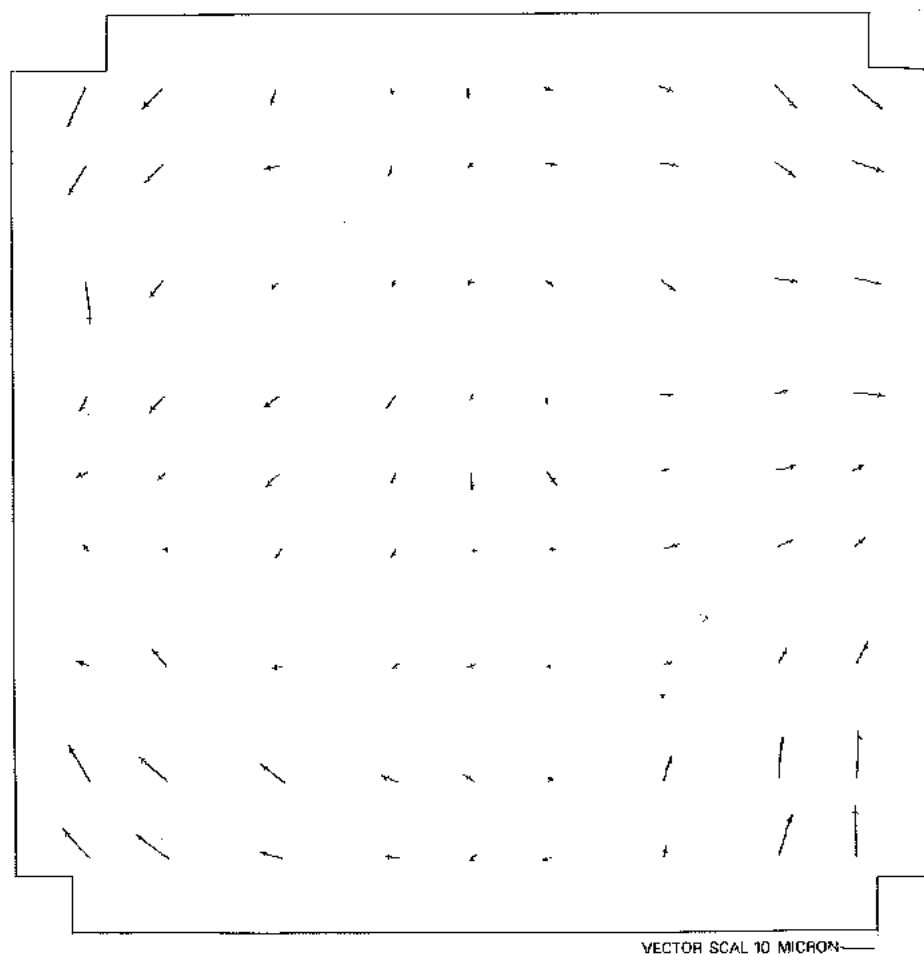
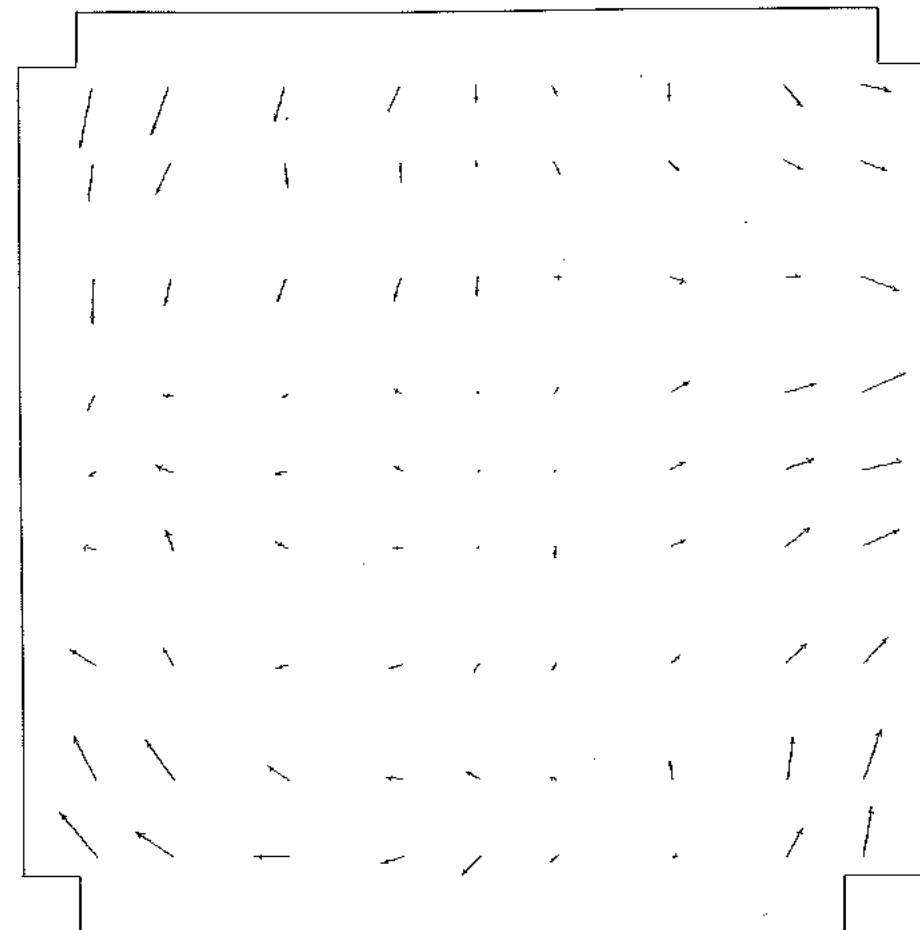


Figure 3 — Residual distortion-plate 412



VECTOR SCALE 10 MICRON

Figure 4 — Average distortion-strip 6



VECTOR SCALE 10 MICRON

Figure 5 — Average distortion-strip 8

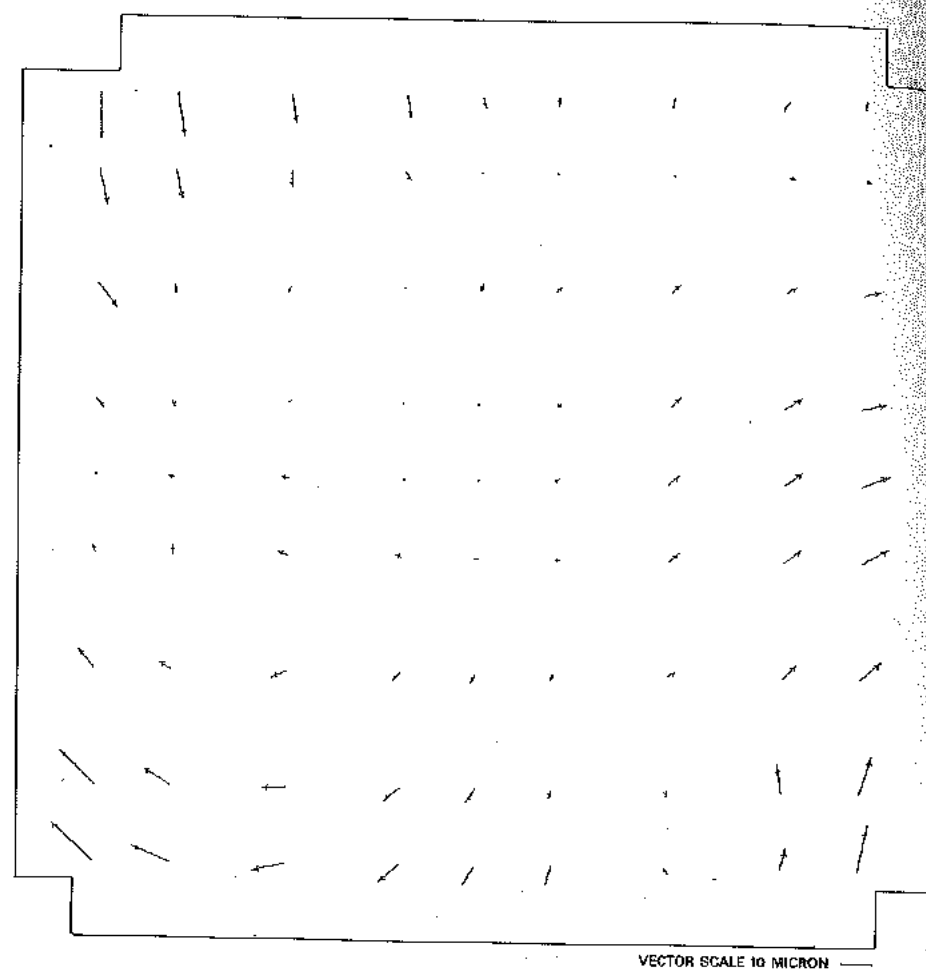


Figure 6 - Average distortion-strip 10

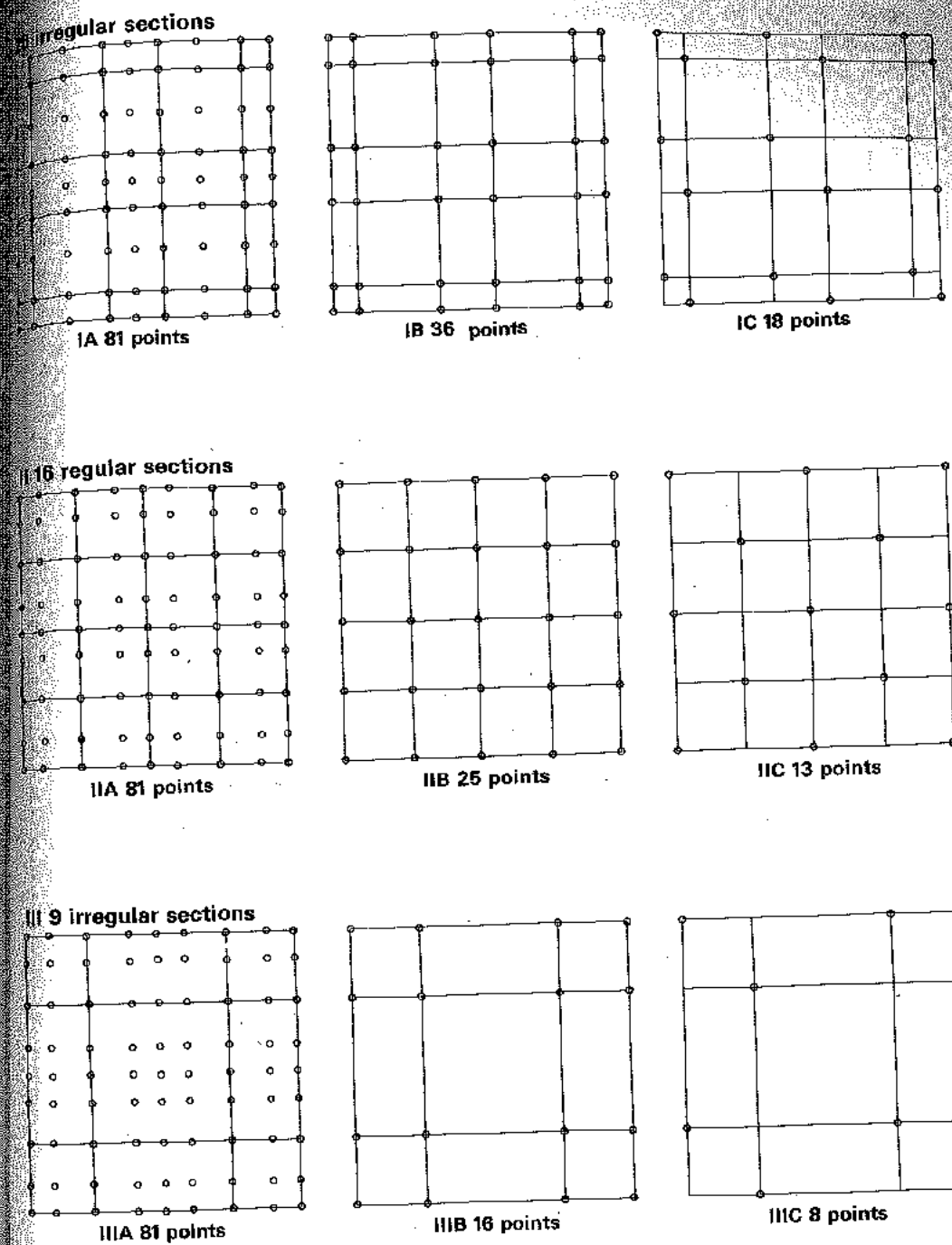


Figure 7 - Division of plate into sections

Table 1 - Section means per strip, in microns

Sect.	x-component					y-component				
	St. 6	St. 8	St. 10	St. 12	St. 14	St. 6	St. 8	St. 10	St. 12	St. 14
1	-0.3	-1.6	-0.5	-8.6	-5.9	-12.9	-12.1	-10.2	-14.2	-9.6
2	0.6	0.4	1.5	-2.6	-2.5	-5.7	-4.8	-1.6	-1.7	0.8
3	3.4	3.8	3.1	3.0	1.3	-0.6	0.6	4.2	6.7	6.6
4	6.7	7.9	7.5	11.9	8.8	-4.0	-3.8	-0.3	1.9	0.2
5	7.3	8.4	9.5	16.6	12.2	-8.1	-9.7	-7.0	-4.4	-6.0
6	2.6	4.1	4.4	10.2	7.0	-5.3	-8.9	-5.7	-7.1	-9.0
7	2.0	3.7	2.4	5.5	3.6	-1.3	-3.0	0.4	-0.7	-2.8
8	1.1	2.9	1.6	1.5	1.7	-0.8	1.5	1.9	0.7	-2.3
9	-1.7	-0.4	0.0	-4.1	-2.1	-6.0	-3.9	-4.0	-7.7	-8.2
10	-2.3	-5.5	-3.0	-7.8	-6.8	-7.5	-8.0	-6.2	-11.6	-9.6
11	-3.8	-9.1	-6.0	-7.9	-6.6	-2.1	-3.4	-3.7	-9.3	-7.2
12	-3.2	-4.0	-3.6	-4.2	-1.9	-0.5	0.6	-1.5	-5.4	-5.8
13	0.0	1.4	-0.5	-0.6	1.3	1.9	1.0	0.3	-2.9	-3.2
14	0.8	2.1	0.3	3.3	3.2	1.4	-3.4	-1.1	-4.4	-3.7
15	0.7	1.9	-0.9	1.4	3.2	1.7	-0.7	0.0	-1.9	-1.9
16	3.5	3.0	-1.7	-2.3	-1.1	4.4	6.1	5.2	5.9	7.0
17	3.6	3.2	-0.5	-0.4	-1.2	4.1	3.4	4.1	3.5	5.3
18	-0.5	0.2	-0.2	0.2	-0.1	2.0	1.2	0.4	1.0	0.7
19	-3.7	-5.2	-2.7	-3.4	-3.3	-0.4	0.8	-1.4	-1.5	-1.8
20	-6.3	-8.0	-3.2	-1.5	-2.7	0.9	0.4	-1.1	-1.4	-2.9
21	-7.1	-6.2	-0.1	3.3	0.0	4.6	4.5	1.7	7.9	3.3
22	-4.5	-3.4	0.3	1.4	-0.5	3.2	4.9	1.5	7.7	4.4
23	-1.6	0.6	-0.9	-1.8	-3.6	1.0	5.4	3.1	6.9	4.6
24	2.5	3.6	-1.3	-2.5	-4.7	3.2	7.5	6.7	9.4	9.1
25	4.7	3.8	-0.9	-3.8	-6.3	7.1	12.3	10.3	13.3	15.3
Mean	3.0	3.8	2.3	4.4	3.7	3.6	4.5	3.3	5.6	5.3

tion, the plate was sub-divided into sections. The average distortion within each section was denoted as the systematic distortion component and the variation within each section as the random component. In this experiment, three main section divisions were examined, whereby the 20cm-square-grid was divided into respectively 25, 16 and 9 sections (see figure 7), while the following point distributions were tested:

25 sections: 81, 36 and 18 points

16 sections: 81, 25 and 13 points

9 sections: 81, 16 and 8 points

Using the average plate of strip 10, the section means were determined separately for the 9 section divisions, and the variations within each section were computed. From the results, the following conclusions can be drawn:

- d. for a given number of sections (either 25, 16 or 9), the number of points used to define the systematic component is not significant. Maximum vector errors of 2.8μ , 1.8μ and 2.5μ , respectively, were found between the use of 81 or 18 points (in the case of 25 sections), 81 or 13 points (16 sections), 81 or 8 points (9 sections), respectively, to establish the section means.
- e. it is obvious that the "random component" will increase with an increase in the size of the section, but the differences found in this investigation between using either 25, 16 or 9 sections were hardly significant. For the minimum number of points used for each case (18, 13 and 8 points, respectively), the maximum vector errors increased from 7.1μ (25 sections) to 11.30μ (9 sections).

Under point c above, mention was made that, although the distortion patterns remained fairly constant throughout a strip, there was some variation from strip to strip, and particularly between the two outer strips 6 and 14. In order to show this in numerical form, the section means were computed, for a division into 25 sections and using 18 points, for each strip, and these means were compared in table 1. From table 1 it can be seen that there are differences up to 11μ between section means computed from average plates of separate strips and that it is thus better not to adopt an average distortion pattern for the whole block.

To conclude this investigation, it was decided to simulate a practical example whereby the residual distortion would be determined for a number of plates in the strip, say at the beginning, middle and end, followed by the computation of the average distortion per point (18 points) and per section (25 sections). These section means would now be considered indicative of the systematic component for all plates in the strip, and it then remained to be seen whether the residual random components of individual plates are tolerable or not.

Strip 10 was selected since its section means as given in table 1 were obtained from the point averages of 25 plates. The standard deviations of the random components of three plates were as follows:

Table 2

Strip	Plate	n	Stand. Dev.	
			x	y
10	302	169	3.0 μ	3.2 μ
10	362	169	2.9	3.8
10	412	169	2.8	3.7
10	Total	507	2.9 μ	3.6 μ

In assessing these results, they should be compared with the 2.6 and 2.8 μ found above (under point b) for the difference (the "noise") between the distortions in individual plates of strip 10, and the average distortions, using 81 reference points rather than 18.

Conclusion: the rather small decrease in accuracy obviously justifies this sub-division into sections and the adoption of the section means as representative of the distortion within that local area.

Recommendations on the use of the reseau: (1) Before commencing with an aerial triangulation, select at least three plates in each strip, preferably at the beginning, middle and end, and measure a limited but selected number of reseau points in each plate (i.e., see figure 12, where 8, 13 or 18 points could be measured depending on whether one desires 9, 16 or 25 sections, respectively). (2) The analyses of these three plates per strip will either confirm that the distortion is small and uniform, in which case the reseau need not be used at all during the triangulation phase, or that the distortion is significant and irregular and that it must be corrected. In the latter case, one will first have to examine whether the division into sections is sufficient — this will automatically yield the minimum number of points required to define the systematic component within each section, and only this limited number of points need then be measured in each plate during the triangulation phase.

2.2. Use of Reseau in Analytical Stereo-restitution

In his paper as presented at the Image Deformation Conference, June 1971, Ottawa (Sources of Image Deformation, Photogrammetric Engineering 1971, pages 1259–1265), H. Ziemann describes all sources of image deformation. "A number of examples are given for deformation of unusual size caused by different sources. These examples demonstrate that a dense centrally projected reseau exposed simultaneously with the original negative is the only solution that permits checking the image deformation at any time caused by any source. A reseau can also be utilized to correct any deformation of the image to an extent limited only by its density, if analytical procedures are used".

A systematic procedure whereby the distortion in each observed point in each individual photograph is located, and corrected for, will of course require some more effort in the observational, as well as in the computational, procedure.

In analytical stereo-restitution, either of single models (on behalf of numerical determination of terrain coordinates of selected points such as signalized cadastral boundary corners, etc.), or of strips and blocks (in aerial triangulation), points are observed on a comparator.

In the case of aerial triangulation on a stereocomparator, the number of points to be observed per stereo-model is usually:

monocular observations: 2 x 4 fiducial marks

stereoscopic observations: 6 orientation points, 6–8 minor control points, 2 x 2 scale-transfer points.

According to the statistics, collected at ITC from analytical triangulation of more than 5,000 stereo-models, the observation time on a stereocomparator is about 45 minutes per stereo-model (including teabreaks, etc., etc.).

In order to be able to trace, and to correct for, distortion errors in the points observed, immediately after the usual stereoscopic observation of a point, the nearest reseau cross in each of the two photographs may be observed (monocularly).

For highest accuracy, in each photograph the four reseau crosses that surround the observation point concerned, should be measured and the distortion of the point be interpolated from the distortions found in the four crosses. As, however, with a centimetre-reseau, the nearest cross is never more than 5 mm away in each coordinate direction, we considered it sufficient to observe in each photo only the nearest cross.

The additional time for the monocular observations of the nearest cross in each photo has appeared to be 27 seconds per point observed; for a normal stereo-model in the case of aerial triangulation thus an additional 7 $\frac{1}{2}$ minutes is required.

Experiments

In ITC, analytical aerial triangulation has been carried out on the Wild Stereocomparator of three of the four sub-strips of models with 60 % forward overlap, that could be formed from the photographs of strip 10 which had 90 % forward overlap: strips 10–1, 10–3 and 10–4, respectively, with 24, 23 and 22 models, respectively. The triangulation was carried out with contact diapositives on glass.

The measuring and computation programmes of the ITC Stereo-Reseau method have been selected in such a way that all reseau-cross observations can be eliminated from the original observations. Each stereo-model etc. can thus be computed with or without use of the reseau observations.

Results after strip-adjustment according to Ackermann:

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Strip	Plate	n	Stand. Dev.	
			x	y
10	302	169	3.0 μ	3.2 μ
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The additional time for the monocular observations of the nearest cross in each photo has appeared to be 27 seconds per point observed; for a normal stereo-model in the case of aerial triangulation thus an additional 7 $\frac{1}{2}$ minutes is required.

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In ITC, analytical aerial triangulation has been carried out on the Wild Stereocomparator of three of the four sub-strips of models with 60 % forward overlap, that could be formed from the photographs of strip 10 which had 90 % forward overlap: strips 10–1, 10–3 and 10–4, respectively, with 24, 23 and 22 models, respectively. The triangulation was carried out with contact diapositives on glass.

The measuring and computation programmes of the ITC Stereo-Reseau method have been selected in such a way that all reseau-cross observations can be eliminated from the original observations. Each stereo-model etc. can thus be computed with or without use of the reseau observations.

Results after strip-adjustment according to Ackermann:

Table 3 — Short bridging distances

Strip	Total nr. of models	Nr. of models bridged between ground control		With use of reseau crosses		Without use of reseau crosses	
		Plan	Height	Plan (cm) X Y	Height (cm)	Plan (cm) X Y	Height (cm)
10-1 first part	9	1.5	1.5	22 22	51	24 28	40
10-1 sec. part	9	1.5	1	19 15	36	18 18	23
10-3	11	2	2	27 29	30	23 23	46
10-4	14	3.5	2	32 34	53	28 26	69
				25 25	42	23 24	45

Conclusions: no significant difference between the two cases.

For these very short bridging distances:

$\sigma_X = \sigma_Y = 24 \text{ cm} = 9 \text{ microns}$ at the scale of photography

$\sigma_H = 43 \text{ cm} = 0.10 \text{ ‰}$ of the flying height

Table 4 — Longer bridging distances

Strip	Total nr. of models	Nr. of models bridged between ground control		With use of reseau crosses		Without use of reseau crosses	
		Plan	Height	Plan (cm) X Y	Height (cm)	Plan (cm) X Y	Height (cm)
10-1 first part	9	4.5	4.5	25 28	64	28 32	55
10-1 sec. part	9	4.5	4.5	24 26	49	25 25	26
10-3	11	5.5	5.5	34 33	67	28 31	87
10-4	14	7	7	33 39	49	27 26	75
				29 32	57	27 29	61

Conclusions: no significant difference between the two cases.

For these longer bridging distances:

$\sigma_X = 28 \text{ cm}$, $\sigma_Y = 30 \text{ cm} = 10 \text{ microns}$ and 11 microns respectively, at the scale of photography

$\sigma_H = 59 \text{ cm} = 0.14 \text{ ‰}$ of the flying height

Absolute accuracy of numerical restitution.

The same camera was used to photograph the test field Rheidt (close to Bonn) at a scale 1 : 10,500, both before and after taking the reseau photography of the Oberschwaben block.

Some models of this test field were observed on the Wild Stereocomparator. Each model was oriented absolutely with the help of 14 control points. With some 50 further check points, the results as shown in table 5 were found.

Table 5 — Absolute accuracy of numerical restitution

Model	With use of reseau crosses	Without use of reseau crosses
39/43	$\sigma_X = 0.09 \text{ m} = 9 \mu$ $\sigma_Y = 0.08 \text{ m} = 8 \mu$ $\sigma_H = 0.13 \text{ m} = 0.08 \text{ ‰} Z$	$\sigma_X = 0.08 \text{ m} = 8 \mu$ $\sigma_Y = 0.08 \text{ m} = 8 \mu$ $\sigma_H = 0.16 \text{ m} = 0.10 \text{ ‰} Z$
43/48	$\sigma_X = 0.08 \text{ m} = 8 \mu$ $\sigma_Y = 0.07 \text{ m} = 7 \mu$ $\sigma_H = 0.12 \text{ m} = 0.075 \text{ ‰} Z$	$\sigma_X = 0.07 \text{ m} = 7 \mu$ $\sigma_Y = 0.09 \text{ m} = 9 \mu$ $\sigma_H = 0.12 \text{ m} = 0.075 \text{ ‰} Z$

Conclusion: no significant difference between the two cases

The reseau, of course, can only be used for the location and correction of the distortion of the aerial photographs that is caused by an unflatness of the emulsion at the moment of exposure, by film distortion and by the photographic processing of the negatives and positives. Lens errors and refraction cannot be corrected for by the use of the reseau.

The above results show that, in the Oberschwaben reseau photography, the distortion due to unflatness, film distortion and photographic processing is so small that the improvement of the geometry of the images, effected by the correction for that distortion in each point observed, is just compensated by the additional errors in the pointing to the reseau crosses (together with the variation in the distortion over distances up to 5 mm).

3. Reseau Investigations at Stuttgart University

3.1. K. Kraus (Phot. Eng. 1972) introduced a new definition of the irregular (or random) and systematic components of the film deformation. Contrary to the analysis described in section 2.1 above, the new definition works without subdividing the photograph into smaller units.

Kraus states that, "in the description of a vector field, the following items are of particular interest:

- Absolute magnitude of the vectors
- Separation of systematic and irregular components
- The range of systematic effects and their variation as a function of distance.

It is assumed that no correlation occurs between the x and y component of the two-dimensional vectors. We then search for concepts or terms describing mathematically the three items, separately for x - and y -directions.

The absolute magnitude of the reseau image errors l_i in the x - or y -direction can be described by the variance $V = M [l_i l_i]$, where M means mean square value. The variance contains the irregular as well as the systematic components of the vector field.

We also introduce the covariance $C(s_{ik}) = M [l_i l_k]$, which is the mean-square value of the cross product of the errors $l_i l_k$ of all reseau points P_i and P_k . The covariance contains the systematic part of the vector field, the irregular part being filtered out. The empirically determined covariances (for some groups of approximately equal distances s) are a perfect indicator on how the common systematic effects between the points decrease with increasing distance between them.

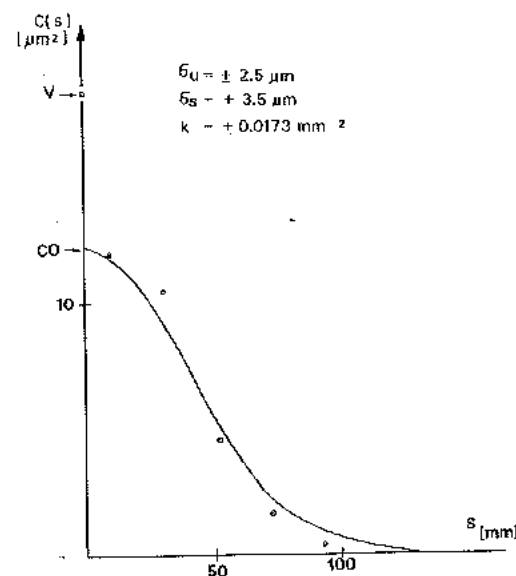


Figure 8

Figure 8 shows first the variance V and the empirical covariances $C(s_{ik})$ for a number of distances s of the y -component of the vector field of measured values of reseau points of

Plate 358, Oberschwaben. The empirical covariances can be approximated by a continuous function, the covariance function, for which the well-known Gaussian curve is the most appropriate:

$$C(s) = CO \cdot \exp \{-k^2 \cdot s^2\}.$$

The maximum value CO is the covariance between points which are infinitely close together ($s=0$). The constant k is responsible for the rate of decrease of the covariance function with increasing distance.

The conclusion is that three constants are sufficient to describe the behaviour of the components of a vector field. These are: the variance V , the maximum value CO of the covariance function, and the constant k .

For the y -components of the vector field of Plate 358, we found the values:

$$V = 18.4 \mu^2 \quad CO = 12.2 \mu^2 \quad k = 0.0173 \text{ mm}^{-2}$$

From these parameters the numerical values for the three terms by which we describe the three basic features of a vector field can be derived directly:

- the mean square value of the absolute magnitude of the y -components was:

$$\sigma_{y_a} = \sqrt{V} = 4.3 \mu$$

- the mean square value of the systematic component was:

$$\sigma_{y_s} = \sqrt{CO} = 3.5 \mu,$$

while the mean square value of the irregular component was:

$$\sigma_{y_u} = \sqrt{V - CO} = 2.5 \mu$$

- the constant $k = 0.0173$ is a representative value for the behaviour of the systematic effects depending on the distances."

For the linear least-squares interpolation, for the details of which one is referred to Phot. Eng., Oct. 1972, pages 1016-1029, it is exactly the above 3 constants that are required. Important practical features of the method are:

- the method is independent of the number and distribution of the reseau points observed;
- the irregular errors of measurement, together with the short period variations in the distortion over distances of some millimeters only, are not transferred to the image points which have to be corrected (interpolated), but instead are filtered out.

3.2. The method was applied on the monocular reseau observations, carried out in ITC, of Plate 358 of strip 10 (with 524 reseau points observed). The problem that was researched, was the problem of how many reference points (reseau point observations) were needed to correct the systematic component of the film deformation.

To that end, firstly 147 reseau points (distributed as a 2cm-grid), then 52 (a 4cm-grid) and next 25 (a 5cm-grid) were used as reference points, and the other points interpolated. All three distributions proved to be very effective, the latter one only very slightly less than the former.

Next, reference points were selected along the edges of the photograph only, these points thus are equivalent to fiducial marks. It then appeared that, for the photograph concerned (Plate 358), the systematic film deformation could be caught rather well in the case of using 27 reference points along the edges. However, when using only 8 or, still worse, 4 reference points only, the results were very poor. "The conclusion is (thus) justified that the conventional fiducial marks in air survey cameras are not suited to correct the systematic film deformation".

3.3. The method was also applied on the measurements on the two test-models of Rheidt, executed at ITC. The results are shown in the following table 6 in which the earlier results of table 5 (section 2.2 above) are added in brackets.

Table 6

Model	With use of reseau	Without use of reseau *)
39/43	$\sigma_X = 6.2 \mu (8.8)$ $\sigma_Y = 5.8 \mu (8.0)$ $\sigma_H = 0.075 \text{ ‰ } Z (0.08)$	$\sigma_X = 7.8 \mu (7.8)$ $\sigma_Y = 7.2 \mu (7.9)$ $\sigma_H = 0.097 \text{ ‰ } Z (0.10)$
43/48	$\sigma_X = 6.8 \mu (8.0)$ $\sigma_Y = 5.5 \mu (7.0)$ $\sigma_H = 0.064 \text{ ‰ } Z (0.075)$	$\sigma_X = 6.6 \mu (7.2)$ $\sigma_Y = 6.0 \mu (9.1)$ $\sigma_H = 0.071 \text{ ‰ } Z (0.075)$

*) The small differences between corresponding values are primarily caused by the fact that earlier only 14 points were used for the absolute orientation, while Kraus used all ground check points.

"Earlier a method was used by which the measured value of the closest reseau point is directly transferred to the image point to be corrected. While that method caused no improvement, the present method shows a significant improvement of the accuracy. For practical applications, the improvement of 20 % of model 39/43, which originally was less accurate, is especially important.

According to these results, the reseau in combination with least-squares interpolation seems to yield 7μ in planimetry and $0.075 \text{ ‰ } Z$ in height, which is independent of the magnitude of the original film deformation".

4. Latest Experiments

4.1. Introduction

The experiments have been concluded at the ITC with new glass diapositives of the original negatives, with positives on polyester film and also with "artificially aged" film positives.

4.2. Stereoscopic Measurements

The abbreviations used to denote the different variants of the two test-models of Rheidt, nrs. 39/43, 43/48 as obtained from the new diapositives, are summarised in table 7.

In the models GL, FI, AA and AB, the floating mark used was a dot of $\phi = 40 \mu$. In models AC, a dot of $\phi = 20 \mu$, surrounded by a ring of $\phi = 150 \mu$ was used. In total, five versions for each of the 2 models were obtained. Observation of a model consisted of the measurement of each model point, followed by the monocular measurement of the closest reseau point. Since the signalised model points are situated in groups of three, the same reseau point is measured three times.

Table 7 - Summary of available variants of each model

Abbreviation	Type of photography
GL	Diapositive on glass
FI	Diapositive on film
AA	Diapositives on film, exposed to a temperature of 30° C during approx. 15 minutes
AB	Diapositives on film, exposed to a temperature of 40° C during approx. 45 minutes
AC	Diapositives on film; Photos 39, 43 and 48 were treated differently: Photo 39: Washed during 1 night; dried at 20° C Photo 43: In fixing bath for 1 night; washing 1 hr; dried at 25° C Photo 48: Washed for 1 hr; dried at 40° C

4.2.1. Computation

As before, the basic data for each model consists of a list of image coordinates in the comparator system. Four reseau points in the corners of the photographs were used as fiducial marks and the comparator coordinates were affinely transformed into the fiducial system with the help of only these 4 reseau points.

Three sets of data were now obtained: one set without reseau points, one including all measurements of reseau points and the third including only one measurement per observed reseau point. This latter data set was obtained by deleting the second and third observation at each reseau point.

Table 8 summarises the magnitude of the film deformations in the observed reseau points, from which it can be concluded that the aging procedures did not deteriorate the geometric quality of the films to that extent as might have been expected.

Table 8 — R. m. s. value of film deformation in μm at photoscale. Values obtained in model 39/43 from 21, in model 43/48 from 28 reseau points, after affine transformation of measurements into the system defined by 4 fiducials.

Photo	39		43	
	x	y	x	y
GL	5	4	8	5
FI	6	4	9	7
AA	4	2	12	5
AB	7	8	7	8
AC	4	5	11	8

(a) Model 39/43

Photo	43		48	
	x	y	x	y
GL	6	6	9	8
FI	7	6	11	8
AA	5	5	10	13
AB	7	12	16	13
AC	7	6	11	12

(b) Model 43/48

Next, the known deformations in the reseau points were used to interpolate corrections in the other points. Following the idea of Kraus, for this purpose a programme using linear prediction (least-squares interpolation) was written.

The correlation function in this programme can either be computed from the known film deformations in a subroutine, or a pre-specified function can be entered. Basically, the photo is split in several zones in such a way that not more than 30 reseau points are used for an interpolation.

Reseau points from adjacent zones are included in interpolations of a zone, if these reseau points are within a specified distance from the boundary line. This is to assure continuity of the interpolated corrections. The separation into zones is necessary to avoid excessively large correlation matrices.

Within each zone a trend polynomial of specifiable order is computed (with one to ten coefficients). The residuals left after the fit of the polynomial to the Δx and Δy deformations are used as input to the linear prediction.

The correlation functions used are of the isotropic type:

$$\text{Corr.} = C_0 / (1 + d^2 / C_1^2) \quad (1)$$

where d is the distance between 2 correlated points.

This function has proved to be successful in many applications, e. g. interpolation of gravity anomalies, and in DTMs.

Attempts failed to compute the coefficients C_0 and C_1 from the residuals per zone. Presumably the number of reseau points was too small per zone. Use of a function computed for the whole of the photograph, or even from a part of it, has been proposed by Kraus (Z. Vermess.-wes., 1971/6). These functions are however not representative of the residuals left per zone after trend subtraction ("centering"). Therefore an optimum correlation function was experimentally found by "trial and error"; different functions were tried out, and the best one chosen.

Interpolation of Δx and Δy corrections has been done independently. Although the programme allows for the consideration of cross correlation between Δx and Δy , the simultaneous interpolation of Δx , Δy did not bring any improvement and so it was excluded from further consideration.

All three sets of image coordinates — without, with one measurement, with three measurements per observed reseau point — were used for relative and absolute orientation, applying the maximum number of available points as orientation points.

Relative and absolute orientation were carried out with more than 50 points in the case of model 39/43, and with more than 70 points in the case of model 43/48. The residuals left after absolute orientation were divided by the scale, and represent the outcome of the experiment. This is discussed next.

4.2.2. Discussion of Results

The first step was the optimization of correlation function (1). Table 9 summarises the results obtained. Coefficients C_0 , C_1 were varied systematically in the interpolation. Figure 9 shows graphical representations of some of the alternatives which were tried out. The values for C_0 , C_1 providing the smallest r. m. s. residuals in the 3 coordinate directions were found to be

$$C_0 \approx .50 - .70$$

$$C_1 \approx 50 - 70 \text{ mm}$$

Table 9 — Experimental optimization of correlation function $C_0/(1 + d^2/C_1^2)$. Trend polynomial of 2nd order (6 unknowns). Values in μm at photoscale

C_1 (mm)		10			30			50			70		
R. m. s. Value		ΔX	ΔY	ΔH	ΔX	ΔY	ΔH	ΔX	ΔY	ΔH	ΔX	ΔY	ΔH
C_0	1	5.3	4.1	9.2	7.4	5.8	12.1	38.3	29.8	44.8			
	.85	4.7	4.3	9.2	4.6	4.3	9.1	4.6	4.2	8.9	4.5	4.2	8.9
	.70	4.6	4.2	9.2	4.5	4.2	9.0	4.5	4.2	8.9	4.5	4.2	9.0
	.50							4.5	4.3	9.1	4.5	4.3	9.1

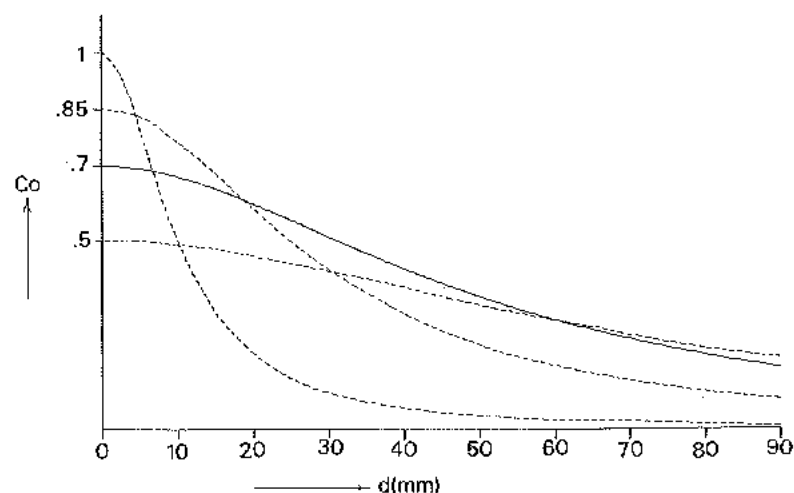


Figure 9 — A sample of correlation functions tested. The thicker line denotes the function ($C_0 = .7$, $C_1 = 50$) that gave the optimum result

Table 9 is computed from model AA 43/48, using one measurement per observed reseau point. Similar computations were carried out for other models, providing the same conclusion. The trend polynomial was chosen to be of 2nd order (6 unknowns).

Taking the optimum correlation function, the number of unknowns of the trend polynomial was varied. The results are shown in table 10. Obviously there is no point in choosing more than a 2nd order trend polynomial per zone.

Table 10 — Comparison of three trend polynomials, with 3, 6 and 10 coefficients respectively, for AC 43/48

		ΔX	ΔY	ΔH
No. of coefficients	3	4.7	4.1	10.0
	6	4.5	4.2	8.9
	10	4.6	4.4	8.9

In table 11, the r. m. s. values of the lack of coplanarity after relative orientation are given. Although there is a noticeable tendency for deteriorating stereoscopy with the aging of the photographs, the magnitude of differences is fairly small.

Table 11 — Lack of coplanarity after relative orientation, with and without use of reseau. Given are r. m. s. values in μm

Model	Reseau	GL	FI	AA	AB	AC
39/43	without	5.3	6.1	6.2	6.5	6.3
	with	5.9	6.0	6.6	6.5	5.7
43/48	without	5.4	5.8	6.7	7.6	7.0
	with	5.1	5.1	6.9	6.5	6.7

Table 12 — R. m. s. residuals ΔX , ΔY , ΔH in μm at photoscale, after absolute orientation. X-axis of terrain system approx. parallel with photo y-axis

	ΔX	ΔY	ΔH	$\Delta H/Z$
GL	7.2	7.1	12.8	.084 ‰
FI	7.3	6.9	10.1	.066 ‰
AA	5.6	5.3	11.4	.074 ‰
AB	7.0	6.0	12.7	.083 ‰
AC	7.9	5.9	10.9	.072 ‰

(a) without reseau, 39/43

	ΔX	ΔY	ΔH	$\Delta H/Z$
GL	4.6	5.5	9.8	.064 ‰
FI	5.5	5.2	9.8	.064 ‰
AA	4.4	5.0	9.6	.063 ‰
AB	4.5	4.8	8.9	.059 ‰
AC	4.1	4.1	10.0	.066 ‰

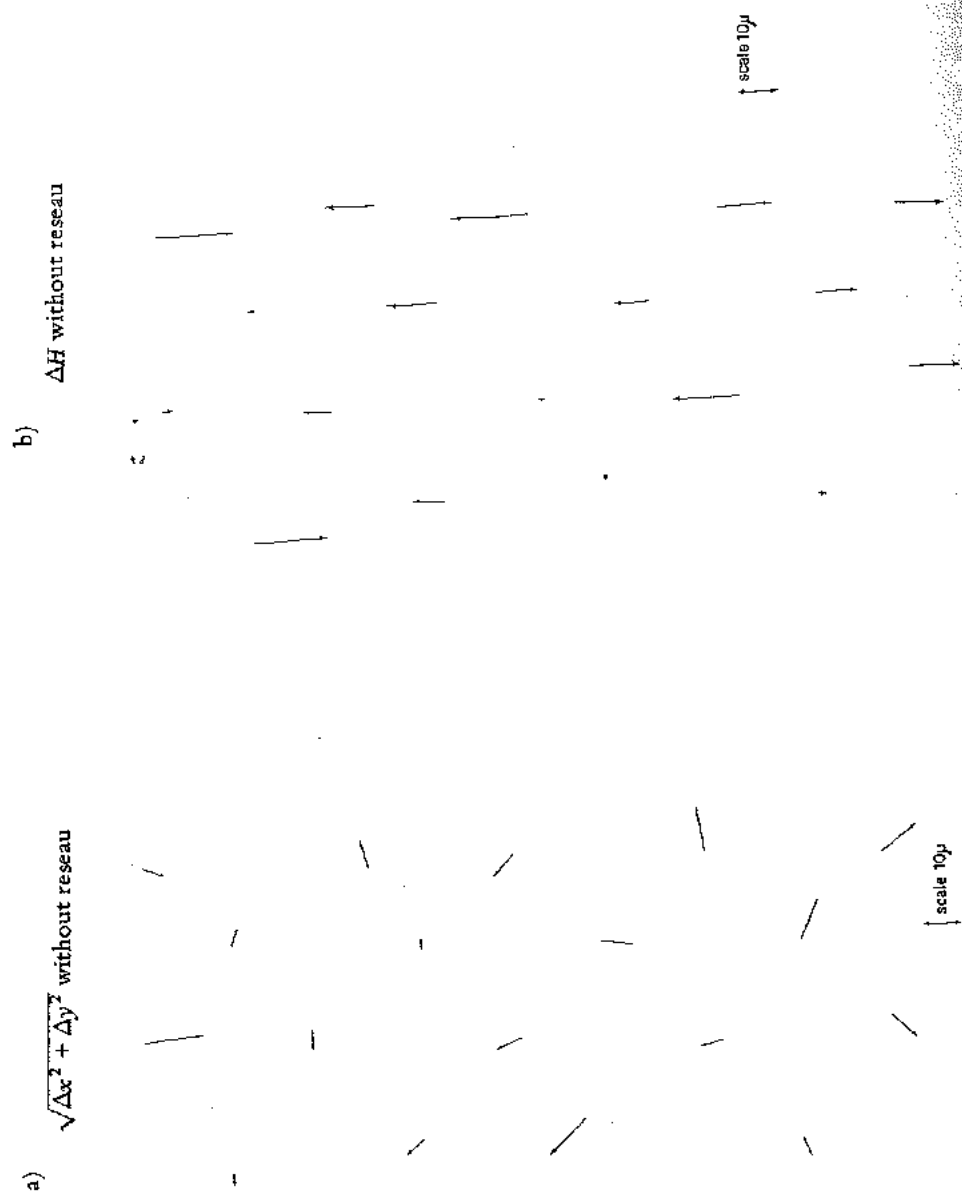
(b) with reseau, 39/43

	ΔX	ΔY	ΔH	$\Delta H/Z$
GL	6.7	6.8	12.4	.082
FI	5.2	5.9	13.3	.087
AA	5.9	6.0	12.3	.081
AB	7.1	5.8	14.1	.093
AC	6.1	4.8	15.5	.101

(c) without reseau, 43/48

	ΔX	ΔY	ΔH	$\Delta H/Z$
GL	6.7	6.0	10.9	.072
FI	4.8	6.6	10.9	.072
AA	5.5	5.6	9.6	.063
AB	5.7	5.7	10.0	.066
AC	4.5	4.2	8.9	.059

(d) with reseau, 43/48



c) $\sqrt{\Delta x^2 + \Delta y^2}$ with reseau

d) ΔH with reseau

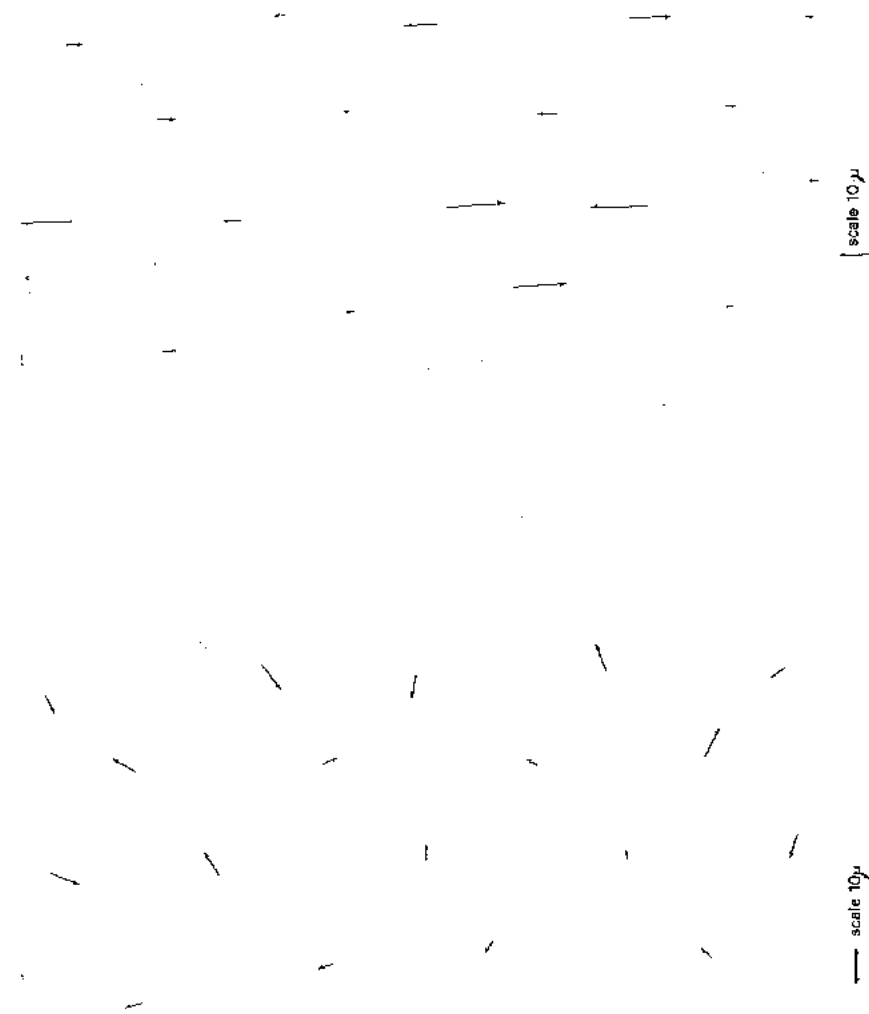


Figure 10 — Vector diagrams of residuals after abs. orientation, model 39/43, version AB

Table 12 shows the main result of the experiment, namely the r. m. s. residuals after absolute orientation of all 5 variants of each model, with and without use of the reseau. In all cases, except for Y of model FI 43/48, improvement could be obtained by the use of the reseau, with a maximum of almost a factor 2 for H in model AC 43/48.

This table refers to the case of a single measurement per observed reseau point. Almost identical results are obtained if all measurements of reseau points are included. Differences are only of the order of fractions of μm . Therefore, these results are not further discussed. There is no point in observing a reseau point more than once, if linear prediction is applied for interpolation.

For model 39/43, the use of reseau leads to a fairly uniform accuracy, whatever procedure of aging was taken. The results are close to the limit of the photogrammetric state of art. It is interesting to note, that no correction for lens distortion or refraction was applied, since the purpose of the experiment was only to demonstrate the use of the reseau. This in particular shows the geometric performance of the photography to be rather impressive.

In model 43/48, version AC gives results superior to glass or film diapositives. This does not justify the conclusion that artificial aging should become a standard photogrammetric procedure. The superiority of the results is purely due to the fact that a different floating mark was used, which allows especially planimetry to be measured with a higher accuracy. This effect is more pronounced in model 43/48 than in model 39/43, since the signalisation of control points causes problems especially in the former model.

For the case of model AB 39/43, a graphical plot of planimetric and height residuals was prepared. Figures 10a, b show the error vectors for planimetry and height without, and figures 10c, d with use of the reseau. The distribution of errors is clearly the same, but their magnitude is reduced by the reseau.

Finally, table 13 lists the maximum residual per case and coordinate direction, with and without reseau. These maxima are reduced by the reseau, however to a much smaller extent than the r. m. s. residuals.

Table 13 — Extreme residual after absolute orientation, without and with use of reseau, for model 43/48. Values in μm at photoscale

	ΔX		ΔY		ΔH	
	without	with	without	with	without	with
CL	- 25	- 22	- 18	- 18	- 32	- 32
FI	13	12	19	- 18	- 40	- 31
AA	16	- 14	18	- 17	- 32	- 25
AB	- 18	- 19	11	- 12	- 42	- 29
AC	- 14	10	17	- 11	- 42	- 25

4.2.3. Conclusion

The above experiment clearly demonstrates that the use of the reseau provides a uniform and excellent accuracy, irrespective of the film deformation introduced by artificial aging procedures. Without the use of reseau, the results were far more inhomogeneous from one version to another than with the reseau. The differences of the results between different aging methods, and between glass and film, are reduced to fractions of microns if reseau corrections are applied.

The absolute magnitude of the film deformations in the particular case of the two models under study was rather small. Post-processing of the developed films did not deteriorate the geometric quality to that extent as expected or hoped, so that a demonstration of the usefulness of reseau with "poor quality" photography was not spectacular. However, it could be clearly proved that the reseau is indispensable if the ultimate potential of the photogrammetric process is to be exploited.

4.3. Monocular Investigations

The extension of the monocular reseau investigations was concentrated on two aspects, namely the influence of film aging and film storage on the geometric stability and a comparison of the stability of glass and film diapositives.

To this end, 15 diapositives on both glass and polyester film were printed from the original film negatives. These sets of 15 positives were selected at the beginning, middle and end of each of the 5 reseau strips. The three glass and film positives of the Rheidt test area were, of course, also included in the investigation, as were the "artificially aged" film positives.

The measurements were restricted to the 81 point grid in each plate, these then being conformally transformed using all points.

The following main conclusions could be drawn from the results:

- There was no significant difference in either the pattern or the magnitude of the distortion of the 15 "new" glass diapositives compared to those prepared some three years earlier.
- The differences between the residual distortions of the sets of glass and film diapositives were insignificantly small i. e. in the order of the pointing accuracy.
- The attempts to "artificially age" the film positives by alternately heating and soaking these were unsuccessful.

In general, the conclusion reached from this material was that the deformation of the original film negatives was not adversely affected by storage and that diapositives on polyester film are certainly not inferior to those on glass as far as geometric stability is concerned.

RÉSUMÉ:

Les bandes 6, 8, 10, 12 et 14 du bloc grand'angulaire d'Oberschwaben ont également été photographiées avec la chambre à réseau de Zeiss (chambre RMK AR 15/23). A l'ITC, 25 diapositives sur verre de la bande 10, et 3 de chacune des bandes 6, 8, 12 et 14, ont été mesurées monoculairement sur un comparateur (voir figures 1-6). Par bande, les résultats étaient presque identiques sauf pour un «jeu» de 2.5 à 3 microns en x et y de chaque point de réseau. Ce jeu correspond pratiquement à l'erreur de mesure. Pourtant, pour les bandes différentes, les résultats étaient légèrement différents d'où l'on pourrait constater un systématisme significatif différent entre les bandes extrêmes 6 et 14.

Pour décrire les erreurs systématiques par photographie, ou par bande, on peut diviser la photographie en sections, mesurer un nombre de points, et calculer l'erreur par section (voir figure 7). Pour la photographie à réseau d'Oberschwaben, le choix de l'une quelconque des 9 combinaisons (nombre de sections, nombre de points mesurés) révélait pratiquement aucune différence pour le but fixé. Une possibilité de correction pour les erreurs systématiques (dus à la déformation du film et à la non-planité de la couche à l'instant de l'exposition) serait la suivante: mesurer — monoculairement — trois diapositives, une au début, une au milieu et une à la fin de chaque bande. Selon l'analyse de la figure 7, 25 points par diapositive suffiront. Si les trois diapositives montrent une distortion uniforme — comme c'était le cas pour Oberschwaben — on pourrait corriger toutes les diapositives de la bande de ce systématisme. Si non, il faudrait mesurer un nombre approprié de points de réseau dans chaque diapositive individuelle.

Parallèlement avec l'investigation monoculaire, un test sur stéréorestitution de photographies à réseau a été exécuté dans lequel, après chaque observation sur un point stéréoscopique, on a mesuré le point de réseau le plus proche, à gauche et à droite, sur un stéréocomparateur. Chaque couple a été calculé deux fois, une fois en utilisant les observations des points de réseau (où la déviation constatée dans le point de réseau le plus proche est utilisée comme correction au point stéréoscopique) et une fois sans utilisation des observations des points de réseau.

Pour la triangulation de quelques bandes, après compensation selon Ackermann, les résultats, indiqués dans les tables 3 et 4, ont été trouvés: une précision très grande, indépendante de l'utilisation ou de la non-utilisation des observations des points de réseau.

Aussi pour la restitution de deux couples couvrant le champ d'essai de Rheidt (avec un grand nombre de points de contrôle signalisés) on n'a pas pu constater une différence significative entre les deux cas: utilisation ou non-utilisation des observations des points de réseau.

K. Kraus (Stuttgart), cependant, en appliquant une autre méthode pour la correction des points stéréoscopiques (c. a. d. une correction pas seulement basée sur la déviation constatée dans le point de réseau le plus proche, mais sur les déviations dans tous les points de réseau mesurés: «méthode d'interpolation linéaire selon le principe des moindres carrés») a trouvé un résultat un peu plus favorable (voir la table 6).

Finalement, à l'ITC, les observations ont été répétées avec des diapositives sur film, aussi que sur verre. Les diapositives sur film ont été «mal-traitées» afin de créer, artificiellement, un matériel moins bon que le matériel photographique original d'Oberschwaben. Ceci pour démontrer la valeur de la photographie à réseau.

Discussion about the Subject 11 under the Chairmanship of Prof. E. Gotthardt

Chairman: Thank you for your presentation Mr. Visser. Who would like to open the discussion?

Amer: I would like to know which strip adjustment procedure was used since this is of importance when comparing adjustments carried out with and without the use of the reseau?

Visser: We used the Ackermann strip adjustment procedure of 1961, which uses piece-wise polynomials.

Kupfer: Mr. Visser comes to the conclusion that the fiducial marks are not suited to correct the film deformation but I think this has to be proved in each flight.

Chairman: Can Prof. Kupfer show us that better results are obtainable from non-reseau cameras?

Visser: I consider the reseau camera to be as good as any other.

Kupfer: So far we have only examined non-reseau cameras but now intend investigating the reseau camera.

Ligterink: Have you computed the variance factor for longer distances?

Visser: Dr. Leberl has so far used distances of up to 9 cm in the plate.

Förstner: What does S represent in fig. 8 of your paper?

Visser: S is the distance between points used to determine the covariance function. The example shows the y -component but a similar graph is also prepared for the x -component.

Ackermann: What is the effect of assuming a negligible cross correlation and treating the x and y corrections separately?

Leberl: In order to determine this we assumed a certain amount of cross correlation in one instance but the results were terrible. The problem, however, is one of interpolation and it is quite possible that polynomials would give a better result.

Chairman: I have always had some doubts about the least squares interpolation procedure.

Kupfer: If there is a systematic trend present I would consider it advisable to start with regression polynomials since these would give a better result than with a least squares interpolation.

Leberl: I consider that we should not do complete injustice to the problem of linear prediction although I admit that it does depend on the experience and intuition of the user, since it is often difficult to determine the covariance function.

Visser: I would still advocate the use of the reseau to correct the systematic effects of film deformation.

Leberl: The effect of film deformation is extremely small.

Jerie: There are a number of other error sources which cannot be eliminated by means of a reseau.

Förstner: Mr. Visser claims that the reseau will not only control the effect of film distortion but also that of a poor flattening of the emulsion and of the photographic processing.

Visser: The reseau does however not control atmospheric influences such as refraction and air turbulence.

Holm: If I remember correctly, the photography was flown on 8 different days and so could not this be used to determine the influence of atmospheric conditions?

Furthermore, could not the different film types used be included in the investigation?

Kupfer: I am unaware of the fact that different film types were used in photographing the Rheidt test area.

Ackermann: The different film types used refer to the photography of the Oberschwaben block, but no double coverages were flown. The fact that the flying took a total of 8 days spread over a period of two months is largely due to poor weather conditions and due to the difficulties encountered in obtaining pin-point photography.

Chairman: I am afraid that I have to close this session now and I want to thank you all for your contributions.

Compensation of Systematic Errors by Analytical Block Adjustment with Common Image Deformation Parameters

(with 7 Figures)

By H. Bauer, Hannover

SUMMARY:

The compensation of systematic errors by bundle adjustment with common deformation parameters has been demonstrated.

First the algebraic model of the deformation parameters has been described. Then computer results of the bundle adjustment with common parameters for the Oberschwaben test with different arrangement of control points and with different deformation parameters followed. The increase of accuracy of this method is 20–60 % in Oberschwaben test.

RÉSUMÉ see p. 324

The accuracy of bundle adjustment is doubled by the common parameters in the Oberschwaben test as against an ordinary bundle adjustment. The idea and the computer-program for the bundle adjustment with additional parameters were developed by Dr. Jürgen Müller [1]. I only cooperated in the selection of the correction terms and supervised these tests.

1. Systematic Errors in Block Adjustment

The quotient $\sigma_{\text{check}} / \sigma_0$ (root mean square error at the controlpoints divided by standard deviation of an image coordinate observation) of a bundle adjustment in the Oberschwaben test is $18.4/5.01 = 3.7$. Theoretically the quotient should be about one. The large difference between the theoretical value one and the real value 3.7 is a sign of the important influence of systematic errors in a block adjustment. Kubik [2] conducted theoretical investigations to discover which types of photo-deformations have a particularly powerful effect on the results of the adjustment. Kubik supposed deformation of the square photo to a rectangle, parallelogram, trapezoid and a star (figure 1). He found out that the planimetric accuracy is especially affected by rectangle and parallelogram deformations and the accuracy of height by stelliform deformations of the photos.

To increase the accuracy of block adjustment it is necessary to allow for the systematic errors. This is possible in the following way:

1. Calibrating the camera by a test area
2. Using a reseau-camera
3. Repeating the flight
4. Additional parameters in the bundle adjustment.

All the methods presuppose that the systematic errors are constant during the whole flight. The first three possibilities of allowing for the systematic errors require additional plotting expense and allow only for systematic errors of photogrammetric origin. The use of additional parameters does not involve additional plotting expense and the system complexes that can be dealt with may be of photogrammetrical or terrainian origin.

It seems to me very important to suppose that systematic errors may be present in the ground control points also. For example trigonometrical points are frequently ground control points in Germany. In Northern Germany these trigonometrical points have not been established for numerical surveying but only for the production of the map 1 : 25 000. Normal terrestrial surveying could not reveal defects in the trigonometrical net. Today the plane insertion of the photogrammetric blocks is a method of measuring that requires an exact geometric quality of the coordinates of the ground control points.

2. The Theory of the Additional Parameters

The theory is very simple. The error equations of the central perspective between the photo and the terrain are extended by additional free parameters. The equations then run:

$$v = A_1x + A_2y + A_3z - 1$$

x are the coordinate unknowns

y are the orientation unknowns

z are the unknown correction parameters.

What demands should the correction terms fulfil?

1. The correction term should be effective.
2. The correction term should be simple and as short as possible.
3. The terms should not be correlated with each other.
4. The terms should not be correlated with the orientation unknowns (coordinates of the perspective centres and orientation angles).
5. The interior orientation of the photos (focal length and principal point) should not be changed.
6. For practical work terms are preferable which are not affected by rotation by 180° . In this case it is not necessary to distinguish the direction in which the strips were flown, and the position of the photos in the comparator (direction of triangulation) has no influence on computation.

We have experimented with the following terms:

1. radial and independent of direction

$$\Delta r' = a_1 r' (r'^2 - r_0'^2)$$

$$= a_2 r' (r'^5 - r_0'^5)$$

$$r_0' = 100 \text{ mm}$$

2. radial and dependent on direction

$$\Delta r' = a_5 r' \cos 2(a - \epsilon_1)$$

$$= a_6 r' \cos 4(a - \epsilon_2)$$

$$= a_{11} r' \cos (a - \epsilon_3 - \chi_1)$$

ϵ = free reference direction

χ = flight direction

3. tangential and dependent on direction

$$\Delta t' = a_{14} r' \cos 2(a - \epsilon_4)$$

$$= a_{16} r' \cos 4(a - \epsilon_5)$$

4. shearing

$$x' = 0$$

$$y' = a_{19} x'$$

5. affinity

$$x' = -a_{20} x'$$

$$y' = +a_{20} y'$$

The program allows four terms to be computed at the same time. For this reason we combined the terms to correction groups with four unknown parameters.

group 1 — radial with r'^3 , r'^6 and $r' \cos 2(a - \epsilon_1)$

group 2 — shearing, affinity and tangential with $r' \cos 2(a - \epsilon_4)$

group 3 — radial and tangential with $r' \cos (a - \epsilon_3 - \chi_1)$

group 4 — radial and tangential with $r' \cos 4(a - \epsilon)$.

Group 4 proved to be ineffective, it will not receive any further mention. Group 3 effects a shifting of the principal point and therefore is not suitable. The results will not be discussed here.

3. Results with the Correction Terms in the Test Area

3.1. Test Area Oberschwaben

As the first test area five strips of Oberschwaben were used, which were all flown over on the same day. For one test series the horizontal ground control points were arranged only on the edge of the block, bridging five models. In the second test series the horizontal ground control points are distributed all over the area at intervals of about two models.

The vertical control points form chains crossing the area bridging five models in the first series and two models in the second.

The second test area is the whole block of Oberschwaben with eight strips and a sidelap of 20 % (figure 2). The distribution of the ground control points corresponds to the first test area.

The comparator data used are the evaluation of the wideangle flight with 20 % sidelap by the IfAG, Frankfurt, with a Zeiss PSK.

3.2. Computer Results

All the results are based on the same data. To demonstrate the increase in accuracy achieved by the correction groups, the standard deviation of the checkpoints of the bundle adjustment without additional parameters is set at 100 %.

With the first arrangement of the control points the bundle adjustment with the first correction group (radial corrections) resulted in a 20 % increase in accuracy (figure 3). The proportions of the radial corrections with r'^3 (independent of direction) and with $r' \cos 2(\alpha - \epsilon)$ (dependent on direction) are of approximately equal size.

The additional second correction group with parameters for shearing, affinity and tangential corrections shows an extreme increase in accuracy (figure 3). Among the four parameters the shearing dominates.

The second denser distribution of the control points yielded results similar to the first distribution of control points. Due to the larger quantity of control points the results are more exact from the start (figure 3). But the accuracy of the first test series is not surpassed. It is most interesting that the same accuracy as with both correction groups 1 and 2 is also attained with the second correction group alone (figure 3). The whole correction is then divided between the shearing term and the affinity term. I believe the shearing that is having an effect here is caused by a systematic error in the ground control points, because otherwise there would be a correction of $10.7 \mu\text{m}$ at the edge of the photo.

On the basis of the results of the first test area we chose a standard correction group with four terms, the shearing and three terms with radial effect. Three radial terms are used in particular with a view to increasing the accuracy in height. We hoped that we could account for the planimetric deformations with the shearing alone, because the investigations of Kubik [2] show that the radial corrections could eliminate the effect of the affinity for a certain extent (figure 4).

The standard correction group was used for the whole block of Oberschwaben. With the denser distribution of control points the accuracy is about the same as in the first test series although here the standard correction group is used while there the correction groups 1 and 2 were applied. The increase in accuracy achieved by the standard correction group is 32 %, in relation to an adjustment without correction.

If the horizontal control points are only established at the edge of the block bridging five models, the accuracy of the easting coordinates is significantly lower than that of the northing coordinates. This effect did not appear in the first test block. With regard to the flight the difference between the whole block of Oberschwaben and the first block is that the three strips were flown over on a different day than the first five strips. The flight direction of the individual strips is irregular. This additional information was at first not considered in the adjustment because the error analysis of a bundle adjustment with the standard correction group shows that the low accuracy in the easting coordinates follows from a contraction of the block in the middle of the longitudinal sides (figure 5).

The theoretical investigations of Kubik show that system complexes of this kind are due to an affinity. To check this statement, we established a correction group with three terms, with correction terms for affinity, shearing and radial with r'^3 . The results confirmed the supposition. The new correction group was able to remove the system complexes. It was possible to increase the accuracy by about 67 % as against an adjustment without correction (figure 3).

In comparison with the planimetric accuracy the height accuracy in existing tests could be improved only slightly, namely by about 20 %. We hoped to reach more by a 60 % sidelap. In the Oberschwaben test this experiment was without success. I believe the reason is that the accuracy of the measurement of the intermediate strips is different from that of the measurement by the IfAG, Frankfurt. To check the increase of height accuracy using a 60 % sidelap with homogeneous data we conducted the Steinbergen test.

3.3. Steinbergen Test

The test area has a size of $6 \times 6 \text{ km}$. The photoscale is $1 : 8\,000$. The arrangement of the strips and the ground control points is shown on figure 6.

With a sidelap of 60 % without correction terms, $\sigma_{\text{check height}} = 17.8 \mu\text{m}$, that is 14 cm, with standard correction group $\sigma_{\text{check height}} = 10.7 \mu\text{m}$, corresponding to 8.6 cm (figure 7). This is an increase in accuracy of about 40 %. For a sidelap of 20 % the result with correction group was $24.4 \mu\text{m}$ height accuracy and without correction terms $26.0 \mu\text{m}$. This shows that for good accuracy of height a sidelap of 60 % is necessary. Only a sidelap of 60 % gives the stability to a block that correction terms for height yield a good increase in accuracy.

4. Conclusion

It has been possible to show adding standardized parameters to the error equations of bundle adjustment produces a considerable increase in accuracy. A prerequisite for the success of the method is the constancy of the systematic errors. To obtain good results it is desirable:

- to carry out the flight on one day,
- to prescribe a regular flight performance,
- to use only one camera,
- to commission only one institute with the plotting.

Besides the photogrammetric system complexes it will always be necessary to reckon with systematic terrestrial errors.

The method of the additional parameters does not separate photogrammetrical and terrestrial system complexes. It is a practical method which yields good results at little expense.

References

- [1] J. Müller: Blocktriangulation mit Verbesserung der inneren Orientierung. — Bul 1971, S. 107.
- [2] K. Kubik: The Effect of Systematic Image Errors in Block Triangulation. — ITC A 49, 1971.

RÉSUMÉ:

La réduction des erreurs systématiques dans la compensation par faisceaux au moyen de paramètres supplémentaires ayant été exposée, le modèle algébrique des paramètres de déformation a été établi.

Ensuite, pour les essais «Oberschwaben» les résultats des compensations par faisceaux avec paramètres supplémentaires obtenus à partir de répartitions variées des points d'appui et de paramètres de déformation différents ont été mentionnés.

Par ce procédé, on a obtenu une amélioration de précision de 20 à 60 % dans les essais Oberschwaben.

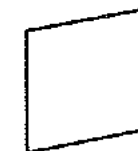
TYPES OF SYSTEMATIC IMAGE ERRORS BY KUBIK

square photo

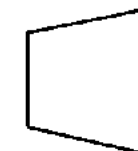
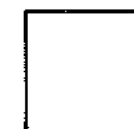


deformed
by affinity

effect
planimetric errors

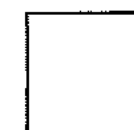


deformed
by shearing



deformed
to a trapezoid

without
dominate effect



radial deformed
to a star

effect height errors

Figure 1

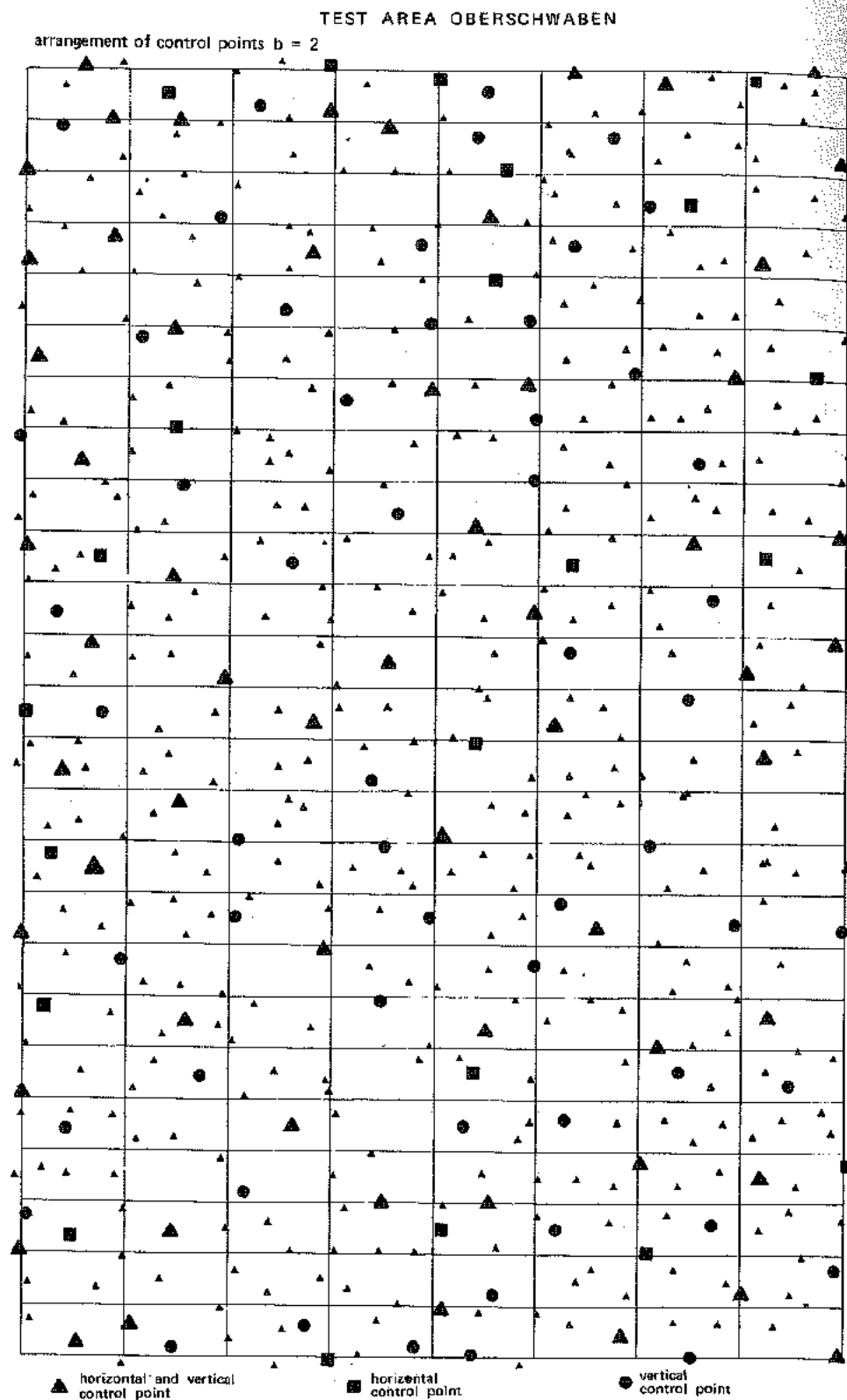


Figure 2

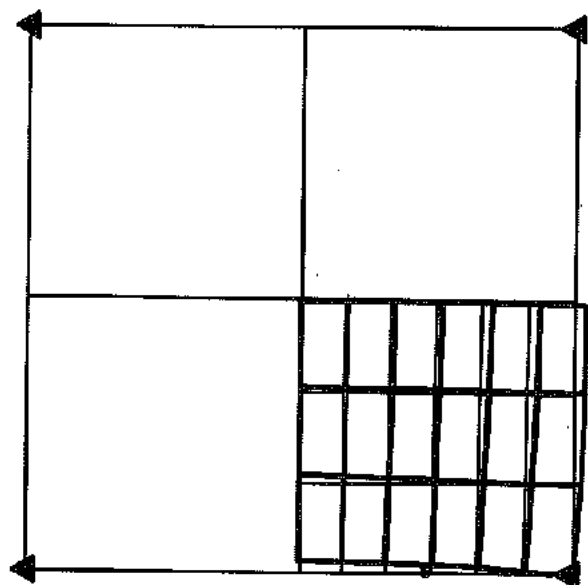
RESULTS OF THE TEST "OBERSCHWABEN"

ground control points	1. Test area, 5 stripes										2. Test area, the whole block, 8 stripes									
	1. series, 18 horizontal 40 height					2. series, 57 horizontal 81 height					1. series, 40 horizontal 75 height					2. series, 84 horizontal 121 height				
	1	2	3	1 + 2 (shearing)	1 (radial)	1	2	3	4	1 + 2 (shearing)	1	2	3	1	2	3	1	2	3	1
test																				
connection group																				
σ_0 standard deviation	5.01	4.72	4.09	5.79	5.13	5.4	5.5	5.4	5.5	5.74	4.83	4.17	6.21	4.90	4.30					
σ Check UTM East	18.4	15.4	5.9	10.7	9.1	5.9	5.8	5.9	5.8	24.6	18.2	7.9	11.6	8.5	5.7					
UTM North	18.4	13.6	5.8	9.9	6.8	5.8	5.8	5.8	5.8	16.9	7.5	5.9	10.4	6.4	5.7					
height	19.6	16.1	14.7	15.7	15.6	14.4	15.6	15.6	15.6	23.4	19.8	13.2	14.5	13.9	13.2					
σ Check $\frac{\sigma}{\sigma_0} = \mu$	3.7	3.1	1.4	1.8	1.4	1.3	1.4	1.3	1.2	3.6	2.8	1.7	1.8	1.5	1.3					
increase of accuracy (check plan)	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%					
max. correction in the image for $r = 100$ mm	100	79	32	100	81	86	86	86	86	100	62	33	100	68	52					
		2.2	8.5		3.7	10.3	14.1	10.0	10.3		10.0	10.3		10.1	10.3					

Figure 3

planimetric coordinate errors

by radial image deformation



by affine image deformation

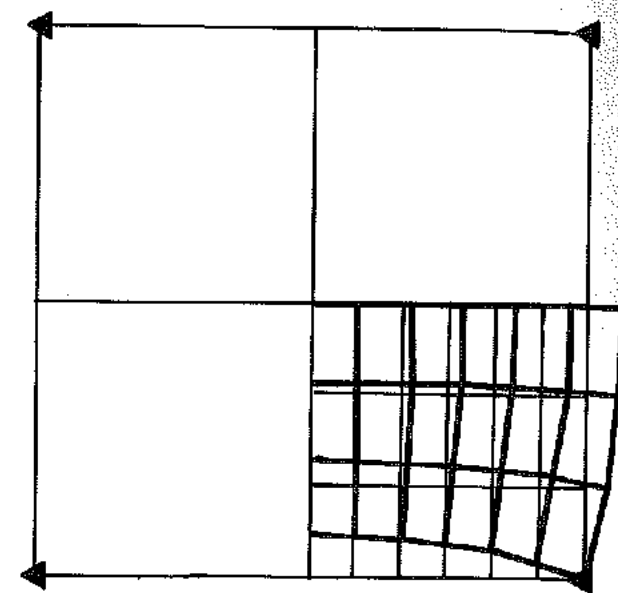
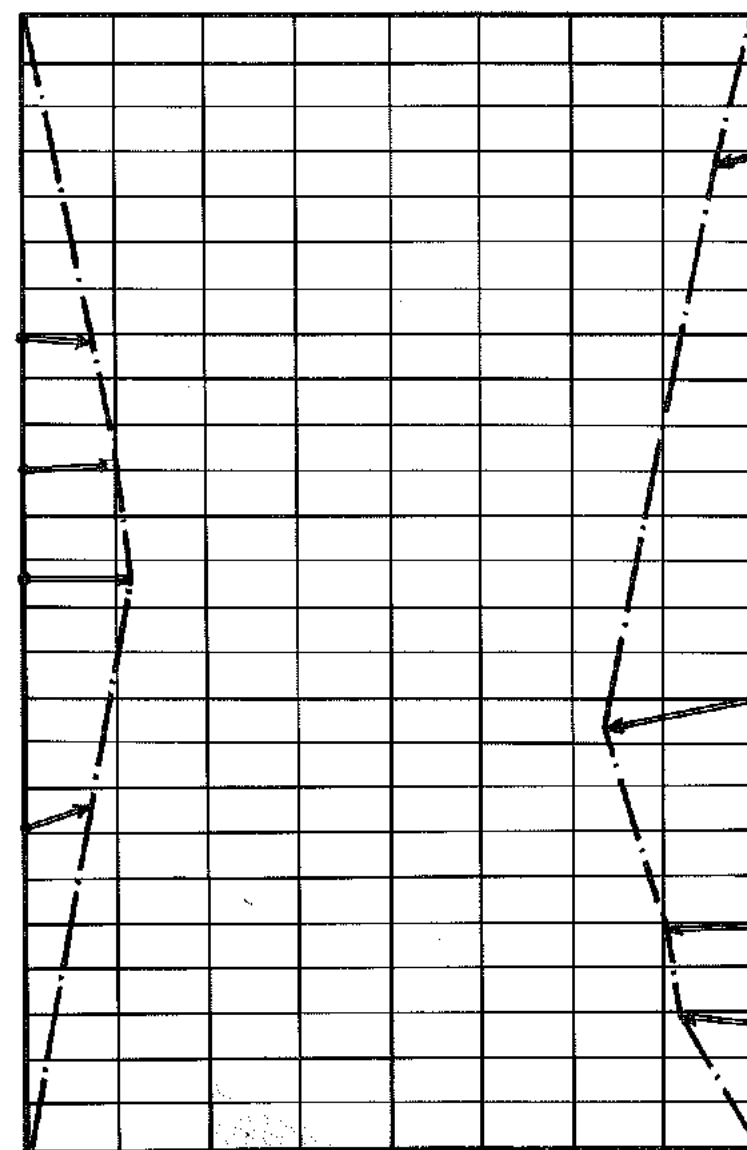


Figure 4

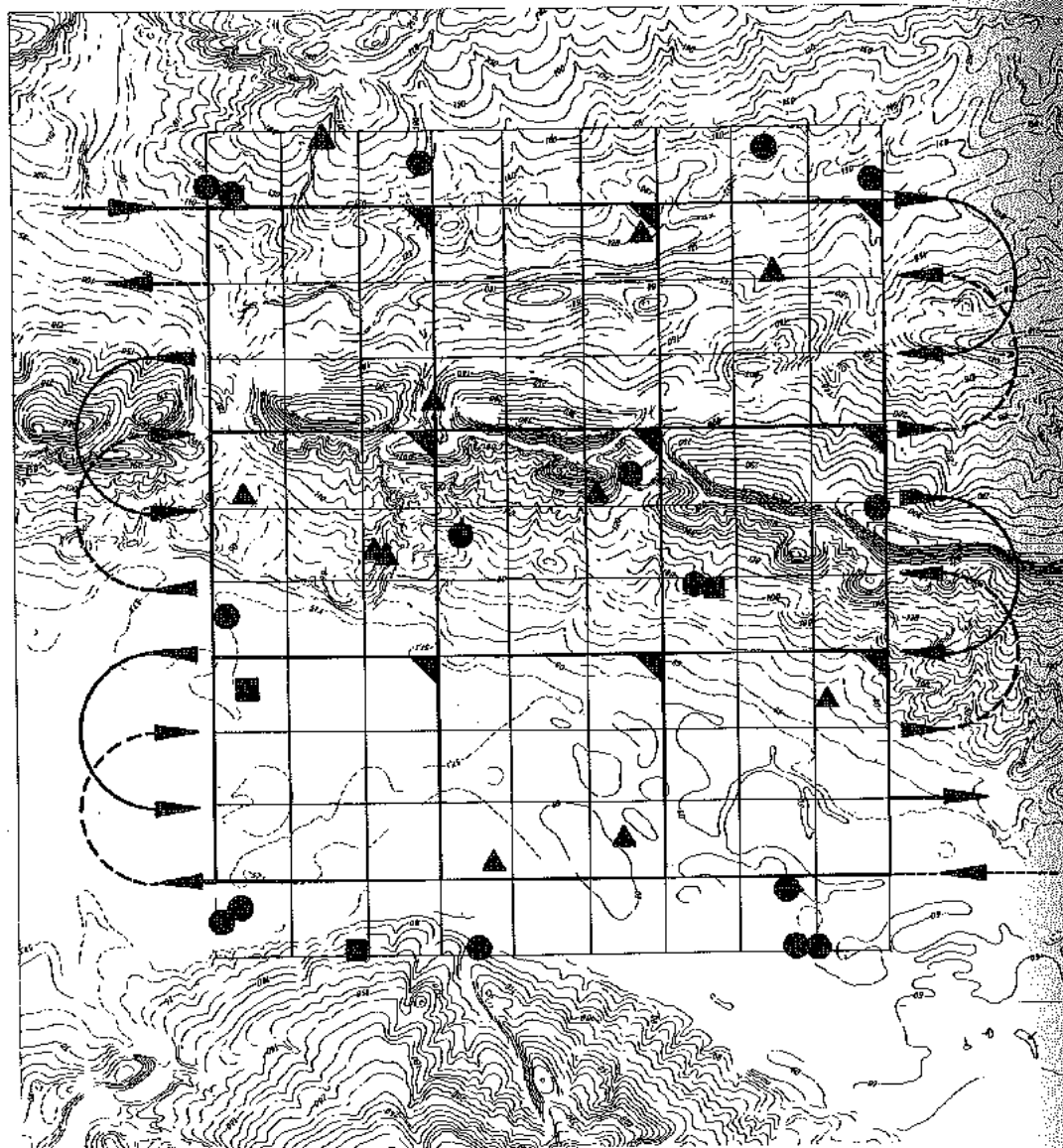


At the periphery the largest errors remained

1m
35μm

scale of the error vectors

Figure 5



TEST "STEINBERGEN"

FLIGHT PLANNING AND
ARRANGEMENT OF CONTROL POINTS

- VERTICAL CONTROL POINT
- HORIZONTAL CONTROL POINT
- ▲ VERTICAL AND HORIZONTAL CONTROL POINT

Figure 6

RESULTS OF THE TEST "STEINBERGEN"

test	sidelap %	standard deviation σ_0 μm	σ_{check}		number of check points	increase of accuracy in percent	remarks
			μm	cm			
1	60	5,3	10,7	8,6	55	41	standard correction group
2	20	6,1	24,4	19,5	53	94	" " "
3	60	5,9	17,9	14,3	58	69	without correction group
4	20	5,2	28,0	20,8	54	100	" " "

Figure 7

Discussion about the Subject 12 under the Chairmanship of Prof. E. Gotthardt

Chairman: Thank you for your interesting presentation Dr. *Bauer*. To whom may I give the floor?

Jerie: These results are certainly very impressive but I would like to ask Dr. *Bauer* as to what considerations were used in arriving at the final correction terms?

Bauer: I was only concerned with the effectiveness of the correction terms.

Jerie: Is there not a certain amount of danger involved in applying corrections successively?

Bauer: The correlation was always computed between successive corrections and whenever this was large we disregarded the latter correction.

Jerie: Is it not impractical to use a single set of correction terms for a whole block since, for example, a block cannot be flown in a single day?

Bauer: This of course depends upon the size of the block.

Ebner: In your paper you mention that the ratio $\sigma_{\text{check}}/\sigma_0$ was found to be 3.7 with dense perimeter control. Now, why should this ratio be theoretically 1, since this surely depends upon the block size and the control distribution and in your case should be 0.75? I agree that it is impressive that you reduce this ratio to 1.7, but this is still twice as large as the theoretical. We can therefore conclude that discrepancies still remain with the inclusion of additional parameters and I believe that a proper selection of the type and number of parameters to be added will become an important topic for further research.

Bauer: Regarding the theoretical ratio of $\sigma_{\text{check}}/\sigma_0$ I meant that it should be approximately equal to one. I agree that we have not eliminated all systematic errors with our additional parameters and that further investigations are necessary.

Ackermann: I consider Dr. *Bauer's* paper quite remarkable in illustrating how the results can be improved with the inclusion of additional parameters and I certainly believe this to be the beginning of a series of investigations on this topic. Regarding the fact that the height accuracy of the Oberschwaben block was not improved by increasing the side overlap from 20 % to 60 %, I do not believe this to be due to differences in the observations of the two centres, Vienna and Frankfurt. It is quite possible that the two sets of photographs have different types of systematic errors. In a similar way also the systematic errors within each of the blocks are not necessarily constant.

Bauer: I agree that there may be different effects in the Vienna and Frankfurt blocks. In the Frankfurt block we took the flight direction into account but this did not improve the results.

Förstner: If one has a test field one can determine the ideal set of parameters but how can this problem be solved in practice?

Kupfer: I think that one of the problems is the fact that we do not have a homogeneous set of data in large blocks. Hence the data has to be homogenized and this could be achieved by incorporating a test field in the project area which is photographed during each flying mission. The points in this test area need not even be signalized.

Chairman: I am afraid that our time is up and that I have to close this interesting session after thanking Dr. *Bauer* and the other speakers for their presentations and you for your contributions.

Final Session

General Discussion under the Chairmanship of Prof. A. J. van der Weele

Chairman: I wish to welcome you all to this final session where we will try to evaluate the results of this symposium and see whether recommendations can be made for future research efforts.

The papers presented to this symposium can be divided into three main categories:

- those dealing with programme I, the experimental confirmation of theoretical predictions regarding various methods of aerial triangulation,
- those related to programme II, the investigation of the error properties of a single photograph, and
- a set of miscellaneous papers related to the determination of image errors by means of test fields and reseau photography.

In order to bring some line into the discussion I propose to start with the first programme and would like to ask Prof. *Ackermann* to summarise his conclusions on this.

Ackermann: The main aim of this programme was to check the validity of theoretical accuracy predictions regarding strip and block triangulations, naturally hoping that a confirmation would enable practising photogrammetrists to rely on these hypotheses. In addition, we hoped to obtain empirical results of a number of problems which cannot be determined theoretically such as accuracy comparisons of WA and SWA photography, the use of signalized against natural or artificial points, etc.

The programme has not yet been completed, but of the provisional results, some agree fairly well with the predictions, others agree to a reasonable extent and a third group is contradictory to the expectations. We have purposely restricted ourselves to just presenting the actual results to this symposium, thus allowing each delegate to draw his own conclusion.

As far as our programme for the immediate future is concerned, we intend to complete the computations and will then continue with a modified programme on the basis of the results and this meeting's recommendations.

Chairman: It is now up to the audience to comment on these results and to offer suggestions.

Proctor: In my opinion, a number of interesting results have been presented but a few anomalies have still left a degree of uncertainty. Regarding the comparison of the independent model and bundle adjustments, I think that the results are unfavourably influenced by the significant proportion of systematic errors present. This was illustrated in an invited paper by Mr. *Anderson* to Commission III at the last ISP Congress, where the adjustments of a simulated block of random errors gave completely different results to those when systematic errors were introduced.

With regard to the use of 60 % side overlap, I wonder whether we are taking enough account of the fact that the perspective centres restrain fictitious tilts in the direction of the strip axes. In this respect, perhaps a bundle adjustment with 60 % side overlap might reduce the height control requirements?

Ebner: As far as the accuracy ratio between blocks with 20 % and 60 % side overlap is concerned, theoretically one expects an improvement with the latter by a factor 2. This expectation was not met in the Oberschwaben block but was however substantiated in the Steinbergen test as reported by Dr. *Bauer*.

Chairman: What about Mr. *Proctor's* question on the perspective centres?

Ebner: It is true that theoretically one should not expect any difference between the x and y directions in the bundle adjustment and this is certainly not the case with an independent model block adjustment.

Jerie: This is only true theoretically since parallaxes are measured within a strip in the x direction and points have to be transferred in the y direction.

Ebner: The use of 60 % side overlap is only one of the possibilities. Another would be to simultaneously adjust two blocks flown at right-angles to each other.

Chairman: Would the systematic errors then be automatically eliminated in the adjustment?

Ebner: Yes, they would.

Chairman: One of the cases where the experimental results contradict the theoretical concerns the comparison of the independent model and bundle adjustments.

Is this mainly due to the increased sensitivity of the bundle adjustment to the presence of systematic errors?

Ackermann: I would like to make a comment on the significance of the results. One must bear in mind that the whole Oberschwaben test is only two blocks and its results are therefore certainly of limited statistical significance. We knew this at the outset and so one could question the justification for conducting this type of test. Nevertheless, I consider it most useful, as it allows, in any case, a number of significant conclusions to be drawn.

With regard to the problem of systematic errors, it is obvious that they are quite large in the Oberschwaben block. We do not know how representative these errors are for photography in general, but I think it is important to note that large systematic errors can and do occur. We therefore have to refine our adjustment methods in order to prevent poor results under such circumstances. As to the sensitivity of the bundle adjustment to the presence of systematic errors, I think this is substantiated by the results. The effect has been predicted by *Kubik*.

Visser: With regard to a possible affinity of the Oberschwaben photography I can report that the reseau photography had extremely small deformations, the differential scale and differential azimuth being $0.06^{0}/_{00}$ and $0.00^{0}/_{00}$ in the parameters derived for individual photographs. Now, perhaps a certain amount of affinity results from the ground control as is suspected by Dr. *Bauer*?

Chairman: The ground control has not been checked.

Ackermann: I consider it a fairly serious accusation of the state authorities that the ground control is that poor. Regarding the affinity, there are other sources which are generally neglected, namely lens distortion and map projection. Only the mean distortion of the four diagonals has been corrected for, whilst map projection has, at this stage, not been taken into account. Hence, before investigating the feasibility of refining the adjustment, we first intend to correct the overall affinity of the film and the affinity caused by lens distortion and map projection. We expect an improvement in the results by, perhaps, a factor 2.

Chairman: As regards the absolute accuracy of ground control networks, when the primary triangulation of the Netherlands was included in the European adjustment, differences of up to 50 cm were found although one expected these to be in the order of a few cm. This however did not affect the internal accuracy.

Now going back to Oberschwaben, if the control is suspect one should obtain the same pattern of residuals in both the WA and SWA photography.

Ackermann: A few comparisons have been made and no serious effects were noticed over short distances. Dr. *Bauer* suspects systematic errors in the control over the whole area, but these would have to be in the order of 2 m to give this degree of affinity.

Kupfer: With regard to the affinity caused by lens distortion, we started to investigate this but dropped it because the effect is so slight. One must also bear in mind that in the camera calibration, the principal point of symmetry is only determined to an accuracy of 0.2 mm.

Bauer: My results indicate a shear in the Oberschwaben block which could be caused by control errors in the order of 50 cm.

Förstner: I think it is fairly easy to blame the ground coordinates but may I remind you that this particular area was chosen due to the reliability of the control. I do not think that the absolute errors exceed some 15 cm.

Chairman: In view of the checks normally applied, I do not think that anyone really doubts the quality of the control.

Another problem raised was why the height accuracy of the SWA photography is inferior to that of the WA. Could this be partly due to the fact that film unflatness has a greater effect in SWA photography?

Lindig: Even the SWA measurements are inferior.

Jerie: This is probably due to the poorer resolution.

Ackermann: The initial theoretical investigations from which a superior height accuracy for SWA photography was inferred were only based on geometrical considerations and took no account of physical and instrumental effects.

It is obvious from this test that these other influences have a detrimental effect on the accuracy. Other factors could be the type of terrain, the oblique inclination of the rays and refraction.

Chairman: Why is the height accuracy of the SWA photography inferior to the WA in the Oberschwaben block and yet there is very little difference between the two in the Rheidt test area?

Ackermann: The results are not entirely contradictory since the Oberschwaben results refer to aerial triangulation points, always located in the danger areas of the models.

Leberl: Are there significant differences in the effect of systematic errors on WA and SWA photography?

Konecny: I suspect that film unflatness is one of the most important influences and this certainly has different effects.

Meissl: If the film is not flat, the displacement of an image point may be fairly large but the angular error will be small. May I also comment on the density of control points in a planimetric block adjustment. The theoretical predictions indicate that a sufficient degree of accuracy can be obtained with only corner control. I agree with this if we only have random errors and the distortion is conformal, but is it still valid if systematic errors are present which do not influence the boundary of the block?

Ackermann: No, in this case corner control will not be sufficient. Coming back to the question of film unflatness, since comparator observations are vertical to the plate the total displacement will show up.

Meissl: I agree, but this will be corrected in the reconstruction. I believe refraction to be a serious source of error, particularly in a heterogeneous area such as Oberschwaben.

Bormann: As far as SWA lenses are concerned, since the physical centering is not more accurate one can expect linearly a larger decentering distortion. Regarding the camera calibrations themselves, these are carried out in laboratories at room temperatures and the distortions are certainly not related to those during an actual flight.

Kupfer: As I have mentioned before, the effect of lens distortion was negligible in our investigation. I believe film unflatness and film distortion, both during exposure and due to processing, to be more serious effects.

Proctor: I know how difficult the camera calibration observations are with a WA camera and imagine the situation is worse with a SWA camera, so perhaps the latter are less reliable?

Ackermann: One of the reasons for only correcting the average lens distortion was the fact that we did not want to introduce debatable refinements to the data at such an early stage of the programme.

Lindig: If large differences are found in the distortions measured along 4 semi-diagonals, would it not be necessary to obtain information on what happens in between by perhaps measuring more diagonals?

Ackermann: Certainly; we need in any case more sophisticated interpolation methods for distortion (although preliminary trials did not give much improvement).

Förstner: Is it not possible to calibrate the distances between fiducial marks to a higher degree of accuracy in order to determine the affinity?

Chairman: A wrong impression would be created if these distances were given more accurately since these four fiducials cannot sufficiently describe the affinity.

Ligterink: I have examined a number of calibration reports with distortions measured along 8 semi-diagonals and very little difference was found with distortions interpolated from the normal 4 semi-diagonals.

Chairman: I would now like to ask Prof. Ackermann to summarise his intended programme for continuing these investigations.

Ackermann: The investigations will continue on 3 levels. We first intend to complete the block adjustments and will complement the results with additional information such as maximum errors and the reliability of the results.

In the second stage all corrections are to be applied which are known a priori (film distortion, lens distortion, map projection, test field calibration).

The third stage will concern accuracy tests with block adjustment programmes which include additional parameters for systematic errors.

Of importance here is, of course, the number of parameters to be added. Parallel to this, we intend to conduct special investigations into the error properties of individual photographs, such as the determination of the correlation between image coordinates both within a photograph and between adjacent photographs.

Jerie: As such, this seems to boil down to three main proposals, namely the use of the reseau, test fields and correction terms. I regard the reseau as a type of insurance to be only used if something goes wrong. Now, as to the other two proposals, I do not regard these as alternatives but consider that these should be used in combination with each other.

Ebner: I agree to this. We still have a problem though of how to treat the additional parameters in the adjustment. These could be considered as free unknowns but this could

be dangerous since they could then assume an infinite magnitude. Two things have to be borne in mind with the introduction of additional unknowns, namely that these additional parameters should be independent of each other and of the original parameters. An alternative could therefore be to introduce these as observation equations.

Jerie: Your latter solution is more of a trick than a principle and these unknowns are certainly not normally distributed. A suitable compromise would be to limit the number of additional parameters introduced.

Ebner: I agree that they are not measured values but do not consider it absolutely necessary that they be normally distributed.

Meissl: I also do not consider the hypothesis of a normal distribution necessary for a least squares adjustment. As to the efficiency of Dr. *Ebner's* proposal, one has to allow the pseudo observations to vary by a considerable amount and this may have an adverse effect on the adjustment. Great care must therefore be taken in utilising this proposal.

Wrobel: Could we not utilise the parameters determined from the test field calibration? In this respect I would consider it useful to discuss the operational aspects of such a proposal, e. g. what happens when one moves away from a test field?

Chairman: I think that you are again stressing the problem and consider that these questions will be answered by the further experiments.

Ackermann: One of the advantages of the proposal is that additional parameters can be added at will without disturbing the system, even if these are not determined.

Chairman: I think that we now have to discuss the second programme of the Oberschwaben test and I would like to ask Prof. *Wiser* to summarise his conclusions.

Wiser: As far as our immediate tasks are concerned, we first have to complete the programme. Future research will be mainly directed towards finding an experimental connection between the various procedures used such as reseau, test field, common parameters and latent parallaxes to determine the accuracy of a single photograph. As such we will compare the results, model by model, performed on the same material and also intend to compare the efficiency of various methods such as the reseau and the latent parallax procedure.

Chairman: These results could also be supplemented with those obtained from programme I. Is there any further discussion on this programme or on the miscellaneous topics covered by the papers of *Kupfer*, *Visser* and *Bauer* which have been raised a number of times during this discussion?

Ligterink: I would like to know how the strips were connected in the case of a 60 % side overlap?

Bauer: We used a double line of tie points.

Amer: How does the bundle adjustment with correction parameters compare with the independent model adjustment without parameters?

Ackermann: The improvement is significant, but surely we can add parameters to the models too.

Chairman: If there is no further discussion I think we can close this session. My impression is that a few problems have been solved but that others have been raised. We are probably headed in the right direction and I am pleased that the discussions have brought forward a number of suggestions for the completion of the programme. In this respect, one of the tasks will be to combine the findings of the three parts of the programme.

Thank you for your attention, ladies and gentlemen, and for your participation in the discussions.

Closing Address

In his closing address, the President of the OEEPE, Prof. *Solaini*, concluded that the aim of the symposium had been fulfilled in that the number of new problems raised would enable the organisation to refine its research programme. Prof. *Solaini* then proceeded to thank all those connected with the organisation of the symposium, Prof. *Wiser*, Prof. *Ackermann*, Ir. *Verlaine* and the Institut Géographique Militaire.

He also expressed his appreciation towards the chairmen of the sessions, those who presented papers and those who participated in the discussions. Prof. *Solaini* concluded with the hope that this OEEPE initiative had been appreciated and that this would only be the first of a number of international symposia organised by the OEEPE.

Symposium

sur

Recherches expérimentales sur la précision de l'aérottriangulation

Résultats des essais Oberschwaben

12., 13., 14 juin 1973, Bruxelles

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